A Probabilistic, Facility-Centric Approach to Lightning Strike Location

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Abstract

A new probabilistic facility-centric approach to lightning strike location has been developed. This process uses the bivariate Gaussian distribution of probability density provided by the current lightning location error ellipse for the most likely location of a lightning stroke and integrates it to determine the probability that the stroke is inside any specified radius of any location, even if that location is not centered on or even with the location error ellipse. This technique is adapted from a method of calculating the probability of debris collision with spacecraft. Such a technique is important in spaceport processing activities because it allows engineers to quantify the risk of induced current damage to critical electronics due to nearby lightning strokes. This technique was tested extensively and is now in use by space launch organizations at Kennedy Space Center and Cape Canaveral Air Force Station. Future applications could include forensic meteorology.
Nomenclature

\( \theta \) = angle between the collision plane coordinates and the rotated collision plane coordinates

\( \rho_{xz} \) = correlation coefficient of \( x \) and \( z \)

\( \sigma_{x'} \) = standard deviation of \( x \)-coordinate of the diagonalized covariance ellipse in the rotated coordinate system

\( \sigma_{z'} \) = standard deviation of \( z \)-coordinate of the diagonalized covariance ellipse in the rotated coordinate system

\( \sigma_x \) = standard deviation of \( x \)

\( \sigma_z \) = standard deviation of \( z \)

\( \sigma_{x'} \) = standard deviation of \( x \) in the rotated coordinate system

\( \sigma_{z'} \) = standard deviation of \( z \) in the rotated coordinate system

\( \mu_K \) = \( x \)-coordinate of target circle in the \((X', Z')\) coordinate system

\( \mu_H \) = \( z \)-coordinate of target circle in the \((X', Z')\) coordinate system

\( A \) = collision cross-sectional area (nautical miles\(^2\), nmi\(^2\))

\( d\theta \) = the angle between two points on the target ellipse

\( dH \) = integration step

\( H \) = intermediate variable in the lightning probability algorithm

\( P \) = probability

\( \text{pdf} \) = probability distribution function

\( r_A \) = radius of circle of cross-sectional area, \( A \) (nautical miles, nmi)

\( R \) = radius of ellipse (nmi)

\( R_1 \) = distance to first point on target ellipse (nmi)

\( R_2 \) = distance to second point on target ellipse (nmi)

\( x \) = horizontal rectangular coordinate in the collision plane (nautical miles, nmi)

\( x' \) = horizontal rectangular coordinate in the rotated collision plane (nmi)

\( x'' \) = transformation variable that circularizes the \( x \)-component of the probability ellipse (nmi)

\( x_e \) = nominal distance of closed approach of two colliding objects (nmi)

\( W \) = intermediate variable in the lightning probability algorithm

\( z \) = vertical rectangular coordinate in the collision plane (nautical miles, nmi)

\( z' \) = vertical rectangular coordinate in the collision plane (nautical miles, nmi)

\( z'' \) = transformation variable that circularizes the \( z \)-component of the probability ellipse (nmi)
Introduction

The ability to accurately estimate the probability that an individual nearby cloud-to-ground lightning stroke was within a specified distance of any specified spaceport processing facility at Kennedy Space Center (KSC) or Cape Canaveral Air Force Station (CCAFS) is important to processing payloads and space launch vehicles before launch. Such estimates allow engineers to decide if inspection of electronics systems aboard satellite payloads, space launch vehicles, and ground support equipment is warranted due to induced currents from that stroke. If induced current damage has occurred, inspections of the electronics are critical to identify required fixes and avoid degraded performance or failure of the satellite or space launch vehicle. However, inspections are costly both financially and in terms of delayed processing for space launch activities. As such, it is important these inspections be avoided if not needed.

At KSC/CCAFS, one of the main purposes of the Four Dimensional Lightning Surveillance System (4DLSS) (Murphy, 2008, Roeder, 2010) is detection of nearby strokes and determination of their peak current to support decisions to inspect electronics (Flinn, 2010a, Flinn, 2010b, Roeder, 2005). The high frequency of lightning occurrence in East Central Florida combined with the large amount of complex sensitive electronics in satellite payloads, space launch vehicles, and associated facilities makes those decisions critically important to space launch processing. The 4DLSS provides the data for 50th percentile location error ellipses for the best location for each stroke, which is then scaled to 95th or 99th percentile ellipses depending on customer requirements. This error ellipse is necessarily centered on the best location of the lightning stroke. The 4DLSS, however, has not been able to provide the probability of the stroke being within a customer specified distance of a point of interest. This paper presents a new method to convert the 4DLSS 50th percentile location error ellipse for best location of any stroke into the probability that the stroke was within any radius of any facility at CCAFS/KSC. This technique could be adapted for use with National Lightning Detection Network (NLDN) data. This new probabilistic facility-centric technique is a significant improvement over the stroke-centric location error ellipses the 45th Weather Squadron (45WS) has provided in the past. This technique is adapted from a method of calculating the probability of debris collision with spacecraft (Chan, 2008, Leleux, 2002, Patera, 2001).

Methodology

Background

In spacecraft collision probability and other applications, at the instant of “nominal” closest approach, the position uncertainty of the collision object relative to the asset being protected is described by a bivariate Gaussian probability density function (pdf) (Chan, 2008, Patera, 2001, Alfano, 2006, Alfano 2007), as shown in the following equation

\[
f(x, z) = \frac{1}{2\pi \sigma_x \sigma_z \sqrt{1 - \rho_{xz}^2}} e^{-\frac{\left(\frac{x}{\sigma_x}\right)^2 - 2\rho_{xz}\left(\frac{x}{\sigma_x}\right)\left(\frac{z}{\sigma_z}\right) + \left(\frac{z}{\sigma_z}\right)^2}{2(1 - \rho_{xz}^2)}}
\]  

(1)

where \(\sigma_x\) and \(\sigma_z\) = the standard deviations of \(x\) and \(z\), \(\rho_{xz}\) = correlation coefficient of \(x\) and \(z\), \(x\) and \(z\) are the designations for the rectangular coordinates in the collision plane.

The probability of collision (Eq. 2) is given by the two-dimensional integral, where \(A\) is the collision cross-sectional area which is a circle with radius, \(r_A\) (Chan, 2008).
\[ P = \int_A f(x, z) dx dz \] \hspace{1cm} (2)

There is no known analytical solution to the above integral when the two standard deviations \( \sigma_x \) and \( \sigma_z \) are not equal. The solution is found by performing a numerical integration of the two dimensional Gaussian pdf (Chan, 2008, Patera, 2001, Alfano, 2006, Alfano 2007).

