Full-Scale Turbofan-Engine Turbine-Transfer Function Determination Using Three Internal Sensors

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Noise-source separation techniques, using three engine-internal sensors, are applied to existing static-engine test data to determine the turbine transfer function for the currently sub-dominant combustion noise. The results are used to assess the combustion-noise prediction capability of the Aircraft Noise Prediction Program (ANOPP) and an improvement to the combustion-noise module GECOR is suggested. The work was carried out in response to the NASA Fundamental Aeronautics Subsonic Fixed Wing Program's Reduced-Perceived-Noise Technical Challenge.

I. Introduction

The reduction of aircraft noise is critical for enabling the anticipated large increase in future air traffic. Noise generated in the jet engine core, by components such as the compressor, combustor, and turbine, can be significant contributors to the overall noise signature at low-power conditions, typical of approach flight. At high engine power settings during takeoff, jet and fan noise have traditionally dominated over core noise for existing engines. However, current design trends and expected technological advances in engine-cycle design as well as noise-reduction methods are likely to reduce non-core-noise contributions even at engine-power points higher than approach. The result of such changes will be to elevate the overall importance of core noise. New airport regulations are likely to require additional noise reductions, thus emphasizing the need for further reductions in core noise.

The present paper is concerned with the combustion noise component of the core noise. The unsteady combustion process is the source of pressure, entropy, and vorticity fluctuations. The noise frequency is set by the unsteady combustion process and its peak value is generally believed to fall in the range of 400–500 Hz. Combustion noise is of either the direct or indirect type. A fraction of the pressure disturbances are acoustic pressure fluctuations with the balance being hydrodynamical unsteadiness. The former is what is referred to as direct combustion noise. Its spectrum is modified by the combustor geometry as well as pressure feedback on the unsteady combustion process itself. The direct combustion noise is reduced due to transmission effects during its propagation through the turbine stages. The combustor entropy (temperature) fluctuations are convected downstream with the local mean velocity and get converted to acoustic pressure fluctuations in the turbine and other regions of rapid flow change. This is the indirect process of turbomachinery combustion noise generation. This is potentially a very effective mechanism and occurs at all turbine stages. The indirect noise occurs in the same basic frequency range as the direct noise, but their spectral-distribution shapes could be quite different. Figure [1] illustrates the dual paths of combustion noise. Note that the direct and indirect noise contributions are correlated at the source because both are caused by the unsteady heat addition. The relative importance of direct and indirect combustion noise contributions is still an unresolved issue.

Direct measurement of turbofan-engine combustion noise is difficult because of the presence of jet noise in the frequency range of interest. Since flight effects reduce jet noise more than combustion noise, combustion noise can be a significant contributor to aircraft approach noise but may be masked by jet noise under the corresponding static-engine test condition. To overcome this obstacle, researchers developed coherence techniques utilizing engine-internal as well as far-field measurements to identify the far-field combustion noise component. Modal analyses were also carried out to determine the source and propagation characteristics of combustion noise.
Hultgren and Miles\cite{Hultgren2006} discussed noise-source separation techniques for application to engine test-stand data and assessed a current prediction method\cite{Royalty2013,Schuster2013} applied to a static-engine test\cite{Hultgren2006}. The current paper is an extension of this work in that a determination of the turbine transfer function is attempted.

### II. Data Analysis

#### A. Static-Engine Test Data

Data obtained from the NASA/Honeywell Engine Validation of Noise and Emission Reduction Technology program\cite{Hultgren2006} (EVNERT) is used herein to assess the turbine transfer of direct combustion-noise and the creation of indirect combustion noise through entropy/blade-row interactions in the turbine. This static engine test activity was carried out in Honeywell Aerospace’s San Tan outdoor test facility from 2005 to 2007. The program used the Honeywell TECH977 research engine, which is typical of a business-jet application in the 6,000–8,000 lbs thrust class.

