1. BACKGROUND

At optical wavelengths and from the vantage point of space, the multiple scattering cloud medium obscures one’s view and prevents one from easily determining what flashes strike the ground. However, recent investigations have made some progress examining the (easier, but still difficult) problem of estimating the ground flash fraction in a set of N flashes observed from space [Koshak (2010), Koshak and Solakiewicz (2011), and Koshak (2011)].

In the study by Koshak (2011), a Bayesian inversion method was introduced for retrieving the fraction of ground flashes in a set of flashes observed from a (low earth orbiting or geostationary) satellite lightning imager. The method employed a constrained mixed exponential distribution model to describe the lightning optical measurements. To obtain the optimum model parameters, a scalar function of three variables (one of which is the ground flash fraction) was minimized by a numerical method. This method has formed the basis of a Ground Flash Fraction Retrieval Algorithm (GoFFRA) that is being tested as part of GOES-R GLM risk reduction.

Figure 1 summarizes the basic functionality of the GoFFRA, and Figure 2 highlights the mathematical attributes of the Bayesian retrieval process.

2. THE GROBNER INITIALIZATION

Note that the estimative initialization scheme provided in Figure 2 involves only two moments (the mean and the standard deviation); it also just initializes the ground flash fraction (alpha) to its centerline value 0.5. By including the third moment, (skewness, γ1) of the lightning optical characteristic, we arrive at a set of 3 polynomials as shown below:

\[
\begin{align*}
\alpha \mu_1 + (1 - \alpha) \mu_2 &= \mu \\
(\mu_1 - \mu_2) \sigma_1 + (\mu_2 - \mu_3) \gamma_1 &= 0 \\
(\mu_1 - \mu_2) \chi_1 + (\mu_2 - \mu_3) \chi_2 &= 0
\end{align*}
\]

Solving this system without any guess-work can be accomplished using Grobner bases. [For perspective, if the equations were linear the Grobner bases method would reduce to Gaussian Elimination common in linear algebra.] We employ Mathematica to find the Grobner bases of this system; the Mathematica utility is called GroebnerBasis which uses an efficient version of the Buchberger algorithm to compute the polynomial bases. We obtain a total of 11 polynomials that define the Grobner bases. Of these, we pick the three easiest to solve, which are:

This set of 3 equations has a solution (which is also a solution to the original set of polynomials) given by:

\[
\begin{align*}
\alpha &= \frac{-B - \sqrt{B^2 - 4AC}}{2A} \\
\mu_2 &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} \\
\mu_1 - \mu_2 &= \frac{-B - \sqrt{B^2 - 4AC}}{2A} \\
\alpha &= \mu_1 - \mu_2 \\
\gamma_1 &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} \\
\chi_1 &= \frac{-B - \sqrt{B^2 - 4AC}}{2A} \\
\chi_2 &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} \\
\lambda &= \frac{\mu_1 - \mu_2}{\mu_1 - \mu_2}
\end{align*}
\]

This represents an analytic solution. It will be used to replace the estimative initialization scheme discussed in Figure 2. We expect it to improve the Bayesian retrieval results.

3. REFERENCES


