Test-Anchored Vibration Response Predictions for an Acoustically Energized Curved Orthogrid Panel with Mounted Components

A rich body of vibroacoustic test data was recently generated at Marshall Space Flight Center for component-loaded curved orthogrid panels typical of launch vehicle skin structures. The test data were used to anchor computational predictions of a variety of spatially distributed responses including acceleration, strain and component interface force. Transfer functions relating the responses to the input pressure field were generated from finite element based modal solutions and test-derived damping estimates. A diffuse acoustic field model was applied to correlate the measured input sound pressures across the energized panel. This application quantifies the ability to quickly and accurately predict a variety of responses to acoustically energized skin panels with mounted components.

Favorable comparisons between the measured and predicted responses were established. The validated models were used to examine vibration response sensitivities to relevant modeling parameters such as pressure patch density, mesh density, weight of the mounted component and model form. Convergence metrics include spectral densities and cumulative root-mean squared (RMS) functions for acceleration, velocity, displacement, strain and interface force. Minimum frequencies for response convergence were established as well as recommendations for modeling techniques, particularly in the early stages of a component design when accurate structural vibration requirements are needed relatively quickly. The results were compared with long-established guidelines for modeling accuracy of component-loaded panels. A theoretical basis for the Response/Pressure Transfer Function (RPTF) approach provides insight into trends observed in the response predictions and confirmed in the test data.

The software developed for the RPTF method allows easy replacement of the diffuse acoustic field with other pressure fields such as a turbulent boundary layer (TBL) model suitable for vehicle ascent. Structural responses using a TBL model were demonstrated, and wind tunnel tests have been proposed to anchor the predictions and provide new insight into modeling approaches for this environment.

Finally, design load factors were developed from the measured and predicted responses and compared with those derived from traditional techniques such as historical Mass Acceleration Curves and Barrett scaling methods for acreage and component-loaded panels.
Test-anchored vibration response predictions for an acoustically energized curved orthogrid panel with mounted components

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Anchored Vibration Response

Agenda

• Introduction
• Test Article Finite Element Model (FEM) Pedigree
• Study the Frequency Range of Interest from a Convergence Perspective for Component Interface Forces and Strain
  Comparison to Vibration Convergence as Acceleration, Velocity, and Displacement
• Methodology for Two Approaches
  – The Acceleration/Pressure Transfer Function Method (APTF) or Direct Method
  – The Response Matching Method (RMM) – Modal Matrix Form
• Array of Models and Patch Densities Used for the Sensitivity Studies
• Acceleration, Strain, and Interface Force Response Sensitivity
  – Patch Density (A Patch is a set of nodes defined for response analysis treated as a unit for applied pressure. Patch density must be refined for adequacy in the Frequency Range of interest)
  – Mesh Density (As the number of elements increases, the element size gets smaller)
  – Vehicle Panel Modeling Approach Comparison
• Conclusions for Frequency Range of Interest Convergence Assessment
• Conclusions for Methodology Comparisons
• Conclusions for Approach Sensitivities
• References
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Introduction

- Finite Element Analysis is one of the premier tools for calculating not only displacement, velocity, and acceleration response, but also forces and strains.
- Working with Structural Finite Element Models to estimate the force and strain response from energies applied over a surface interface with a fluid medium is a developing field for which few experimental validations have been supplied.
- Recent measured structural response results from ground Acoustic Tests Conducted at Marshall Space Flight Center should help to fill that gap.
- Additionally, the correlated FEMs produced for the test project may allow us to evaluate response interface forces for several configurations of equipment mounted to a flight like vehicle panel test article.

- This Presentation will Summarize
  - *Response Estimates from a Highly Correlated FEM.*
  - *Several analysis approach choices necessary to converge to the measured solution*
- A series of FEM’S is presented which demonstrate sensitivities to some popular modeling approaches employed in developing FEMS of launch vehicle orthogrid panels.
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Introduction

• The presentation will help to answer the following questions:
  How does APTF Response Compare with Measured Response?
  – Acceleration PSD, Strain Cumulative RMS?
  – Strain PSD, Strain Cumulative RMS? Measurements from the two orthogonal Rosette Strain channels are carefully aligned with the corresponding FEM element response output.