![Honeywell TECH977 turbofan engine-internal sensors](image)

The engine-internal instrumentation in EVNERT configuration 35 included high-temperature pressure sensors with air cooling in a combustor ignitor port (CIP1) and at the turbine exit (T551 and T552). Pressure time histories at the internal sensors CIP1, T551, and T552 herein. The EVNERT data acquisition system had a sampling rate of 65,536 Hz and a duration of about 70 s, leading to time histories with just over 4.5 million data points. Each time series is analyzed here using an FFT length of 8192 points (corresponding to an 8 Hz frequency resolution or bin width), Hanning windowing, and a 50 percent data-segment overlap. The narrow-band auto spectra, resulting from $M = 1117$ averages, then can be summed up to yield the corresponding 1/3-octave sound-pressure level (SPL). The 130° 1/3-octave far-field total SPL result was found to be in full agreement with the Honeywell provided 1/3-octave SPL data\cite{Royalty2013}.

Royalty and Schuster\cite{Royalty2013} analyzed the acoustic modes in the combustor for a different arrangement of the EVNERT turbofan engine than considered herein. In that configuration, the fan was replaced by a water brake in order to remove fan sources from the total noise signature. The no-fan configuration could be operated up to a power setting corresponding to the approach condition of 60% corrected fan speed. The combustor internal instrumentation consisted of a circumferential array of 16 equally-spaced pressure probes. They\cite{Royalty2013} (see their Fig. 19) found that for low frequencies most of the acoustic energy was associated with the plane wave ($m = 0$) mode, that the first circumferential mode ($m = \pm 1$) was dominant in the frequency range of 500–1000 Hz, and that higher circumferential modes ($m = \pm 2, m = \pm 3, m = \pm 3, \ldots$) sequentially became the most significant feature at successively higher frequencies, where $m$ is the azimuthal wave number or mode order. One can observe in their figure that at 500 Hz, the plane wave mode is about 8 dB and 5 dB below the total acoustic level at the 48% and 60% power settings, respectively. They also reported that the higher modes ($m \neq 0$) were not present in the far-field data. It is concluded here that this indicates
that the non-plane-wave modes are cut-off in the turbine/duct downstream of the combustor for this particular engine. This is in agreement with the results of Hultgren and Miles\cite{Hultgren_2020} in which coherent combustion noise was only found for frequencies less than about 400+ Hz.

## B. Frequency Response Functions

The goal of the present work is to determine the frequency response functions $H_{uv}(f)$

\[
H_{uv}(f) \equiv \frac{G_{uv}(f)}{G_{uu}(f)} = \frac{G_{xy}(f)}{G_{uu}(f)} = H_{xy}(f)[1 + N_u(f)],
\]

\[
H_{uu}(f) \equiv \frac{G_{uu}(f)}{G_{uu}(f)} = \frac{G_{xz}(f)}{G_{uu}(f)} = H_{xz}(f)[1 + N_u(f)],
\]

\[
H_{vv}(f) \equiv \frac{G_{vv}(f)}{G_{uu}(f)} = [1 - \delta(f)]G_{yz}(f)/G_{vv}(f),
\]

\[
H_{ww}(f) \equiv \frac{G_{ww}(f)}{G_{uu}(f)} = [1 - \delta^*(f)]G_{y^z}(f)/G_{ww}(f),
\]

where $f$ is the frequency and the star denotes complex conjugation. $G_{\alpha\alpha}$ and $G_{\beta\beta}$ denote the one-sided auto-spectrum and cross-spectrum of the signals $\alpha$ and $\beta$, where $\alpha$ and $\beta$ are dummy indexes. The one-sided auto-spectra $G_{uu}(f)$, $G_{vv}(f)$, and $G_{ww}(f)$ represent the combustion-noise component of the total noise signature $G_{xx}(f)$, $G_{yy}(f)$, and $G_{zz}(f)$ at each of the measuring stations. $H_{xy}(f) \equiv G_{xy}(f)/G_{xx}(f)$ and $H_{xz}(f) \equiv G_{xz}(f)/G_{xx}(f)$ can be thought of as ‘directly measured’ frequency transfer functions;

\[
N_u(f) = \frac{G_{mm}(f)}{G_{uu}(f)}
\]

is a real and positive quantity representing the noise to signal ratio at the originating measuring station and is not directly obtainable; and

\[
\delta(f) = \frac{G_{no}(f)}{G_{y^z}(f)}
\]

is a measure of the relative strength of the possible correlation between the $n(t)$ and $o(t)$ signals. $\delta(f)$ describes the influence of broadband turbine noise on the results and is assumed to be small in the frequency range of interest here. The frequency response functions are commonly expressed as $H_{\alpha\beta} = |H_{\alpha\beta}| \exp(i\phi_{\alpha\beta})$, where $|H_{\alpha\beta}|$ and $\phi_{\alpha\beta} = \arg H_{\alpha\beta}$ are referred to as the gain and phase factors, respectively.