The geometry used for spaceflight collision probability can also be used for estimation of the probability of an individual nearby lightning stroke contacting the surface within a specified distance of a specified point of interest as shown in Fig. 1. In Fig. 1, \( \alpha \) is the heading of the semi-major axis of the lightning location uncertainty ellipse from true north and \( \theta \) is the angle between the semi-major axis of the lightning location uncertainty ellipse and line connecting the center of the lightning uncertainty ellipse and the center of the area of interest. Two methods of integrating the above probability were tested and gave identical results. The first solution method was based on an algorithm by Patera (2001) and the second solution method was based on an algorithm by Chan (2011). Chan’s algorithm ran much faster and therefore was selected as the algorithm for the 45WS lightning probability program.

\[ \text{Area around point of interest} \]
\[ \text{1-\sigma uncertainty ellipse around lightning strike} \]
\[ \text{P(LatP, LonP)} \]
\[ \text{S(LatS, LonS)} \]

\textbf{The Numerical Integration Technique (Chan, 2011)}

This numerical integration technique is one in which the miss distance is given by a non-central chi distribution with unequal variances (Chan, 2011). The covariance matrix corresponding to the bivariate Gaussian pdf in Eq. 1 is (Chan, 2008):
When the correlation coefficient, $\rho_{xz}$, is not zero, there are undesirable off-diagonal terms that overly complicate the calculation. In order to eliminate these terms, the coordinate system $(x, z)$ is rotated to a new coordinate system $(x', z')$ such that the major and minor axes of the ellipse associated with the covariance are aligned along the coordinate axes and the new covariance matrix is (Chan, 2008):

$$C' = \begin{bmatrix} \sigma_x^2 & 0 \\ \rho_{xz} \sigma_x \sigma_z & \sigma_z^2 \end{bmatrix}$$

The angle, $\theta$, between the two coordinate systems is (Chan, 2008):

$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{2 \rho_{xz} \sigma_x \sigma_z}{\sigma_x^2 - \sigma_z^2} \right)$$

The KSC/CCAFS 4DLSS system does not provide the covariance matrix, but instead provides the semi-major axis, semi-minor axis, and the orientation of the semi-major axis of the 50% location error ellipse relative to north. Therefore the angle, $\theta$, in Eq. 5 is found using geometry where $\theta$ is the angle between the semi-major axis of the lightning location uncertainty ellipse and line connecting the center of the lightning uncertainty ellipse and the center of the area of interest.

In the $(x', z')$ system, the Eq. 1 pdf becomes (Chan, 2008)

$$f(x', z') = \frac{1}{2\pi \sigma_x \sigma_z} e^{-\frac{1}{2} \left( \frac{x'}{\sigma_x} \right)^2 - \left( \frac{z'}{\sigma_z} \right)^2}$$

and Eq. 2, the collision probability becomes (Chan, 2008)

$$P = \frac{1}{2\pi \sigma_x \sigma_z} \int_A e^{-\frac{1}{2} \left( \frac{x'}{\sigma_x} \right)^2 - \left( \frac{z'}{\sigma_z} \right)^2} dx' dz'$$

where

$$A' = A, r_{A'} = r_A, x' = x e \cos \theta, z' = x e \sin \theta$$

For spacecraft collision, $x_e$ is the nominal distance of closest approach of the two colliding objects and $(x'_p, z'_p)$ are the coordinates of the spacecraft relative to the debris. For lightning strike probability, $x_e$ (the distance between the position of the center of the strike location ellipse and the position of the target area) is calculated using the Haversine distance formula.
The standard deviations in the new rotated coordinate system are calculated by dividing the semi-major and semi-minor axes of the 50% lightning positional confidence ellipse by the scaling constant used to scale standard error to the 50% confidence level. The scaling constant is:

\[ k = \sqrt{-2 \times \ln(1 - 0.50)} \]  

(9)

The probability is given by (Chan, 2011)

\[
P = \frac{1}{2\sqrt{2\pi} \sigma_H} \int_{0}^{W} \left[ e^{-\left(\frac{H-H^2}{2\sigma_H^2}\right)} + e^{-\left(\frac{H+H^2}{2\sigma_H^2}\right)} \right] \left\{ \text{erf} \left( Z_1 \right) + \text{erf} \left( Z_2 \right) \right\} dH
\]

(10)

where

\[
Z_1 = \left[ \sqrt{(W - H^2)} - \mu_K \right] \sqrt{2} \sigma_K
\]

(11)

\[
Z_2 = \left[ \sqrt{(W - H^2)} + \mu_K \right] \sqrt{2} \sigma_K
\]

The parameters \( \mu_K \) and \( \mu_H \) are the coordinates of the target circle in the \((X', Z')\) coordinate system; and \( \sigma_K \) and \( \sigma_H \) are the standard deviations of the diagonalized covariance ellipse shown in Eq. 4. The derivation of equations (10) and (11) above is shown in further detail in (Chan, 2011). A detailed example of the calculations using a real-world case is provided in Appendix-A. The Excel Visual Basic code to implement these calculations is shown in Appendix-B.

**Evaluation**

The probability that any lightning strike is within any radius of any point of interest would be extremely difficult to estimate intuitively. As a result, given the high impact of the decisions on space launch operations, the tool developed for this application was extensively tested. Tests were conducted and are discussed in the following sections: 1) known mathematical solutions, and 2) examination of real-world events. The new technique passed all of the tests. Tests were also conducted to assure the probabilities calculated using the algorithm of Chan (2011) matched probabilities calculated using the algorithm of Patera (2001). The probabilities calculated from the two algorithms were identical and thus are not shown here.

**Test Set 1**

The first set of testing compared the lightning strike probability calculated using the 45WS lightning strike spreadsheet (which uses an adaptation of the numerical integration algorithm by Chan, 2011, to the corresponding circular probability from the CRC Handbook of Tables for Probability and Statistics (Beyer, 1968). Table 1 shows the probability from the new numerical integration technique for various inputs and the corresponding correct probability from the CRC Handbook. The values matched to within a tenth of a percent. These errors in the final digit may be due to round-off error.
Table 1. Calculated probability vs. CRC Handbook probability for various inputs

<table>
<thead>
<tr>
<th>Semi-major axis (nmi)</th>
<th>Semi-minor axis (nmi)</th>
<th>Heading from true North</th>
<th>Point Of Interest Latitude</th>
<th>Point Of Interest Longitude</th>
<th>Strike Latitude</th>
<th>Strike Longitude</th>
<th>Radius around Point Of Interest (nmi)</th>
<th>Calculated probability</th>
<th>CRC Handbook probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>15</td>
<td>28.6082</td>
<td>-80.6041</td>
<td>28.6995</td>
<td>-80.6041</td>
<td>3</td>
<td>0.095</td>
<td>0.095</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>15</td>
<td>28.6082</td>
<td>-80.6041</td>
<td>28.631</td>
<td>-80.6041</td>
<td>3</td>
<td>0.453</td>
<td>0.452</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>15</td>
<td>28.6082</td>
<td>-80.6041</td>
<td>28.608</td>
<td>-80.6041</td>
<td>3</td>
<td>0.500</td>
<td>0.499</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>15</td>
<td>28.6082</td>
<td>-80.6041</td>
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<td>1</td>
<td>0.200</td>
<td>0.200</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>15</td>
<td>28.6082</td>
<td>-80.6041</td>
<td>28.6995</td>
<td>-80.6041</td>
<td>1</td>
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<td>28.608</td>
<td>-80.6041</td>
<td>2</td>
<td>0.937</td>
<td>0.938</td>
</tr>
</tbody>
</table>