What can we say about convergence of solution with increase in number of patches across the surface?
  – Convergence from below. Need adequate patch density to achieve adequately conservative solution. High frequency.
  – Convergence from above. Need adequate patch density to prevent or avoid over estimates of the solution. Low frequency.

What does the Convergence of RMS Velocity and RMS Displacement look like? Can either of these be used to indicate the convergence of RMS strain or RMS force?
  – The convergence evaluation for strain includes both analysis estimates and measured test channels.
  – Since Force was not measured directly in the acoustic test series, it was necessary to produce the force frequency band convergence cumulative RMS from the analysis estimate.
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Test Article Finite Element Model Pedigree

• High-fidelity Finite Element Models (FEMs) were developed to support a recent test program at Marshall Space Flight Center (MSFC). Admirable correlation was shown for five configurations of the Test Article.
  
The FEMs correspond to test articles used for a series of acoustic tests.
  
Modal survey tests were used to validate the FEMs for five acoustic tests (a bare panel and four different mass-loaded panel configurations).
  
Modal survey tests did test-validate the dynamic characteristics of the FEMs distinguishable modes used for acoustic test excitation studies.

• Correlated with dense array of response points

Modal Survey Tests: 32 tri-axial accels for the empty fixture, 77 tri-axial accels for the bare panel, and 85 tri-axial accels for each of the mass-loaded panel configurations.

Bare Panel Test and Analysis Correlation Summary

<table>
<thead>
<tr>
<th>TEST MODE</th>
<th>TEST FREQ</th>
<th>FEM MODE</th>
<th>FEM FREQ</th>
<th>MAC</th>
<th>CROSS-ORTHOG</th>
<th>FREQ DIFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>101.21</td>
<td>1</td>
<td>97.20</td>
<td>0.96</td>
<td>-0.99</td>
<td>-4.0%</td>
</tr>
<tr>
<td>2</td>
<td>110.01</td>
<td>2</td>
<td>106.67</td>
<td>0.95</td>
<td>0.97</td>
<td>-3.0%</td>
</tr>
<tr>
<td>3</td>
<td>150.75</td>
<td>5</td>
<td>143.04</td>
<td>0.93</td>
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</tr>
<tr>
<td>4</td>
<td>159.82</td>
<td>7</td>
<td>155.33</td>
<td>0.94</td>
<td>-0.98</td>
<td>-2.8%</td>
</tr>
<tr>
<td>5</td>
<td>186.64</td>
<td>9</td>
<td>181.98</td>
<td>0.90</td>
<td>0.90</td>
<td>-2.5%</td>
</tr>
<tr>
<td>6</td>
<td>207.79</td>
<td>10</td>
<td>198.37</td>
<td>0.86</td>
<td>0.92</td>
<td>-4.5%</td>
</tr>
<tr>
<td>7</td>
<td>208.64</td>
<td>11</td>
<td>202.65</td>
<td>0.51</td>
<td>-0.54</td>
<td>-2.9%</td>
</tr>
<tr>
<td>8</td>
<td>218.95</td>
<td>14</td>
<td>211.19</td>
<td>0.92</td>
<td>0.98</td>
<td>-3.6%</td>
</tr>
<tr>
<td>9</td>
<td>242.66</td>
<td>17</td>
<td>233.86</td>
<td>0.75</td>
<td>-0.86</td>
<td>-3.6%</td>
</tr>
<tr>
<td>10</td>
<td>251.70</td>
<td>18</td>
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<td>-3.4%</td>
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<td>281.13</td>
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<td>268.45</td>
<td>0.39</td>
<td>0.60</td>
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<tr>
<td>12</td>
<td>291.97</td>
<td>26</td>
<td>283.18</td>
<td>0.75</td>
<td>-0.68</td>
<td>-3.0%</td>
</tr>
<tr>
<td>13</td>
<td>319.69</td>
<td>30</td>
<td>307.89</td>
<td>0.88</td>
<td>0.90</td>
<td>-3.7%</td>
</tr>
</tbody>
</table>