Consequently, the ‘true’ turbine gain factor is always underpredicted by the ‘directly measured’ one, i.e., $|H_{uv}(f)| \geq |H_{xy}(f)|$ and $|H_{uw}(f)| \geq |H_{xz}(f)|$. The ‘true’ turbine phase factors, however, are identical to the ‘directly measured’ ones since both are obtainable from the same cross-spectrum, i.e., $\phi_{uv}(f) = \phi_{xy}(f) = \arg G_{xy}(f)$ and $\phi_{uw}(f) = \phi_{xz}(f) = \arg G_{xz}(f)$.

Figure 5 illustrates the relationships between the signals measured by the engine-internal sensors CIP1, T551, and T552. The signals $u(t)$, $v(t)$, and $w(t)$ represent the coherent acoustic combustion-noise signal at the three sensors as functions of time $t$. These ‘desired’ signals cannot be directly obtained by themselves because of the presence of the random uncorrelated ‘noise’ signals $m(t)$, $n(t)$, and $o(t)$ at the different measuring stations. However, the downstream signals $v(t)$ and $w(t)$ are uniquely determined by the previous-station signal $u(t)$ and the impulse-response functions $h_{uv}$ and $h_{uw}$, respectively. The measurable signal, $x(t)$, $y(t)$, or $z(t)$, at each sensor is the sum of the ‘desired’ and corresponding ‘noise’ signals. The signals $m(t)$, $n(t)$, and $o(t)$ can be taken as mutually uncorrelated as well as uncorrelated with the combustion-noise signal $u(t)$, $v(t)$, or $w(t)$ at all the stations. The signal $m(t)$ is to a large extent caused by hydrodynamical pressure fluctuations (pseudo sound) in the combustor and possibly also higher acoustic modes present in the combustor but cut-off in the downstream tail pipe and can potentially be quite large. The signals $n(t)$ and $o(t)$ are mainly due to acoustic pressure fluctuations from other noise sources and can actually be correlated if turbine broadband noise is also present at the frequencies of interest.
To obtain a better estimate for the ‘true’ frequency response functions than given by (1a) and (1b), with \( N_u \) neglected, the one-sided auto-spectra \( G_{uu}(f) \), \( G_{vv}(f) \), and \( G_{ww}(f) \), i.e., the coherent combustion-noise at each of the measuring stations, need to be determined and this is discussed in the next two subsections. The first of the two deals with a two-signal diagnostic technique, commonly referred to as the coherent-output-power method. The next one addresses a three-signal technique.

C. Coherent-Output-Power Method

The basic formulation for the coherent-output-power method is described in the textbook by Bendat and Piersol. If the sensor inside the combustor and the appropriate turbine-exit sensor (Fig. 3) are used in this technique, it follows that the coherent combustion-noise spectrum at the turbine-exit locations are given by

\[
G_{vv}(f) = \frac{|G_{uv}(f)|^2}{G_{uu}(f)} = \frac{|G_{xy}(f)|^2}{G_{yu}(f)} = \gamma_{xy}(f)G_{yy}(f)[1 + N_u(f)] \tag{4a}
\]

\[
G_{ww}(f) = \frac{|G_{uw}(f)|^2}{G_{uu}(f)} = \frac{|G_{xz}(f)|^2}{G_{zu}(f)} = \gamma_{xz}(f)G_{zz}(f)[1 + N_u(f)] \tag{4b}
\]

regardless of the output noise \( G_{nn}(f) \) or \( G_{oo}(f) \). \( \gamma_{\alpha\beta} = |G_{\alpha\beta}|/\sqrt{G_{\alpha\alpha}G_{\beta\beta}} \) is the coherence.

Note that ignoring \( N_u \) in (3) is equivalent to replacing \( G_{uu} \) with the measured \( G_{xx} \). The latter is a positive-biased estimate for the unknown input spectra \( G_{uu} \). In view of the certain presence of nonpropagating pressure fluctuations in the combustor, i.e., \( G_{mm} \neq 0 \), ignoring \( N_u \) in (4) is quite likely to underpredict the actual coherent output spectrum. Eqs. (1a), (1b) and (4), all with \( N_u = 0 \), will be referred to as the two-signal method results in what follows.