**Test Set 2**

The second type of testing analyzed six real-world lightning strikes near Space Launch Complex 39A on 3 August 2009. Figure 2 shows the spreadsheet used to generate the lightning report for those six strikes. Additional data on three of these six strikes are in Table 2. These strikes were selected because the closest point on the lightning position uncertainty ellipse was within 0.45 nautical miles of Launch Complex 39A, which is the key radius for assessing the need to inspect electronics for induced current damage to the Space Shuttle. Figures 3 through 5 are Google Maps depictions of three of these six strokes. In Figures 3-5, the black ellipse is the 99% lightning location uncertainty ellipse. The white circle is a 0.45 nrni radius around the point of interest. The probabilities for a small area around a facility, even for a nearby stroke, may appear to be surprisingly low. For example, figures 3 and 4 respectively illustrate a 53.8% and 7.7% probability that the lightning strike occurred within the area of interest while figure 5 shows that a strike just 0.65 nautical miles away had only a 1.1% probability of being within the 0.45 nautical mile radius of Launch Complex 39A. All calculated probabilities are consistent with these real-world events.
The KSC Electromagnetic Environmental Effects (EEE) Panel requested six more real-world lightning strikes be investigated. These were recently investigated lightning strikes near Launch Complexes 39A or 39B where there was camera verification of the location of the strike. The EEE Panel wanted to compare the results of the new facility-centric probabilistic technique to these cases where the true answers were known unambiguously. The data used for this analysis are in Table 3. Both 4DLSS and National Lightning Data Network (NLDN) cases were examined, depending upon which sensor system recorded the stroke. CGLSS strokes were obtained from 45WS 4DLSS. The NLDN usually provided flash data, so NLDN return stroke data were purchased as special StrikeNet reports from Vaisala Corporation. This was done to match the return strokes routinely provided by 4DLSS. Figures 6 through 8 show the probability results from these cases. In Figures 6-8, the black ellipse is the 99% lightning location uncertainty ellipse. The white circle is a 0.45 nmi radius around the point of interest. As with the previous real-world tests, all calculated probabilities were consistent with these additional real-world events.

Table 2. Input values used for scenarios shown in Figures 3 through 5.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Semi-major axis of 50% confidence ellipse (km)</th>
<th>Semi-major axis of 50% confidence ellipse (km)</th>
<th>Confidence</th>
<th>Heading from true North of 50% confidence ellipse (°)</th>
<th>Point of interest latitude (°N)</th>
<th>Point of interest longitude (°W)</th>
<th>Strike latitude (°N)</th>
<th>Strike longitude (°W)</th>
<th>Radius around point of interest (nmi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.4</td>
<td>0.2</td>
<td>0.99</td>
<td>300.7</td>
<td>28.60827</td>
<td>-80.6041</td>
<td>28.6114</td>
<td>-80.6113</td>
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</tr>
<tr>
<td>4</td>
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<td>0.99</td>
<td>293</td>
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<td>28.6178</td>
<td>-80.6069</td>
<td>0.45</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>0.1</td>
<td>0.99</td>
<td>20.3</td>
<td>28.60827</td>
<td>-80.6041</td>
<td>28.5995</td>
<td>-80.6113</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Figure 3. Google Maps visualization of the 99% confidence uncertainty ellipse for one of the closest lightning strikes to Complex 39A on 03 August 2009.

Figure 4. Google Maps visualization of the 99% confidence uncertainty ellipse for a lightning strike near Complex 39A on 03 August 2009.
Figure 5. Google Maps visualization of the 99% confidence uncertainty ellipse for nearby lightning strike to Complex 39A on 03 August 2009.

Table 3. Input values used for scenarios shown in Figures 6 through 8.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Semi-major axis of 50% confidence ellipse (km)</th>
<th>Semi-major axis of 50% confidence ellipse (km)</th>
<th>Confidence</th>
<th>Heading (from true North)</th>
<th>Point of interest latitude ('N)</th>
<th>Point of interest longitude ('W)</th>
<th>Strike latitude ('N)</th>
<th>Strike longitude ('W)</th>
<th>Radius around point of interest (nmi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.6</td>
<td>0.4</td>
<td>0.99</td>
<td>82</td>
<td>28.60827</td>
<td>-80.6041</td>
<td>28.6069</td>
<td>-80.6087</td>
<td>0.45</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>0.4</td>
<td>0.99</td>
<td>95</td>
<td>28.60827</td>
<td>-80.6041</td>
<td>28.6057</td>
<td>-80.6085</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
<td>0.1</td>
<td>0.99</td>
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<td>-80.6275</td>
<td>28.6275</td>
<td>-80.6202</td>
<td>0.45</td>
</tr>
</tbody>
</table>
Figure 6. Illustrates a probability of 69.1% of a lightning strike of amplitude -43.0 kA detected by NLDN occurring 0.26 nmi from the center of Launch Complex 39A on 8/16/2009.

Figure 7. Illustrates a probability of 74.7% of a lightning strike of amplitude -71.4 kA detected by NLDN occurring 0.28 nautical miles from the center of Launch Complex 39A on 10/14/2009.
Other Applications

The techniques and methods described in this paper clearly have application reaching far beyond the space program uses for which they were designed. The list of potential applications is many and varied and would be of interest to anyone seeking information pertaining to probability of lightning strike locations, such as the power industry, aviation, or any industry sensitive to electrical overloads. This methodology is also applicable to forensic meteorology [13], where the question of whether lightning struck at or near a particular location is an issue in litigation. This technique could be applied to NLDN or any other cloud-to-ground lightning detection system that provides location error ellipses.

Conclusion

A probabilistic facility-centric approach to lightning location has been developed to calculate the probability that any nearby cloud-to-ground lightning stroke occurred within any radius of any point of interest. In practice, this provides the probability that a nearby lightning stroke was within a key distance of a facility, rather than within the error ellipses centered on the stroke. This process uses the bivariate Gaussian distribution of probability density provided by the current lightning location error ellipse for the most likely location of a lightning stroke and integrates it to determine the probability that the stroke is inside any specified radius. This new facility-centric technique was tested extensively and is much more useful to the space launch customers.
Appendix A – Example Lightning Probability Calculation

This appendix is an example of calculating the probability of any lightning stroke with a known error ellipse being within a circle of any radius around any point. It is provided to clarify the calculation process. An example calculation is shown in Table 4.

This example is a real-world event from a lightning strike near the Space Shuttle launch pad 39A at 02:35 GMT on 16 August 2009 (ref. Figure 6). Although the lightning data usually are from the cloud-to-ground component of the Four Dimensional Lightning Surveillance System (CG-4DLSS) (Murphy et al 2008, Roeder 2010) in this example a lightning stroke from the NLDN is used. We sometimes use StrikeNet reports that provide stroke data from the NLDN to double check the CG-4DLSS report. The lightning stroke input values are shown in row 1 of Table 3.

The following assumptions are applicable to the example calculation in Table 4. The location of Launch Pad 39A is 28.60827486 north latitude (or 0.499309 radians) and 80.60411653 west longitude (or -1.406807 radians). This is also the center of the circle in which the lightning probability will be calculated. The desired radius for probability of lightning around Launch Pad 39A is 0.45 nautical miles (nmi). This lightning stroke occurred on 16 August 2009 at 0235 GMT.

Table 4. Lightning strike probability calculation process.