MAC = 0.95 / Cross-Orthogonality = -0.96 / Frequency Difference = +0.1%

FEMs Suitable For Estimating Vibroacoustics Response
Anchored Vibration Response

Test Article Configurations

Table 2. Designators for 150 Model Variants (XX EX PXX WX)

<table>
<thead>
<tr>
<th>Placeholder (X)</th>
<th>Designation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>XX</td>
<td>ES</td>
<td>Explicit Shell</td>
</tr>
<tr>
<td></td>
<td>EB</td>
<td>Explicit Beam</td>
</tr>
<tr>
<td>SC</td>
<td>&quot;Smeared&quot; Composite</td>
<td></td>
</tr>
<tr>
<td>EX</td>
<td>1</td>
<td>1 x 1 = 1 Elements/Cell</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2 x 2 = 4 Elements/Cell</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6 x 6 = 36 Elements/Cell</td>
</tr>
<tr>
<td>PXX</td>
<td>01</td>
<td>01 x 01 = 1 Patches/Panel</td>
</tr>
<tr>
<td></td>
<td>02</td>
<td>02 x 02 = 4 Patches/Panel</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>12 x 12 = 144 Patches/Panel</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>20 x 20 = 400 Patches/Panel</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>30 x 30 = 900 Patches/Panel</td>
</tr>
<tr>
<td>WX</td>
<td>0</td>
<td>Component with 0 Increment Plates; ( \Delta W = 0 ) (lb)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Component with 1 Increment Plate; ( \Delta W = 15.4 ) (lb)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Component with 2 Increment Plates; ( \Delta W = 30.8 ) (lb)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Component with 3 Increment Plates; ( \Delta W = 46.2 ) (lb)</td>
</tr>
</tbody>
</table>

Table 1. Panel Assembly Configurations

<table>
<thead>
<tr>
<th>Configuration Name</th>
<th>Description</th>
<th>Primary Mockup Weight (lb)</th>
<th>Increment Plate Stack Weight (lb)</th>
<th>Total Component Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UL</td>
<td>Unloaded (Bare) Panel</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>W0</td>
<td>Panel + Primary</td>
<td>95.0</td>
<td>0</td>
<td>95.0</td>
</tr>
<tr>
<td>W1</td>
<td>Panel + Primary + 1 Incr Plate</td>
<td>95.0</td>
<td>15.4</td>
<td>110.4</td>
</tr>
<tr>
<td>W2</td>
<td>Panel + Primary + 2 Incr Plates</td>
<td>95.0</td>
<td>30.8</td>
<td>125.8</td>
</tr>
<tr>
<td>W3</td>
<td>Panel + Primary + 3 Incr Plates</td>
<td>95.0</td>
<td>46.2</td>
<td>141.2</td>
</tr>
</tbody>
</table>

Figure 4. Typical configuration of the test article
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Acceleration Measurement Locations used for Comparison

Large Mass Simulator

Z direction
Measurements were normal to curved vehicle panel.

X direction
Measurements aligned with the hoop ribs.

Y direction
Measurements aligned with the axial ribs.
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*Acceleration Response Comparison to Measurements*

Responses at location 11z normal to the Vehicle Panel

Bare Panel and Mass Loaded Response: The **dashed** analysis estimate traces compare well with the measured 5 Hz constant bandwidth Acceleration Spectral Density results.

**Comparison of Measured vs. Calculated Liftoff Bare Panel Responses**

**Red Solid** curves represent Measured Data.

**Comparison of Measured vs. Calculated Liftoff Panel Responses: Large Mass Simulator +3**

**Large Mass Simulator**

Bare Panel Results Comparison Above

Large Mass Simulator + 3 increments On Left
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Acceleration Response Comparison to Measurements

Comparison of Measured vs. Calculated Liftoff Panel Responses: Large Mass Simulator +1

Measurement Location 15 near center of Large Mass Simulator +1.

Lump approximation of Increments was used, but the comparisons were reasonable.
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*Strain Measurement Locations Used for Comparison*

All strain gauge locations were three channel rosettes with an axis aligned parallel to rib edge. Another axis was aligned in the z normal to panel direction.
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Strain Response Comparison to Measurements

Comparison of Measured vs. Calculated
Bare Panel Strain Responses, Liftoff

Dashed curves represent Analysis results.