D. Three-Signal-Coherence Method

Chung developed a three-signal coherence technique for microphone flow-noise rejection. This three-signal method also applies to the situation shown in Fig. 3. The coherent auto-spectra at the three sensors, CIP1, T551, and T552, are, hence, given by

\[
|1 - \delta(f)|G_{uu}(f) = \frac{|G_{xy}(f)||G_{xz}(f)|}{|G_{yu}(f)||G_{zu}(f)|} = \frac{\gamma_{xy}(f)\gamma_{xz}(f)}{\gamma_{yu}(f)\gamma_{zu}(f)}G_{xx}(f), \tag{5a}
\]

\[
\frac{G_{ww}(f)}{|1 - \delta(f)|} = \frac{|G_{xy}(f)||G_{yz}(f)|}{|G_{zu}(f)||G_{xy}(f)|} = \frac{\gamma_{xy}(f)\gamma_{yz}(f)}{\gamma_{zu}(f)\gamma_{xy}(f)}G_{yy}(f), \tag{5b}
\]

\[
\frac{G_{ww}(f)}{|1 - \delta(f)|} = \frac{|G_{xz}(f)||G_{yz}(f)|}{|G_{xy}(f)||G_{xz}(f)|} = \frac{\gamma_{xz}(f)\gamma_{yz}(f)}{\gamma_{xy}(f)\gamma_{xz}(f)}G_{zz}(f), \tag{5c}
\]

where the \( \delta \), defined by Eq. (3), reflects the effect of a possible correlation between the \( n(t) \) and \( o(t) \) signals. The inclusion of the \( |1 - \delta| \) factor is a new result. The standard three-signal method is obtained for \( \delta = 0 \).

The strength of the standard three-signal method is that it involves only measured cross-spectra. The measured cross-spectra are affected by extraneous noise only if this noise correlates between measurement locations. This can often be avoided by an appropriate spatial separation of the sensors involved and the three-signal method then provides unbiased estimates of the coherent auto-spectra. In contrast, measured auto-spectra will always include a positive definite contribution from the extraneous noise.

Krejsa considered the situation consisting of two engine-internal sensors—signals \( x(t) \) and \( y(t) \)—and a far-field microphone—signal \( z(t) \)—and obtained (5e), with \( \delta = 0 \), as his far-field result. The three-signal coherence technique used by Krejsa eliminated the bias error in the coherent combustion-noise measurements due to engine-internal nonpropagating pressure fluctuations. It is also possible to separate core noise from jet noise using three far-field microphones since each would pick up correlated core noise and uncorrelated external noise from the jet. As long as the spatial (polar angle) separation of the microphones is large enough, the jet noise at each location can be assumed to be mutually uncorrelated and Eq. (5), with \( \delta = 0 \), would apply. Mendoza et al. analyzed data from the same Honeywell TECH977 static test as considered herein, using a three-signal far-field method, among other multiple-microphone signal-processing techniques. They found that the method worked well in frequency regions where a single engine-internal source was dominant. The method did not perform well for frequencies for which multiple self-correlated internal noise sources were of comparable magnitude, e.g. in the relatively limited frequency range where combustion noise and turbine-broadband noise overlapped.
Combining the results (5) with the ones in (1) shows that the frequency response functions are given by

\[
H_{uw}(f) = |1 - \delta(f)| \frac{G_{yz}(f)}{G_{zx}(f)} \exp[i\phi_{xy}(f)],
\]

(6a)

\[
H_{aw}(f) = |1 - \delta(f)| \frac{G_{yz}(f)}{G_{xy}(f)} \exp[i\phi_{xz}(f)],
\]

(6b)

\[
H_{vw}(f) = \frac{G_{zx}(f)}{G_{xy}(f)} \exp(i\{\phi_{yz}(f) + \arg[1 - \delta(f)]\}),
\]

(6c)

\[
H_{uw}(f) = \frac{G_{xy}(f)}{G_{xz}(f)} \exp(-i\{\phi_{yz}(f) + \arg[1 - \delta(f)]\}) = \frac{1}{H_{vw}(f)}.
\]

(6d)

Eq. (6) with \(\delta = 0\) will be referred to as the three-signal results henceforth. In general, \(\delta\) is a complex quantity, although assumed small here. Consequently, there can be an error in the gain factor for the turbine transfer of combustion noise by using \(\delta = 0\) above, due to the actual presence of broadband turbine noise at the same frequency, but the corresponding phase factor will be unaffected. For the turbine-internal transfer functions, the situation is the opposite.