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Equation and other information</th>
<th>Calculation and Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Convert semi-major and semi-minor axes from km to nmi</td>
<td>1 nmi = 1.852 km</td>
<td>0.6 km = 0.324 nmi semi-major axis</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.4 km = 0.216 nmi semi-minor axis</td>
</tr>
<tr>
<td>2</td>
<td>Calculate distance from lightning stroke to center of circle</td>
<td>Haversine Distance Formula: [ Distance = \text{Earth Radius} \times C ]  [ C = 2 \times \text{Atn}2(\sqrt{1 - A^2}) ]  [ A = \sin(dlat/2) \times \sin(dlat/2) + \cos((\text{target lat}) \times \cos((\text{stroke lat})) \times \sin(dlon/2) \times \sin(dlon/2) ]  [ dlat = \text{latitude difference from target to stroke} = 28.60827^\circ - 28.6069^\circ = 2.39959 \times 10^{-5} \text{ (radians)} ]  [ dlon = \text{longitude difference from circle to stroke} = -80.60411^\circ - 80.6085^\circ = 7.99967 \times 10^{-5} \text{ (radians)} ]</td>
<td>Distance = 3443.920086 * 7.4217 * 10^{-5} = 0.2556 nmi</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C = 2 * \text{Atn}2(1 - 1.377 \times 10^{-9})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 7.4217 * 10^{-5}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A = \sin(1.200 \times 10^{-5}) \times \sin(1.200 \times 10^{-5}) + \cos(0.4993) \times \cos(0.4993) \times \sin(4.000 \times 10^{-5}) \times \sin(4.000 \times 10^{-5}) ]  [ X = (\text{Longitude of Target} - \text{Longitude of Stroke}) \times \cos(\text{Latitude of Stroke}) ]</td>
</tr>
</tbody>
</table>


to eliminate the off-diagonal term in the covariance matrix.

- \( Z = \) Latitude of Target - Latitude of Stroke
- \( \theta = \alpha - ((\pi/2) - \text{Atan2}(X,Z)) \)
- \( \alpha \) is the orientation angle of the 50% lightning positional confidence ellipse
- \( X' = \) miss distance*\( \text{Cos}(\theta \text{ (coordinate system rotation angle)}) \)
- \( Z' = \) miss distance*\( \text{Sin}(\theta) \)

4 Calculate the standard deviations in the new rotated coordinate system.

- \( \sigma_X' = \) Semi-major axis of the 50% lightning positional confidence ellipse /elliptical scaling constant used to scale standard error to the 50% confidence level
- \( \sigma_Z' = \) Semi-minor axis of the 50% lightning positional confidence ellipse /elliptical scaling constant used to scale standard error to the 50% confidence level
- Elliptical scaling constant, \( k \), is:
  \[ k = \sqrt{-2 \ln(1 - \text{probability})} \]

5 Calculate the probability that lightning stroke was within the target area of interest by performing a numerical integration using Simpson’s rule of the lightning uncertainty ellipse over the area of the circle around the target of interest.

\[
p = \frac{1}{2\sqrt{2}\sigma_H} \int_{-\infty}^{\infty} e^{-\left(\frac{H-H_H}{2\sigma_H}\right)^2} e^{-\left(\frac{H+H_H}{2\sigma_H}\right)^2} \frac{\text{erf}(Z_1)}{\sqrt{2k}} \frac{\text{erf}(Z_2)}{\sqrt{2k}} dH
\]

- \( \mu_K \) and \( H_H \) are the coordinates of the target circle in the \((X', Z')\) coordinate system
- \( \sigma_K \) and \( \sigma_H \) are equal to \( \sigma_X' \) and \( \sigma_Z' \) which are the standard deviations of the diagonalized covariance matrix.
- \( W = \) Radius around target
- \( N = \) the number of iterations to perform in the integration (for this example, \( N \) is set to 200).
- \( DH = \) integration step
- B, C, and D are intermediate variables in

\[
W = \frac{0.45^2}{2} = 0.2025
\]

\[
DH = \frac{\sqrt{W}}{N}
\]

\[
DH = \frac{\sqrt{0.2025}}{200} = 0.00225
\]

\[
B = \frac{\sqrt{2} \sigma_X'}{\sqrt{2} \sigma_Z'} = \frac{0.2752}{0.1834} = 0.3891
\]

\[
C = \frac{X'}{B}
\]

\[
C = \frac{0.2556}{0.3891} = 0.6451
\]

\[
D = \frac{1}{2 \sqrt{2\pi \sigma_z}} = \frac{1}{2 \sqrt{2\pi \times 0.1834}} = 0.00225
\]

\[
H = \text{iteration no.} \times DH
\]

\[
H = 199 \times 0.00225
\]

\[
H = 0.4478
\]
the algorithm corresponding to various parts of the probability equation shown above

- \( B = \sqrt{2} \sigma_x \)
- \( C = X' / B \)
- \( D = 1 / (2 \sqrt{2\pi} \sigma_z) \)

- \( A, H, z1, z2, E, F, \text{Erf}(z1), \text{Erf}(z2), Q \) and sum are intermediate variables in the algorithm corresponding to various parts of the probability equation shown above
- A loop is performed for \( j = 1 \) to 199. This example is shown for \( j = 199 \).

<table>
<thead>
<tr>
<th>( A = \sqrt{W - H^2} )</th>
<th>( A = \sqrt{0.2025 - 0.44775^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = 4.494 \times 10^{-2} )</td>
<td>( A = 4.494 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

\( z_1 = A / B - C \)
\( z_1 = 4.494 \times 10^{-2} / 0.3891 - 0.6451 \)
\( z_2 = A / B + C \)
\( z_2 = 4.494 \times 10^{-2} / 0.3891 + 0.6451 \)
\( z_2 = 0.7606 \)

- \( \text{Erf}(z_1) = \) ErrorFunction(z1)
\( \text{Erf}(-0.5296) = -0.5461 \)
\( \text{Erf}(z_2) = \) ErrorFunction(z2)
\( \text{Erf}(0.7606) = 0.7179 \)
\( E = (H - Z')^2 / (2 \sigma_z^2) \)
\( E = (0.4478 - 0.0482)^2 / (2 \times 0.1834^2) \)
\( E = 2.372 \)
\( F = (H + Z')^2 / (2 \sigma_z^2) \)
\( F = (0.4478 + 0.0482)^2 / (2 \times 0.1834^2) \)
\( F = 3.654 \)

\( Q(j) = (e^{-E} + e^{-F}) \times (\text{Erf}(z_1) + \text{Erf}(z_2)) \)

\( Q(199) = (e^{-2.372} + e^{-3.654}) \times (-0.5461 + 0.7179) = 2.0467 \times 10^{-2} \)

\( \text{sum} = \text{sum} + (3 - (-1)^j) \times Q(j) \)

\( \text{sum} = \text{sum} + 2.0467 \times 10^{-2} \)

\( \text{sum} = 844.8952 \)
End Loop

\( \text{sum} = \text{sum} + Q(0) + Q(N) \)
\( \text{sum} = 844.8952 + 2.9361 + 0 = 847.8317 \)

\( \text{Probability} = D \times \text{sum} \times DH / 3 \)
\( \text{Probability} = 1.0874 \times 847.8317 \times 0.00225 / 3 = 847.8317 \times 0.00225 / 3 = \)
\[ E = \frac{(H - Z')^2}{(2 \cdot \sigma^2)} \]
\[ E = \frac{(0.4478 - 0.0482)^2}{(2 \cdot 0.1834^2)} \]
\[ E = 2.372 \]

\[ F = \frac{(H + Z')^2}{(2 \cdot \sigma^2)} \]
\[ F = \frac{(0.4478 + 0.0482)^2}{(2 \cdot 0.1834^2)} \]
\[ F = 3.654 \]

\[ Q(j) = (e^{-E} + e^{-F}) \cdot \left( \text{Erf}(Z1) + \text{Erf}(Z2) \right) \]
\[ Q(199) = (e^{-2.372} + e^{-3.654}) \cdot (-0.5461 + 0.7179) = 2.0467 \times 10^{-2} \]

\[ \text{sum} = \text{sum} + (3 - (-1)^j) \cdot Q(j) \]
\[ \text{sum} = 844.8952 \]