Bare Panel Responses – Analysis compares well with the measured 5 Hz constant bandwidth Strain Spectral Density results for individual legs of the Rosette Strain Gauge.
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Strain Response Comparison to Measurements

Liftoff Panel Strain Responses: Large Mass Simulator +3

A clamped boundary condition was employed for the series of results presented here in, which explains the frequency shift in first few modes.

Clamped BC helped facilitated output of results for large number of cases.

Bare Panel Responses – Analysis compares well with the measured 5 Hz constant bandwidth Strain Spectral Density results for individual legs of the Rosette Strain Gauge.
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Frequency Range of Interest: Measured Strain/Acceleration Results

Dashed curves represent Analysis results.

* Earthquake Analyses make use of Velocity as best measure of Damage Potential

Velocity* is an excellent indicator of the Frequency Band Convergence of Force and Strain Response.

- Red, Green, and Blue, are Acceleration, Velocity, and Displacement Cumulative RMS fractions respectively.
- Magenta is the Force Overlaid (CBUSH Interface Force).
- Black is the Strain Cumulative RMS Overlaid.

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Frequency Range of Interest - Measured Strain Results on Panel

Why is this true?

The Strain and Force Spectral Density Results roll off more quickly than the Acceleration Spectral Density. But the roll off for Velocity is very similar ~ 12 dB/Octave.

**Velocity is an excellent indicator of the Frequency Band Convergence for Force and Strain.**

Strain and Vibration Measured Spectral Density Results from Large Mass Sim +0

Response Data for Channel 10Z & 10A

Velocity Response -Channel 15H, 15A, & 15z

PSD [µε²/Hz] vs Frequency (Hz)

PSD [in²/s²/Hz] vs Frequency (Hz)
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Methodologies

Start

Archive Flight Data

Scaling/Similarity Methods, Band-Averaging

SPL SPEC

Correlated Pressure Field

System FEM Skin Only

Acreage (Skin) APSD

Applied PRESSURE

System FEM Skin + Component

Component APSD

APTF Response from Direct Pressure

H_s (ω) / H_c (ω)

System FEM Skin Only

H_s (ω)

Barrett

Acreage (Skin) APSD SPEC

System FEM Skin + Component

Component APSD

Response Match

Component APSD

W_n / (W_n + W_c)

Empirical Scaling Response

Component APSD SPEC

Component APSD SPEC

Barrett Scaled

Applied Acceleration

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Acceleration/Pressure Transfer Function (APTF) Method (Direct)

- Direct application of the pressure field across the panel can be accomplished by assuming that pressure autospectra defined at the center of each patch occur on the diagonal of the pressure matrix, with pressure cross-spectra appearing on the off diagonal terms (set $P_{bc} = P_{cb}^*$). If spatial functions $\gamma(\omega, R)$ are defined that relate the autospectra to the cross-spectra, then the applied pressure field may be written as:

$$ P_{N_p}(\omega) = \begin{bmatrix} \gamma_{11} \hat{P}_{11} & \gamma_{12} \hat{P}_{12} & \cdots & \gamma_{1N_p} \hat{P}_{1N_p} \\ \gamma_{21} \hat{P}_{12} & \gamma_{22} \hat{P}_{22} & \cdots & \gamma_{2N_p} \hat{P}_{2N_p} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N_p1} \hat{P}_{1N_p} & \gamma_{N_p2} \hat{P}_{2N_p} & \cdots & \gamma_{N_pN_p} \hat{P}_{N_pN_p} \end{bmatrix} $$

Where: $\hat{P}_{bc} = \sqrt{P_{bb} P_{cc}}$

Arising from the inequality requirement for Coherence:

$$ 0 \leq \frac{|P_{bc}(\omega)|^2}{P_{bb}(\omega) P_{cc}(\omega)} \leq 1.0 \quad (1) $$

- The represented stationary gaussian random pressure field with non-zero cross-spectral density can be further defined as a diffuse field if the cross spatial functions are expressed as follows:

$$ \gamma_{bc}(\omega, R_{bc}) = \frac{\sin(R_{bc} \kappa(\omega))}{R_{bc} \kappa(\omega)} $$