E. Implementation

From a purely theoretical point of view, \(0 \leq \gamma_{\alpha\beta} \leq 1\), with \(\gamma_{\alpha\beta} = 0\) meaning that the two signals \(\alpha(t)\) and \(\beta(t)\) are completely uncorrelated and \(\gamma_{\alpha\beta} = 1\) indicating perfectly correlated signals. In practice, only estimates \(\hat{\gamma}_{\alpha\beta}\) of the coherence can be obtained using finite data series. The estimated coherence will, in fact, be nonzero even for completely uncorrelated signals.5, 31, i.e., only the interval

\[
e < \hat{\gamma}_{\alpha\beta} \leq 1
\]

(7)

is meaningful, where

\[
e^2 = 1 - (1 - P_t)^1/(N_s - 1)
\]

(8)

is the \(P_t\)-percent confidence interval if the true \(\gamma_{\alpha\beta}^2\) is zero and \(N_s\) is the number of independent data segments used in obtaining \(\hat{\gamma}_{\alpha\beta}^2\). Welch5 showed, in the context of estimating auto power spectra, that \(N_s\) can be replaced by \(9M/11\), where \(M\) is the number of 50-percent-overlapped segments used in the analysis. Mile31 suggested that a better estimate for the coherence threshold value, or noise floor, \(e\) can be obtained by purposely unaligning the two time series. That is, a time delay is deliberately introduced to ensure that the two resulting finite time series are uncorrelated. The estimated unaligned coherence does not depend on any particular assumptions about the underlying statistical properties of the time series and accounts for any data-segment overlap and algorithms used in the analysis. The unaligned result captures the coherence of any discrete tones present in the signals and also provides an estimate of the minimum observable broadband coherence. Mile31 found that Eq. (8) with \(N_s = M\) provided a good estimate of the noise floor. Following Miles,31 the estimated coherence threshold for the present study is \(e = 0.0518\). If the estimated coherence exceeds the threshold the two time series are coupled. If it is less than the threshold the signals are random and appear independent for that particular number of samples/segments.

In the two-signal (coherent-output-power) method calculations carried out here, the estimated coherence \(\hat{\gamma}_{\alpha\beta}(f)\) is replaced by the threshold value \(e\) if it falls below that value for a particular narrow-band frequency. That is, the estimated narrow-band combustion-noise component, say \(\hat{G}_{uw}(f)\), is simply set to \(e^2\hat{G}_{xz}(f)\) for the frequency in question. Otherwise it is given by (4) with \(\hat{N}_u = 0\).

Mathematically, it follows from Eq. (7) that

\[
e^2 < \hat{\gamma}_{xz}\hat{\gamma}_{yz}/\hat{\gamma}_{xy} < e^{-1}.
\]

(9)

The upper limit of this inequality is an unphysical result in view of Eq. (5) and the fact that \(\hat{G}_{uw}\) cannot be larger than \(\hat{G}_{xz}\). Clearly, an additional discriminator is needed to ensure a physically realistic three-signal combustion-noise estimate. This is provided by the following necessary condition33 for Eq. (5) to be valid:

\[
\Theta = \arg[G_{xz}\text{conj}(G_{yz})/G_{xy}] = 0.
\]

(10)

The standard deviation (in radians) of the estimate for the cross-spectrum phase angle \(\theta_{\alpha\beta} = \arg(G_{\alpha\beta})\) is given by33

\[
\sigma_{\alpha\beta} = \sin^{-1}\sqrt{(1 - \gamma_{\alpha\beta}^2)/2\gamma_{\alpha\beta}^2 N_s}.
\]

(11)
Note that the standard deviation is zero for perfectly correlated signals and increases as the coherence is diminished. Consequently, in the three-signal method calculations carried out here, the estimated narrow-band combustion-noise component for a particular narrow-band frequency, say $\hat{G}_{ww}(f)$, is set to $\epsilon^2 \hat{G}_{zz}(f)$ if, any of the estimated coherence values, $\hat{\gamma}_{xz}$, $\hat{\gamma}_{yz}$, or $\hat{\gamma}_{xy}$, fall below the threshold value $\epsilon$, or if the estimated phase angle $\hat{\Theta} > \hat{\sigma}_{xz} + \hat{\sigma}_{yz} + \hat{\sigma}_{xy}$, where $\hat{\sigma}_{\alpha\beta} = \sin^{-1}\left[1/\max(\hat{\gamma}_{\alpha\beta}^2, \epsilon^2) - 1/2N_s\right]$; otherwise it is given by \ref{5}.