End Loop

\[ \text{sum} = \text{sum} + Q(0) + Q(N) \]
\[ \text{sum} = 844.8952 + 2.9361 = 847.8317 \]

\[ \text{Probability} = D \times \text{sum} \times \frac{DH}{3} \]
\[ \text{Probability} = 1.0874 \times 847.8317 \times 0.00225 / 3 = 0.6914 \]
Appendix B – Excel VBA Macro for 45th Weather Squadron Lightning Spreadsheet that Calculates the Probability of any Nearby Lightning Stroke being Inside any Radius of Any Point of Interest

Conversions Module

Const pie As Double = 3.14159265358979
Const ecc As Double = 0.081819190842622 'eccentricity (e^2)
Const kmPerMile As Double = 1.852

Function toRad(Degrees As Double) As Double
    'Converts degrees to radians
    toRad = Degrees * pie / 180
End Function

Function toDeg(Radians As Double) As Double
    'Converts radians to degrees
    toDeg = Radians * 180 / pie
End Function

Function applyConfidence(length As Double, newSigma As Double, currentSigma As Double) As Double
    'convert length based on confidence interval
    applyConfidence = length * newSigma / currentSigma
End Function

Function revertConfidence(length As Double, newSigma As Double, currentSigma As Double) As Double
    'convert length based on confidence interval
    revertConfidence = length * currentSigma / newSigma
End Function

Function kmToNmi(kilometers As Double) As Double
    'Converts kilometers to nautical miles
    kmToNmi = kilometers / kmPerMile
End Function

Function geodToECEF(Lat As Double, Lon As Double) As Variant
    'Convert lat-lon to EFG
    Dim ret(2) As Double
    ret(0) = getE(Lat, Lon)
    ret(1) = getF(Lat, Lon)
    ret(2) = getG(Lat)
    geodToECEF = ret
End Function
Function EcefToGeod(E As Double, F As Double, g As Double) As Variant
'vert return if EFG coordinate to lat-lon
Dim ret(1) As Double 'array of (lat,lon)
ret(0) = Atn(g / ((1 - ecc ^ 2) * Sqr(E ^ 2 + F ^ 2))) * 180 / pie
ret(1) = Atn2(E, F) * 180 / pie
EcefToGeod = ret
End Function

---

**Functions Module**

Const pie As Double = 3.14159265358979
Const eRad As Double = 3443.920086 'radius of earth in nautical miles
Const ecc As Double = 0.081819190842622 'eccentricity (e^2)

Public Function Atn2(ByVal X As Double, ByVal Y As Double) As Double
Dim at As Double
If X = 0 Then
    If Y < 0 Then
        Atn2 = -pie / 2
    ElseIf Y > 0 Then
        Atn2 = pie / 2
    Else
        Atn2 = 0
    End If
ElseIf Y = 0 Then
    If X > 0 Then
        Atn2 = 0
    ElseIf X < 0 Then
        Atn2 = pie
    End If
Else
    at = Atn(Y / X)
    If X < 0 And Y < 0 Then
        Atn2 = at - pie
    ElseIf X < 0 And Y > 0 Then
        Atn2 = at + pie
    Else
        Atn2 = at
    End If
End If
End Function

Function arcCos(X As Double)
    arcCos = Atn((1 - X ^ 2) ^ 0.5 / X)
End Function
Function addPoints(p1() As Double, p2() As Double) As Variant
    'add two EFG coordinates together
    Dim ret(2) As Double
    ret(0) = p1(0) + p2(0)
    ret(1) = p1(1) + p2(1)
    ret(2) = p1(2) + p2(2)
    addPoints = ret
End Function

Function getEPointDiffs(eLat As Double, eLon As Double, A As Double, B As Double, phi As Double, t As Double) As Variant
    'get EFG differentials for a point on ellipse at t radians
    Dim ret(2) As Double
    Dim xl As Double, y1 As Double
    xl = getX(A, B, 0, 90 - phi, t)
    y1 = getY(A, B, 0, 90 - phi, t)
    ret(0) = -xl * Sin(eLon * pie / 180) - y1 * Sin(eLat * pie / 180) * Cos(eLon * pie / 180)
    ret(1) = xl * Cos(eLon * pie / 180) - y1 * Sin(eLon * pie / 180) * Sin(eLat * pie / 180)
    ret(2) = y1 * Cos(eLat * pie / 180)
    getEPointDiffs = ret
End Function

Function getFocalPointDiffs(eLat As Double, eLon As Double, A As Double, B As Double, phi As Double) As Variant
    'get EFG differentials for focal point of ellipse
    Dim ret(2) As Double
    Dim fociDist As Double, xl As Double, y1 As Double
    fociDist = Sqr(A^2 - B^2)
    xl = fociDist * Cos(toRad(90 - phi))
    y1 = fociDist * Sin(toRad(90 - phi))
    ret(0) = -xl * Sin(eLon * pie / 180) - y1 * Sin(eLat * pie / 180) * Cos(eLon * pie / 180)
    ret(1) = xl * Cos(eLon * pie / 180) - y1 * Sin(eLon * pie / 180) * Sin(eLat * pie / 180)
    ret(2) = y1 * Cos(eLat * pie / 180)
    getFocalPointDiffs = ret
End Function

Function getDistGeod(lat1 As Double, lon1 As Double, lat2 As Double, lon2 As Double) As Double
    'spherical
    Dim rLat1 As Double, rLat2 As Double, rLon1 As Double, rLon2 As Double
    rLat1 = toRad(lat1)
    rLat2 = toRad(lat2)
    rLon1 = toRad(lon1)
    rLon2 = toRad(lon2)
getDistGeod = getN((lat1 + lat2) / 2) * arcCos(Cos(rLat1) * Cos(rLon1) * Cos(rLat2) * Cos(rLon2) + Cos(rLat1) * Sin(rLon1) * Cos(rLat2) * Sin(rLon2) + Sin(rLat1) * Sin(rLon1)) / 180 * pi

End Function

Function greatCircle(lat1 As Double, lon1 As Double, lat2 As Double, lon2 As Double) As Double
' great circle distance in NM
Dim rLat1 As Double, rLat2 As Double, rLon1 As Double, rLon2 As Double
rLat1 = toRad(lat1)
rLat2 = toRad(lat2)
rLon1 = toRad(lon1)
rLon2 = toRad(lon2)
greatCircle = (eRad) * arcCos(Cos(rLat1) * Cos(rLat2) * Cos(rLon2 - rLon1) + Sin(rLat1) * Sin(rLat2))
End Function

Function Haversine(lat1 As Double, lon1 As Double, lat2 As Double, lon2 As Double) As Double
' Calculates distance between two latitude and longitude points by the haversine formula in NM
Dim dlat As Double, dlon As Double, C As Double, A As Double

dlat = toRad(lat1) - toRad(lat2)
dlon = toRad(lon1) - toRad(lon2)
A = Sin(dlat / 2) * Sin(dlat / 2) + Cos(toRad(lat1)) * Cos(toRad(lat2)) * Sin(dlon / 2) * Sin(dlon / 2)
C = 2 * Atn2(Sqr(1 - A), Sqr(A))
Haversine = eRad * C
End Function

Function getX(A As Double, B As Double, H As Double, phi As Double, t As Double) As Double
' get the X cartesian coordinate of a point on the ellipse at t radians
getX = H + A * Cos(t) * Cos(phi * pi / 180) - B * Sin(t) * Sin(phi * pi / 180)
End Function