Where: $R_{bc}$ is the distance between the area CGs of patches b and c,

$$ \kappa(\omega) = \omega / C_o $$

And $C_o$ is the speed of sound through the fluid medium adjacent to the patch material. (2)
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Acceleration/Pressure Transfer Function (APTF) Method (Direct)

- Finally, since the patch autospectra may be expressed as products of frequency-dependent scaling functions $W_{bb}(\omega)$ and an arbitrary reference autospectrum $P_{ref}(\omega)$. (The reference autospectrum selected could be one of the patch autospectra, but this is not required.)

$$P_{N_p}(\omega) = \begin{bmatrix} \gamma_{11}W_{11} & \gamma_{12}W_{12} & \cdots & \gamma_{1N_p}W_{1N_p} \\ \gamma_{12}W_{12} & \gamma_{22}W_{22} & \cdots & \gamma_{2N_p}W_{2N_p} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1N_p}W_{1N_p} & \gamma_{2N_p}W_{2N_p} & \cdots & \gamma_{N_pN_p}W_{N_pN_p} \end{bmatrix} P_{ref}(\omega)$$

Where: $P_{bb}(\omega) = W_{bb}(\omega)P_{ref}(\omega)$

And: $W_{bc}(\omega) = \sqrt{W_{bb}(\omega)W_{cc}(\omega)}$

- The acceleration/pressure transfer function for a single patch may be expressed explicitly as the sum of weighted acceleration/force transfer functions. The weighting factor is the static force $F_k$ at each input location $k$ on the patch due to a unit pressure (from an OLOAD request in NASTRAN SOL 101):

$$H_{a_j/p_b}(\omega) = \sum_{k=1}^{N_b} \left\{ F_k \cdot \sum_{m=1}^{M} \left[ \frac{-\omega^2 \phi_{jm} \phi_{km}}{\omega_m^2 - \omega^2 + i 2 \zeta_m \omega_m \omega} \right] \right\}$$

Where: $H_{a_j/p_b}(\omega)$ is the transfer function between acceleration at point $j$ and pressure $p_b$ on patch $b$, $F_k$ is the static force at point $k$ associated with a unit pressure on patch $b$, $\phi_{jm}$ is the $m^{th}$ mass-normalized mode shape at response point $j$, $\phi_{km}$ is the $m^{th}$ mass-normalized mode shape at point $k$ in the pressure patch, $\omega$ is the circular frequency, $\omega_m$ is the circular natural frequency of mode $m$, $\zeta_m$ is the critical damping ratio for mode $m$, $N_b$ is the number of GRIDs in the pressure patch, $M$ is the number of retained modes.
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Acceleration/Pressure Transfer Function (APTF) Method (Direct)

- The acceleration PSD response at point $j$ to the applied random pressure spectral density on a single patch is the squared magnitude of the acceleration/pressure transfer function in (4) multiplied by the pressure PSD:

$$A_{jb}(\omega) = \left| H_{a_j/p_b}(\omega) \right|^2 P_{bb}(\omega)$$

$$= \left| H_{a_j/p_b}(\omega) \right|^2 W_{bb}(\omega) P_{ref}$$  \hspace{1cm} (5)

- The total response at location $j$ includes the response from the pressure autospectra on all of the patches and also from non-zero pressure cross-spectra between any two patches:

$$A_j(\omega) = \sum_{b}^{N_p} \left| H_{a_j/p_b} \right|^2 P_{bb} + \sum_{b}^{N_p} \sum_{c \neq b}^{N_p} H_{a_j/p_b} H_{a_j/p_c}^* P_{bc}$$

$$= \sum_{b}^{N_p} \sum_{c}^{N_p} H_{a_j/p_b} H_{a_j/p_c}^* P_{bc}$$  \hspace{1cm} (6)

- Expressing eq (6) in terms of the reference pressure spectral density and the spatially dependent cross-spectra of eqs (2) and (3), we obtain:

$$A_j(\omega) = \sum_{b}^{N_p} \sum_{c}^{N_p} \gamma_{bc} W_{bc} H_{a_j/p_b} H_{a_j/p_c}^* P_{ref} = \sum_{b}^{N_p} \sum_{c}^{N_p} \frac{\sin(\kappa R_{bc})}{\kappa R_{bc}} \sqrt{W_{bb} W_{cc}} H_{a_j/p_b} H_{a_j/p_c}^* P_{ref}$$  \hspace{1cm} (7)

Note that the spatial functions $\gamma$ reduce to unity for $b = c$. 