### III. Results

#### A. Turbine Auto Spectra

![Figure 4. Narrow-band (8 Hz) turbine-internal SPL; black and green curves—total noise signature, $G_{yy}$ and $G_{zz}$; red and magenta curves—coherent noise using turbine-internal two-signal method, $G_{(yz)}^{(zz)}$ and $G_{(yz)}^{(yy)}$; blue and cyan curves—coherent noise using combustor-turbine two-signal method, $G_{(xy)}^{(vv)}$ and $G_{(xz)}^{(ww)}$; gray curve—coherence limit, $NOP = \frac{\epsilon^2}{2} (G_{yy} + G_{zz})$; (a): 48 % corrected fan speed (flight idle); (b): 60 % corrected fan speed (approach); (c): 71 % corrected fan speed (cutback); (d): 87 % corrected fan speed (takeoff)\]

Figure 4 shows the narrow-band (8 Hz) SPL results, obtained by using two-signal source-separation techniques, at the turbine-exit locations corresponding to the T551 and T552 sensors for the flight-idle, approach, cutback, and takeoff conditions (48, 60, 71, and 87 percent corrected-fan-speed engine power settings) in panels (a) – (d). The black and green curves show the total noise signatures $G_{yy}$ (T551) and $G_{zz}$ (T552), respectively; see Fig.\ref{3} for the signal-sensor labeling scheme. The gray curve shows the average threshold value for the coherent output power, or noise floor, $NOP = \frac{\epsilon^2}{2} (G_{yy} + G_{zz})$. The red and magenta curves show the coherent noise, $G_{(yz)}^{(yz)} = \gamma_{yz}^2 G_{yy}$ and...
\( G_{ww}^{(yz)} = \gamma^2_{yz} G_{zz} \), educed by using the signals \( y \) and \( z \). These two curves illustrate the coherence between the turbine-exit-sensor signals. The blue and cyan curves show the coherent noise, \( G_{vv}^{(xy)} = \gamma^2_{xy} G_{yy} \) and \( G_{ww}^{(xz)} = \gamma^2_{xz} G_{zz} \), extracted by using the signal pairs \((x, y)\) and \((x, z)\). These latter two curves illustrate the coherence between the signals measured by the combustor sensor (CIP1) and each of the turbine-exit sensors. Allowing for the positive-bias error inherent in two-signal methods, Fig. 4 indicates that the turbine-internal and combustor-turbine methods both yield similar results up to a frequency of about 450 Hz. The increase in coherence seen in the turbine-internal two-signal method curves (red and magenta) occurring at higher frequencies could be an indication of the presence of turbine-broadband noise. However, the deviation between the source-separation results at those frequencies could also be a consequence of an increased bias error in the combustor-turbine method. The former is the more likely scenario, and, in agreement with the results of Hultgren and Miles, it is concluded that coherent combustion noise is present for frequencies up to about 450 Hz at the turbine exit.

B. Turbine-Internal Transfer Functions

![Graphs showing turbine-internal gain factors.](image)

Figure 5. Narrow-band (8 Hz) turbine-internal gain factors; magenta curves—\(|H_{vw}|\); cyan curves—\(|H_{wv}|\); (a): 48 % corrected fan speed (flight idle); (b): 60 % corrected fan speed (approach); (c): 71 % corrected fan speed (cutback); (d): 87 % corrected fan speed (takeoff)

Figures 5 and 6 show the turbine-internal response functions determined using Eqs. 6c and 6d, both with \( \delta = 0 \). Figure 5 shows the gain factor and Fig. 6 shows the phase factor, both for the four engine power settings above in panels (a)–(d). The magenta and cyan curves correspond to \( H_{vw} \) and \( H_{wv} \) results, respectively. These figures indicate that the unsteady pressure field at the turbine exit is dominated by plane waves up to about 350 Hz. Furthermore, a circumferential array with many more pressure sensors, than used in this particular configuration, would be needed to determine the azimuthal structure of the pressure field for higher frequencies.
Figure 6. Narrow-band (8 Hz) turbine-internal phase factors; magenta curves—\( \arg(H_{vw}) \); cyan curves—\( \arg(H_{wv}) \); (a): 48 % corrected fan speed (flight idle); (b): 60 % corrected fan speed (approach); (c): 71 % corrected fan speed (cutback); (d): 87 % corrected fan speed (takeoff)