Function getY(A As Double, B As Double, k As Double, phi As Double, t As Double) As Double
' get the Y cartesian coordinate of a point on the ellipse at t radians
getY = k + B * Sin(t) * Cos(phi * pi / 180) + A * Cos(t) * Sin(phi * pi / 180)
End Function

Function getN(Lat As Double) As Double
' get radius of the earth as a given latitude, adjusted for eccentricity
getN = eRad / (1 - ((ecc ^ 2) * Sin(Lat * pi / 180) ^ 2)) ^ 0.5
End Function

Function getE(Lat As Double, Lon As Double) As Double
End Function
'get E coordinate from lat-lon
getE = getN(Lat) * Cos(Lat * \(\pi/180\)) * Cos(Lon * \(\pi/180\))
End Function

Function getF(Lat As Double, Lon As Double) As Double
'get F coordinate from lat-lon
getF = getN(Lat) * Cos(Lat * \(\pi/180\)) * Sin(Lon * \(\pi/180\))
End Function

Function getG(Lat As Double) As Double
'get G coordinate from lat
getG = getN(Lat) * (1 - ecc^2) * Sin(Lat * \(\pi/180\))
End Function

Function getEPoint(ecLat As Double, ecLon As Double, A As Double, B As Double, phi As Double, t As Double) As Variant,
'get lat-lon of a point on ellipse at t radians
Dim ret(l) As Double
Dim eC() As Double
Dim point() As Double
Dim diffs() As Double
eC = geodToECEF(ecLat, ecLon)
diffs = getEPointDiffs(ecLat, ecLon, A, B, phi, t)
point = addPoints(eC, diffs)
getEPoint = EcefToGeod(point(0), point(1), point(2))
End Function

Function ePointLat(ecLat As Double, ecLon As Double, A As Double, B As Double, phi As Double, t As Double) As Double
'get lat of a point on ellipse at t radians
Dim geod() As Double
geod = getEPoint(ecLat, ecLon, A, B, phi, t)
ePointLat = geod(0)
End Function

Function ePointLon(ecLat As Double, ecLon As Double, A As Double, B As Double, phi As Double, t As Double) As Double
'get lon of a point on ellipse at t radians
Dim geod() As Double
geod = getEPoint(ecLat, ecLon, A, B, phi, t)
ePointLon = geod(1)
End Function

Function getAzimuth(lat1 As Double, lon1 As Double, lat2 As Double, lon2 As Double) As Double
'get azimuth between two points
Dim angle As Double
angle = (Atn2(Cos(toRad(lat1)) * Sin(toRad(lat2)) - Sin(toRad(lat1)) * Cos(toRad(lat2)) * 
   Cos(toRad(lon2) - toRad(lon1)), Sin(toRad(lon2) - toRad(lon1)) * Cos(toRad(lat2))) * 180 / pie
If (angle = 0) Then
   getAzimuth = 360
Else
   getAzimuth = (angle + 360) Mod 360
End If
End Function

Module 2 Code

Dim point As New Strike
Const passWord As String = "PAEc0d3m0nk3y$"

Sub POI_Change()
   Calculate
End Sub

Sub calculateRow()

   Application.EnableCancelKey = xlDisabled
   Worksheets("Results").protect Contents:=False, passWord:=passWord
   Dim ecefO As Double
   Dim interestPoint(1) As Double ' (lat,lon)
   Dim rowNumber_data As Double
   Dim rowNumber_result As Double
   Dim rowData As Variant
   Dim AOI(3) As Double
   Dim lastRow As Double
   If Worksheets("Results").FilterMode = True Then
      Worksheets("Results").ShowAllData
   End If
   lastRow = Worksheets("Results").Cells(Rows.Count, "A").End(xlUp).row
   If (lastRow > 6) Then
      Worksheets("Results").Range("A7:R" & lastRow).Select
      Selection.Delete
   End If

   AOI(0) = Range("latLower")
   AOI(1) = Range("latUpper")
   AOI(2) = Range("lonLower")
   AOI(3) = Range("lonUpper")

   interestPoint(0) = Range("poiLat")
   interestPoint(1) = Range("poiLon")
   ecef = geodToECEF(interestPoint(0), interestPoint(1))
Call point.setStatics(interestPoint, ecef, Range("newSigma"), Range("currentSigma"), Range("radius"))
Dim xlCalc As XICalculation
xlCalc = Application.Calculation
' On Error GoTo CalcBack

lastRow = Worksheets("Data").Cells(Rows.Count, "A").End(xlUp).row
rowNumber_result = 7
For rowNumber_data = 2 To lastRow
    rowData = Worksheets("Data").Range("A" & rowNumber_data & ":I" & rowNumber_data)
    If (rowData(1, 3) > AOI(0) And rowData(1, 3) < AOI(1) And rowData(1, 4) > AOI(2) And rowData(1, 4) < AOI(3)) Then
        Call point.init(rowData(l, 3), rowData(1, 4), rowData(l, 6), rowData(l, 7), rowData(l, 8), True)
        Worksheets("Results").Cells(rowNumber_result, 1) = rowData(l, 1)
        Worksheets("Results").Cells(rowNumber_result, 2) = rowData(l, 2)
        Worksheets("Results").Cells(rowNumber_result, 3) = point.centerAzimuth
        Worksheets("Results").Cells(rowNumber_result, 4) = point.centerRange
        Worksheets("Results").Cells(rowNumber_result, 5) = point.criticalAzimuth
        Worksheets("Results").Cells(rowNumber_result, 6) = point.criticalRange
        Worksheets("Results").Cells(rowNumber_result, 7) = rowData(l, 5)
        Worksheets("Results").Cells(rowNumber_result, 8) = point.isInside
        Worksheets("Results").Cells(rowNumber_result, 9) = rowData(l, 9)
        Worksheets("Results").Cells(rowNumber_result, 10) = point.Prob_simpson
        Worksheets("Results").Cells(rowNumber_result, 11) = point.rangeDifference
        Worksheets("Results").Cells(rowNumber_result, 12) = point.Lat
        Worksheets("Results").Cells(rowNumber_result, 13) = point.Lon
        Worksheets("Results").Cells(rowNumber_result, 14) = point.A
        Worksheets("Results").Cells(rowNumber_result, 15) = point.B
        Worksheets("Results").Cells(rowNumber_result, 16) = point.phi
        Worksheets("Results").Cells(rowNumber_result, 17) = point.criticalLat
        Worksheets("Results").Cells(rowNumber_result, 18) = point.criticalLon
        'Worksheets("Results").Cells(rowNumber_result, 19) = point.Prob_rician
        'Worksheets("Results").Cells(rowNumber_result, 20) = point.Prob_simpson

    rowNumber_result = rowNumber_result + 1
End If
Next rowNumber_data
lastRow = Worksheets("Results").Cells(Rows.Count, "A").End(xlUp).row
Worksheets("Results").Range("A7:A" & lastRow).NumberFormat = "MM/DD/YYYY"
Worksheets("Results").Range("B7:B" & lastRow).NumberFormat = "HH:mm:ss.000"
Worksheets("Results").Range("C7:C" & lastRow).NumberFormat = "000"
Worksheets("Results").Range("D7:D" & lastRow).NumberFormat = "0.00"
Worksheets("Results").Range("E7:E" & lastRow).NumberFormat = "000"
Worksheets("Results").Range("F7:F" & lastRow).NumberFormat = "0.00"
Worksheets("Results").Range("J7:J" & lastRow).NumberFormat = "0.00000000"
Worksheets("Results").Range("K7:K" & lastRow).NumberFormat = "0.00"
Worksheets("Results").Range("L7:M" & lastRow).NumberFormat = "0.0000"
Worksheets("Results").Range("N7:P" & lastRow).NumberFormat = "0.00"
Worksheets("Results").Range("Q7:R" & lastRow).NumberFormat = "0.0000"
Worksheets("Results").Range("S7:T" & lastRow).NumberFormat = "0.000000%"