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Acceleration/Pressure Transfer Function (APTF) Method (Direct)

The Matrix Form Notation for MATLAB is:

\[
A_j(\omega) = \left[ H_{a_j} \sqrt{W} \Gamma \sqrt{W} H_{a_j}^\dagger \right] P_{\text{ref}}(\omega)
\]  

(8)

where \( ^\dagger \) denotes the Hermitian conjugate and

\[
H_{a_j}(\omega) = \begin{bmatrix} H_{a_j/p_1} & H_{a_j/p_2} & \cdots & H_{a_j/p_{N_p}} \end{bmatrix} \quad \text{(a/p transfer functions from eq (4)),}
\]

\[
W(\omega) = \begin{bmatrix} W_{11} & 0 & \cdots & 0 \\
0 & W_{22} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & W_{N_p,N_p} \end{bmatrix} \quad \text{(pressure autospectra scaling functions from eq (3)),}
\]

\[
\Gamma(\omega) = \begin{bmatrix} 1 & \frac{\sin(\kappa R_{12})}{\kappa R_{12}} & \cdots & \frac{\sin(\kappa R_{1N_p})}{\kappa R_{1N_p}} \\
\frac{\kappa R_{12}}{\sin(\kappa R_{21})} & 1 & \cdots & \frac{\sin(\kappa R_{2N_p})}{\kappa R_{2N_p}} \\
\vdots & \vdots & \ddots & \vdots \\
\text{SYM} & \cdots & \cdots & 1 \end{bmatrix} \quad \text{(spatial functions from eq (2)).}
\]
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Acceleration/Pressure Transfer Function (APTF) Method (Direct)

• Dividing both sides by the reference pressure spectral density, \( P_{\text{ref}} \), we obtain the squared transfer function between the total response at location \( \ell \) and the entire diffuse pressure field:

\[
\left| H_{a_j/p}(\omega) \right|^2 = H_{a_j} \sqrt{W} \, \Gamma \sqrt{W} \, H_{a_j}^\dagger = \frac{A_j(\omega)}{P_{\text{ref}}(\omega)}
\]  

(9)

Response Matching Method (Delta Configuration with Component)

• Now consider the case when a component is mounted to the skin. The applied pressure does not change, but the transfer function in (9) developed for the bare skin must be generated for the component-loaded skin. Eqs (4) – (9) are applied at a response location using the modes of the component-loaded skin, and (9) becomes

\[
\left| \tilde{H}_{a_j/p}(\omega) \right|^2 = \tilde{H}_{a_j} \sqrt{W} \, \Gamma \sqrt{W} \, \tilde{H}_{a_j}^\dagger = \frac{\tilde{A}_j(\omega)}{P_{\text{ref}}(\omega)}
\]  

(10)

where the tilde denotes the component-loaded acceleration/pressure transfer function and skin response.

• If the pressure is unknown, but the acceleration on the unloaded skin is known from measured flight data or computational models, the component-loaded response at any point \( \ell \) may be obtained by eliminating \( P_{\text{ref}}(\omega) \) from (9) and (10):

\[
\tilde{A}_j(\omega) = A_j(\omega) \cdot \frac{\left| \tilde{H}_{a_j/p}(\omega) \right|^2}{\left| H_{a_j/p}(\omega) \right|^2}
\]  

(11)

where the acceleration/single-patch pressure transfer functions in eq (9) for the bare skin are obtained from eq (4)
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Response Matching Method (Delta Configuration with Component)

• The acceleration/pressure transfer functions \( \tilde{H}_{a_{i}/p_{b}}(\omega) \) for the component-loaded skin may be obtained from:

\[
\tilde{H}_{a_{i}/p_{b}}(\omega) = \sum_{k=1}^{N_{b}} \sum_{m=1}^{M} \left\{ F_{k} \cdot \sum_{i} \left[ \frac{-\omega^{2} \phi_{i,m} \phi_{k,m}}{\omega_{m}^{2} - \omega^{2} + 2 \zeta_{m} \omega_{m} \omega} \right] \right\} \quad (12)
\]