C. Turbine Transfer Function

Figure 7 shows the turbine-transfer gain factor, determined from Eq. (6a) with \( \delta = 0 \). The frequency range in this figure has been limited to 400 Hz in order not to exceed by too much the frequency range where plane waves are dominant; panels (a)–(d) correspond to the flight-idle, approach, cutback, and takeoff engine-power settings; and the red and blue curves correspond to \( |H_{vw}| \) and \( |H_{wv}| \) results, respectively. The turbine-transfer gain factor squared, expected normally to be less than unity, is simply the acoustic transmission loss across the turbine. Two empirical turbine transmission factors are discussed in Ref. [33].

The first one is the GE-based\(^{34}\) turbine-transmission-loss formula used in ANOPP\(^{26, 27}\)

\[
|H(f)_{A/GE}|^2 = (\Delta T_{des}/T_{ref})^{-4},
\]  

where \( T_{des} \) is the design-point temperature drop across the turbine and \( T_{ref} \) is the reference temperature (ambient temperature, actual or standard sea-level value). Note that the acoustic transmission loss is essentially independent of the engine operating condition. The second formula was developed by Pratt & Whitney\(^{4, 35}\) and can, with a further simplification\(^{33}\) be written as

\[
|H(f)_{PW}|^2 = \frac{(1 + \zeta)^2}{0.8\zeta},
\]  

where \( \zeta \) is the ratio of the characteristic impedances across the turbine, i.e., \( \zeta = \frac{\rho_{te}c_{te}}{\rho_{ti}c_{ti}} \) with \( \rho \) and \( c \) denoting density and speed of sound, respectively, and the subscripts \( 'te' \) and \( 'ti' \) indicating turbine exit and inlet. Both
empirical formulas, \( \text{(12)} \) and \( \text{(13)} \), are frequency independent and the corresponding gain factors are shown in Fig. 7 as brown and green dashed lines, respectively. In particular for frequencies larger than about 150 Hz, the Pratt & Whitney gain factor \( \text{(13)} \) appears to be a better fit with experimental data than the ANOPP implementation \( \text{(12)} \). From a practical point of view, this frequency range is more important than the lower-frequency range \((< 150 \text{ Hz})\) since it is expected to contain frequencies near or at the combustion noise peak.

Figure 8 shows the far-field results in the 130° direction of the three source-separation procedures carried out by Hultgren and Miles \( \text{(25)} \) on the same EVNERT dataset as here, as well as the effects on their ANOPP predictions by replacing the ANOPP turbine-attenuation factor by the simplified Pratt & Whitney formula. The 1/3-octave sound-pressure-level (SPL) results are shown at the four engine power settings of 48, 60, 71, and 87 percent corrected fan speed (flight-idle, approach, cutback, and takeoff conditions) for the 1/3-octave center frequency range of 20 to 1000 Hz. The solid lines represent the original ANOPP 1/3-octave SPL predictions for the total (dark-gray) and combustion (red-brown) noise. The symbols correspond to results computed from the experimental time histories as described in Ref. \( \text{25} \). The black squares, labeled \( G_{zz} \), represent the total noise signature, which is reasonably well predicted by the original ANOPP results. The gray squares, labeled NOP, correspond to the threshold value for the coherent output power and any combustion-noise result below these values would not be meaningful using the number of data segments and source-separation techniques used in their work. The blue, red, and green squares correspond to the combustion noise detected using the three methods labeled ‘2s-cip1’, ‘2s-t551’, and ‘3s’, respectively (see Ref. \( \text{25} \) for details). The dashed curves represent the present post-corrected ANOPP 1/3-octave SPL predictions for the total (dark-gray) and combustion (red-brown) noise using the Pratt & Whitney acoustic turbine-transmission formula \( \text{(13)} \).
Figure 8. Total and combustion noise 1/3-octave SPL versus 1/3-octave center frequency in the 130° direction; symbols and solid lines—data and ANOPP predictions from Ref. 25; dashed lines—ANOPP predictions modified to use the P&W turbine-attenuation; (a): 48 % corrected fan speed (flight idle); (b): 60 % corrected fan speed (approach); (c): 71 % corrected fan speed (cutback); (d): 87 % corrected fan speed (takeoff)

rather than the one build into ANOPP. It is clear from panels (a)–(c) in Fig. 8 that the total noise signature is better predicted by the modified ANOPP results than the original ones. One can also argue that the combustion-noise prediction is also improved by examining the source-separated results in this figure, although the situation is not as clear for frequencies larger or equal to 400 Hz.