If (lastRow > 6) Then

With Worksheets("Results").Range("H7:H" & lastRow).Select
    With Selection.FormatConditions(1).Font
        .Color = -16777024
        .TintAndShade = 0
    End With
    With Selection.FormatConditions(1).Interior
        .PatternColorIndex = xlAutomatic
        .Color = 5263615
        .TintAndShade = 0
    End With
End With
End If

Application.Calculation = xlCalc
Worksheets("Results").EnableAutoFilter = True
Worksheets("Results").protect Contents:=True, passWord:=passWord, AllowFiltering:=True,
AllowSorting:=True, userInterfaceOnly:=True

MsgBox ("Data Calculation Complete")

Exit Sub

CalcBack:
   Worksheets("Results").protect Contents:=True, passWord:=passWord, AllowFiltering:=True,
   AllowSorting:=True, userInterfaceOnly:=True
   Application.Calculation = xlCalc

End Sub

Sub openBrowser()
    Dim rowNumber As Double
    Dim N As Integer
    Dim ecef() As Double
    Dim interestPoint(l) As Double ' (lat,lon)
    Dim showPath As Boolean
    showPath = ActiveSheet.showPath.value

    interestPoint(0) = Range("poiLat")
    interestPoint(1) = Range("poiLon")

    ecef = geodToECEF(interestPoint(0), interestPoint(1))
Call point.setStatics(interestPoint, ecef, Range("newSigma"), Range("currentSigma"), Range("radius"))
rowNumber = Range("nRow")
N = Range("nPoints")
Call point.init(Cells(rowNumber, 12), Cells(rowNumber, 13), Cells(rowNumber, 14), Cells(rowNumber, 15), Cells(rowNumber, 16), False)

Call point.setURL(N, showPath)
'Range("urlString") = point.URL

Set browser = CreateObject("InternetExplorer.Application")

If (Len(point.URL) > 1950) Then
    MsgBox ("Your request contains too many characters. Please select a smaller number of perimeter points or turn on off the path attribute.")
Else
    browser.Navigate (point.URL)
    browser.Visible = True.
End If

End Sub

--------------------

**Probability Module**

Const pie As Double = 3.14159265358979

Function RicianIntegral(U As Double, V As Double, m As Integer)
    'This subroutine computes the Rician Integral Pm when u, v and m are given.
    'It is preferred because both the exponentials are outside the summation loops.
    'This formulation gives essentially the same numerical value as the other one.
    Dim tm As Double, rm As Double, sm As Double, sumTm As Double, sumTmSm As Double
    On Error GoTo overflow
    tm = 1
    rm = 1
    sm = 1
    sumTm = 1
    sumTmSm = 1
    For i = 1 To m
        tm = tm * V / (2 * i)
        rm = rm * U / (2 * i)
        sm = sm + rm
        sumTm = sumTm + tm
        sumTmSm = sumTmSm + tm * sm
    Next i
    RicianIntegral = Exp(-V / 2) * sumTm - Exp(-(U + V) / 2) * sumTmSm
    Exit Function
overflow:

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RicianIntegral = 0
End Function

Function numericIntegral(Xprime As Double, Zprime As Double, SigmaXp As Double, SigmaZp As Double, Xe As Double, Ra As Double, N As Integer)
    Dim xl As Double, zl As Double, r1 As Double, x2 As Double, z2 As Double, r2 As Double
    Dim R As Double, sum As Double, dTheta As Double, thisPhi As Double, Q As Double
    DPhi = 2 * pi / N
    sum = 0
    thisPhi = 0
    xl = (Xprime + Ra) * SigmaZp / SigmaXp
    zl = Zprime
    For i = 1 To N
        thisPhi = thisPhi + DPhi
        x2 = (Xprime + Ra * Cos(thisPhi)) * SigmaZp / SigmaXp
        z2 = Zprime + Ra * Sin(thisPhi)
        r1 = Sqr(xl * xl + zl * zl)
        r2 = Sqr(x2 * x2 + z2 * z2)
        dTheta = Application.WorksheetFunction.Asin((xl * z2 - x2 * zl) / (r1 * r2))
        R = (r1 + r2) / 2
        sum = sum + Exp(-(R / SigmaZp) ^ 2 / 2) * dTheta
        xl = x2
        zl = z2
    Next i
    Q = -sum / (2 * pi)
    If Xe < Ra Then
        numericIntegral = 1 + Q
    ElseIf Xe = Ra Then
        numericIntegral = 1 / 2 + Q
    Else
        numericIntegral = Q
    End If
End Function

Function simpsonIntegral(Xprime As Double, Zprime As Double, SigmaXprime As Double, SigmaZprime As Double, Ra As Double, N As Integer)
    'This program computes the collision probability between two objects.
    'The integration is performed using Simpson's Rule.
    Dim Q(1000) As Double, P As Double
    Dim A As Double, B As Double, C As Double, D As Double, E As Double, F As Double, W As Double
    Dim H As Double, DH As Double
pi = 3.14159265358979

B = Sqr(2) * SigmaXprime
C = Xprime / B
D = 1 / (2 * Sqr(2 * pi) * SigmaZprime)
W = Ra ^ 2
DH = Sqr(W) / N

'Compute Q(I) array from 0 to (N-1)
For j = 0 To (N - 1)
    H = j * DH

    A = Sqr(W - H ^ 2)

    z1 = A / B - C
    z2 = A / B + C

    ErfZ1 = ErrorFunction(z1)
    ErfZ2 = ErrorFunction(z2)

    E = (H - Zprime) ^ 2 / (2 * SigmaZprime ^ 2)
    F = (H + Zprime) ^ 2 / (2 * SigmaZprime ^ 2)

    Q(j) = (Exp(-E) + Exp(-F)) * (ErfZ1 + ErfZ2)

Next j

Q(N) = 0

'Compute the integral by Simpson's Rule
sum = 0
For j = 1 To (N - 1)
    sum = sum + (3 ^ (-1) ^ j) * Q(j)
Next j

sum = sum + Q(0) + Q(N)

P = D * sum * DH / 3

simpsonIntegral = P

End Function

Function ErrorFunction(X)
    'This subroutine computes the Error Function when X is given.
    Dim Z, SqrtPi, tm, sum, Max, m, ErfX, ErfcX
    SqrtPi = 1.77245385090552

    erfX = ErfX
    erfcX = ErfcX

    For j = 1 To (N - 1)
        sum = sum + (3 ^ (-1) ^ j) * Q(j)
    Next j

    sum = sum + Q(0) + Q(N)

    P = D * sum * DH / 3

    simpsonIntegral = P

    End Function
\[ Z = \text{Abs}(X) \]

If \( Z \leq 4 \) Then
\[
\begin{align*}
  \text{tm} &= Z \\
  \text{sum} &= \text{tm} \\
  \text{Max} &= \text{Int}(20 \times Z) + 1 \\
  \text{For } m = 1 \text{ To Max} \\
  &\quad \text{tm} = -\text{tm} \times Z \times Z \times (2 \times m - 1) / (m \times (2 \times m + 1)) \\
  &\quad \text{sum} = \text{sum} + \text{tm} \\
  \text{Next } m \\
  \text{ErfX} &= \text{sum} \times 2 / \text{SqrtPi}
\end{align*}
\]