• Force and moment responses at specified interface elements (e.g., CBUSH) may be obtained in the same fashion by replacing the first mode shape term in eq (12) with the modal forces and moments obtained in a NASTRAN RESTART in SOL 103. The \(-\omega^{2}\) term in the numerator of eq (12) is also dropped. The expression for the response force (or moment) at location is similar to that for acceleration in eq (11):

\[
\tilde{F}_{q}(\omega) = A_{j}(\omega) \cdot \left| \frac{\tilde{H}_{f_{q}/p}(\omega)}{H_{a_{i}/p}(\omega)} \right|^{2} \quad (13)
\]

where \( \tilde{F}_{q}(\omega) \) is the force (or moment) PSD at location \( q \).

• \( \tilde{H}_{f_{q}/p} \) is the transfer function between the total force (or moment) at location \( q \) and the pressure:

\[
\left| \tilde{H}_{f_{q}/p}(\omega) \right|^{2} = \tilde{H}_{f_{q}} \sqrt{W} \Gamma \sqrt{W} \tilde{H}_{f_{q}}^{\dagger} = \frac{\tilde{F}_{q}(\omega)}{P_{ref}(\omega)} \quad (14)
\]

• The individual transfer functions \( \tilde{H}_{f_{q}/p_{b}}(\omega) \) in \( \tilde{H}_{f_{q}} \) between the force at location \( q \) and each single-patch pressure on any patch \( b \) are given by

\[
\tilde{H}_{f_{q}/p_{b}}(\omega) = \sum_{k=1}^{N_{b}} \sum_{m=1}^{M} \left\{ F_{k} \cdot \left[ \frac{\psi_{q,m} \phi_{k,m}}{\omega_{m}^{2} - \omega^{2} + 2 \zeta_{m} \omega_{m} \omega} \right] \right\} \quad (15)
\]

where \( \psi_{q,m} \) is the \( m^{th} \) modal force at location \( q \).
Anchored Vibration Response
Array of Models Used for Sensitivity Studies

The applied pressure patch densities were varied between:

- 01x01 = 1 Patch
- 02x02 = 4 Patches
- ...
- 30x30 = 900 Patches

Generally 12x12 or 15x15 was considered a pretty well converged solution.

<table>
<thead>
<tr>
<th>Description of Modeling Approach</th>
<th>Model Approach</th>
<th>Model Designation</th>
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</thead>
<tbody>
<tr>
<td>Ribs with Shell Elements</td>
<td>Explicit</td>
<td>1</td>
</tr>
<tr>
<td>Smear Ribs as Composite Layer</td>
<td>PCOMP</td>
<td>2</td>
</tr>
<tr>
<td>Ribs with Beam Elements</td>
<td>Beam</td>
<td>3</td>
</tr>
</tbody>
</table>

Three mesh densities were considered:

- Medium Mesh Density 2x2 within one Cell
- Low Mesh Density 1x1 within one Cell
- Medium Density 2x2 within one Cell
- High Mesh Density 6x6 within one Cell
Anchored Vibration Response

Array of Models Used for Sensitivity Studies

Example Patch Densities 04x04 to 30x30 - Shown on Explicit High Mesh Density Vehicle Panel FEM
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Array of Models Used for the Sensitivity Studies

Example Patch Densities 04x04 to 30x30 - Shown on the Smeared High Mesh Density Vehicle Panel FEM
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Acceleration Response Sensitivity Patch Density – Bare Panel

The High frequency Response converges from below as Patch density increases.