Figure 9 shows the turbine-transfer phase factor, determined from Eq. (6b) with $\delta = 0$. In this figure, the frequency range has been limited to 400 Hz in order to only slightly exceed the frequency range where plane waves are dominant; panels (a)–(d) correspond to the flight-idle, approach, cutback, and takeoff engine-power settings; and the red and blue curves correspond to $\text{arg}(H_{uv})$ and $\text{arg}(H_{uw})$ results, respectively. It can be seen that the phase changes by about 4.5 radians from 0 to 200 Hz. This indicates that in this frequency range there is a signal time delay of about 3.6 ms, which strongly implies that the combustion noise is dominated by indirect noise.

Figure 10 shows the ratio of the actual transfer function and the directly measurable transfer function. It clearly indicates the error caused by the noise-to-signal ratio at the combustor location if $H_{xy}$ and $H_{xz}$ are used as approximations for the actual combustion-noise transfer functions $H_{uv}$ and $H_{uw}$. 

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Figure 9. Narrow-band (8 Hz) turbine phase factors; red curves—\( \arg(H_{uv}) \); blue curves—\( \arg(H_{uw}) \); (a): 48 \% corrected fan speed (flight idle); (b): 60 \% corrected fan speed (approach); (c): 71 \% corrected fan speed (cutback); (d): 87 \% corrected fan speed (takeoff)

### IV. Summary and Conclusions

NASA/Honeywell EVNERT\textsuperscript{[28]} full-scale static engine test data has been analyzed by using source-separation techniques in order to determine the turbine transfer of combustor noise. The true combustion-noise turbine-transfer function was educed from the data by using a three-signal approach. The resulting gain factors were compared with the corresponding constant values obtained from ANOPP/GE and Pratt & Whitney empirical acoustic-turbine-loss formulas. It was found that the Pratt & Whitney formula agrees better with the experimental results for frequencies of practical importance.

The far-field 1/3-octave SPL results in the 130° direction of Hultgren and Miles\textsuperscript{[25]} were reexamined using a post-correction of their ANOPP predictions for both the total noise signature and the combustion-noise component. It was found that replacing the standard ANOPP turbine-attenuation function for combustion noise with the Pratt & Whitney one clearly improved the total-noise predictions and also improved the combustion-noise predictions. The latter comparison was not as conclusive as the former due to the inherent difficulty in extracting the combustion-noise component from the total noise signature. However, the former would not be true if the combustion-noise component predictions had not been improved by the attenuation-formula change.

Based on these results, it is recommended that the GECOR combustion-noise module in ANOPP be updated to allow for a user-selectable switch between the current transmission-loss model (12) and the simplified Pratt & Whitney formula (13).
Figure 10. Narrow-band (8 Hz) combustor-noise factor $1 + N_u(f)$; red curve: 48 % corrected fan speed (flight idle); blue curve: 60 % corrected fan speed (approach); magenta curve: 71 % corrected fan speed (cutback); cyan curve: 87 % corrected fan speed (takeoff)

References


**Title and Subtitle**
Full-Scale Turbofan-Engine Turbine-Transfer Function Determination Using Three Internal Sensors

**Abstract**
Noise-source separation techniques, using three engine-internal sensors, are applied to existing static-engine test data to determine the turbine transfer function for the currently subdominant combustion noise. The results are used to assess the combustion-noise prediction capability of the Aircraft Noise Prediction Program (ANOPP) and an improvement to the combustion-noise module GECOR is suggested. The work was carried out in response to the NASA Fundamental Aeronautics Subsonic Fixed Wing Program’s Reduced-Perceived-Noise Technical Challenge.

**Subject Terms**
Turbofan engine; Function transfer; Internal sensors