Typical Bare Panel configuration

When the patch density is inadequate, certain cases produce overestimate of low frequency results.
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Acceleration Response Sensitivity Patch Density – Large Sim +0

Typical Mass Loaded configuration

Ensuring an Adequate Patch Density prevents over-estimates of low frequency response and under-estimates of high frequency response.
Strain Spectral Density

Several modes contribute significantly to RMS response

Typical Bare Panel configuration

Several modes contribute significantly to RMS response
Anchored Vibration Response

Strain Response Sensitivity Patch Density Large Sim +0

Strain Spectral Density
More dominated by single first mode

Typical Mass Loaded configuration
Rolls off sooner

Mass loaded Strain Cases
Provide Tighter grouping

More dominated by single first mode
Anchored Vibration Response

**Force Response Sensitivity Patch Density Large Sim +0**

**Force Spectral Density**

- Dominated by single first mode

**Mass loaded Strain Cases**
- Provide tighter grouping

**Although dominated by single first mode**, secondary rocking modes are a bit more significant than the strain measurements.
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Sensitivity Model Approach Rib-Beam, Smeared PCOMP, Explicit

Acceleration PSD Sensitivity to Model Form (Beams vs Smeared vs Shells):
Bare Orthogrid Panel Model w/ High Mesh and Patch Densities, Liftoff Event, Diffuse Pressure Model, Accelerometer #11 Normal Direction

Acceleration Spectral Density

Typical Bare Panel configuration

"Model Form" sensitivity plot - Beams and Explicit shells Representation of the ribs agree with each other. Smeared model doesn't match as well.
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Response Sensitivity Mesh Density – Bare Panel

Acceleration PSD Sensitivity to Mesh Density:
Bare Orthogrid Panel Model w/ Explicit Ribs, Liftoff Event, Diffuse Pressure Model, Accelerometer #11 Normal Direction

Trials Completed do not yet illustrate the threshold of adequacy for Mesh Density. More trials with Coarser Mesh Densities are needed.
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Conclusions: Frequency Range of Interest Convergence Assessment

- Convergence to total RMS have been presented from both measurements and analytical studies. The convergence data can be used to identify the necessary Frequency Range of Interest for estimating response.
- In particular, the authors were interested to identify and recommend frequency range of interest for Component interface forces appropriate for this assembly. This frequency range of interest can be contrasted with that necessary for Acceleration Vibration Environments:

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</tr>
</tbody>
</table>

- The frequency range of interest for strain and interface force convergence is similar to that for convergence of RMS Velocity. Therefore Velocity PSDs should be a better indicator for conclusions about Component Loads Band Width of Interest than acceleration or displacement PSDs for other representative assemblies.
- The correlated FEMs using the APTF Method to compute response from a Diffuse Acoustic Field assumption provided more than adequate estimates of measured strain and acceleration response. Both the method and the models were therefore well suited for drawing conclusions concerning estimates of Component Loads.
Anchored Vibration Response
Conclusions and Methodology Comparisons

• The early sections of this paper provide evidence that the models and approach used to analytically estimate the structural vibration response from the energies provided during a ground acoustic test performed admirably. Furthermore the approach can be said to be robust because the method provided these very suitable estimates for a compliment of 5 different configurations of the test articles.

• The analytical estimates of vibration response were verified for both acceleration measurements and for strain measurements. The ground test set up and test articles were presented. Additional data was provided to describe the satisfying correlation that was achieved for the models when compared to observations from tap modal tests conducted on the hardware configured in the ground test facility.

• Additionally, the authors have explored the frequency range of interest for component interface forces, strains and stresses. The velocity Spectral density was found to be a better indicator of the frequency of interest for such component loads than either displacement or acceleration spectral density. The cumulative RMS calculated from the velocity spectral density was found to converge over the frequency domain at about the same rate as the interface force and strain results.

• After demonstrating the adequacy of the models and the approach, the later sections of the report were prepared to demonstrate the sensitivity of such a vibration response analysis to several parameters.

• Although mesh density and model form were among the parameters explored, the analysis proved to be more significantly sensitive to the choice of patch density. A large number, 855, permutations of these analyses completed. The different patch densities were explored over models of 3 different mesh densities and configured to represent 5 different configurations of the test article.

• The patch density sensitivity verified that spatial correlation of the pressure field is important to demonstrate the more lively response that is observed (measured) for panels at the coincidence frequency. The frequency range above coincidence also proves to be more responsive when the patch density is fine enough to adequately represent the spatial correlation of the fluid sound pressure wave lengths. The frequency range above the coincidence frequency is said to be acoustically fast.
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References