ElseIf \( Z \leq 5.736 \) Then
\[
\begin{align*}
  \text{tm} &= 1 \\
  \text{sum} &= 1 \\
  \text{For } m = 1 \text{ To 1} \\
  &\quad \text{tm} = -\text{tm} \times (2 \times m - 1) / (2 \times Z^2) \\
  &\quad \text{sum} = \text{sum} + \text{tm} \\
  \text{Next } m \\
  \text{ErfcX} &= \text{sum} \times \text{Exp}(-Z^2) / (\text{SqrtPi} \times Z) \\
  \text{ErfX} &= 1 - \text{ErfcX}
\end{align*}
\]

Else
\[
\text{ErfX} = 1
\]
End If

\[ \text{ErrorFunction} = \text{Sgn}(X) \times \text{ErfX} \]
End Function

---

**Strike Class Module**

Const \( \pi \) As Double = 3.14159265358979
Const eRad As Double = 3443.920086 'radius of earth in nautical miles
Const ecc As Double = 0.081819190842622 'eccentricity (e^2)

Private p_poi_geo(0) As Double 'lat lon point of interest [lat,lon]
Private p_poi_ecef() As Double 'EFG point of interest [E,F,G]
Private p_newSigma As Double 'user defined confidence level
Private p_currentSigma As Double '50% confidence

Private p_strike_geo(1) As Double 'lat lon of strike [lat,lon]
Private p_major As Double 'major axis in NM
Private p_minor As Double 'minor axis in NM
Private p_phi As Double 'stroke orientation in degrees

Private p_a As Double 'scaled major axis in NM
Private p_b As Double 'scaled minor axis in NM
Private p_strike_ecef() As Double 'EFG strike [E,F,G]
Private p_criticalT As Double 'angle of closest point in radians
Private p_critical_geo(0) As Double 'lat lon of closest point [lat,lon]
Private p_critical_ecef(2) As Double 'EFG of closest point [E,F,G]
Private p_aziCenter As Double 'azimuth to strike center
Private p_rangeCenter As Double 'distance to strike center
Private p_aziCritical As Double  'azimuth to closest point
Private p_rangeCritical As Double  'distance to closest point
Private p_rangeDifference As Double
Private p_isInside As String
Private p_URL As String
Private p_foci1() As Double
Private p_foci2() As Double

Private p_prob_rician As Double
Private p_prob_numeric As Double
Private p_prob_simpson As Double

Private p_radius As Double  'radius around point of interest

Public Sub setStatics(poi_geo As Variant, poi_ecef As Variant, newSigma As Double, currentSigma As Double, radius As Double)
    p_poi_geo = poi_geo
    p_poi_ecef = poi_ecef
    p_newSigma = Sqr(-2 * Log(1 - newSigma /100))
    p_currentSigma = Sqr(-2 * Log(1 - currentSigma /100))
    p_radius = radius
End Sub

Public Sub init(Lat, Lon, major, minor, phi, adjustAxis As Boolean)
    On Error GoTo initErr

    p_strike_geo(0) = Lat
    p_strike_geo(1) = Lon
    p_phi = phi
    If (adjustAxis) Then
        'input major and minor are in KM, and unscaled
        p_major = kmToNmi(CDbl(major))
        p_minor = kmToNmi(CDbl(minor))
    Else
        'input major and minor are in NM and scaled
        p_major = CDbl(major)
    End If
    If p_major = 0 Then
        p_major = 0.05
    End If
    If p_minor = 0 Then
        p_minor = 0.05
    End If
    p_a = applyConfidence(p_major, p_newSigma, p_currentSigma)
    p_b = applyConfidence(p_minor, p_newSigma, p_currentSigma)
Else
    'input major and minor are in NM and scaled
    p_a = CDbl(major)

p_b = CDbl(minor)
If p_a = 0 Then
    p_a = 0.1
End If
If p_b = 0 Then
    p_b = 0.1
End If
p_major = revertConfidence(p_a, p_newSigma, p_currentSigma)
p_minor = revertConfidence(p_b, p_newSigma, p_currentSigma)
End If

p_strike_ecef = geodToECEF(p_strike_geo(0), p_strike_geo(1))
Call setT
p_critical_geo = getEPoint(p_criticalT)
p_aziCenter = getAzimuth(p_poi_geo(0), p_poi_geo(1), p_strike_geo(0), p_strike_geo(1))
p_aziCritical = getAzimuth(p_poi_geo(0), p_poi_geo(1), p_critical_geo(0), p_critical_geo(1))
p_rangeCenter = Haversine(p_poi_geo(0), p_poi_geo(1), p_strike_geo(0), p_strike_geo(1))
p_rangeCritical = Haversine(p_poi_geo(0), p_poi_geo(1), p_critical_geo(0), p_critical_geo(1))
p_rangeDifference = p_rangeCenter - p_rangeCritical
Dim diffs() As Double
Dim point() As Double
diffs = getFocalPointDiffs(p_strike_geo(0), p_strike_geo(1), p_a, p_b, p_phi)
point = addPoints(p Strike_ecef, diffs)
p_foci1 = EcefToGeod(point(0), point(1), point(2))
diffs(0) = -diffs(0)
diffs(1) = -diffs(1)
diffs(2) = -diffs(2)
point = addPoints(p_strike_ecef, diffs)
p_foci2 = EcefToGeod(point(0), point(1), point(2))
Call setProbability
Exit Sub
initErr:
    asd = 0
End Sub

Public Property Get A()
    A = p_a
End Property

Public Property Get B()
    B = p_b
End Property

Public Property Get criticalT()
    criticalT = p_criticalT
End Property
Public Property Get criticalLat()
    criticalLat = p_critical_geo(0)
End Property

Public Property Get criticalLon()
    criticalLon = p_critical_geo(1)
End Property

Public Property Get criticalAzimuth()
    criticalAzimuth = p_aziCritical
End Property

Public Property Get centerAzimuth()
    centerAzimuth = p_aziCenter
End Property

Public Property Get criticalRange()
    criticalRange = p_rangeCritical
End Property

Public Property Get centerRange()
    centerRange = p_rangeCenter
End Property

Public Property Get rangeDifference()
    rangeDifference = p_rangeDifference
End Property

Public Property Get URL()
    URL = p_URL
End Property

Public Property Get Lat()
    Lat = p_strike_geo(0)
End Property

Public Property Get Lon()
    Lon = p_strike_geo(1)
End Property
Public Property Get phi()
    phi = p_phi
End Property

Public Property Get Prob_rician()
    Prob_rician = p_prob_rician
End Property

Public Property Get Prob_numeric()
    Prob_numeric = p_prob_numeric
End Property

Public Property Get Prob_simpson()
    Prob_simpson = p_prob_simpson
End Property

Public Property Get isInside()
    Dim dl As Double, d2 As Double, t0 As Double, d3 As Double, t1 As Double
    dl = Haversine(p_foci1(0), p_foci1(1), p_poi_geo(0), p_poi_geo(1))
    d2 = Haversine(p_foci2(0), p_foci2(1), p_poi_geo(0), p_poi_geo(1))
    t0 = getEPoint(0)
    t1 = getEPoint(pie)
    da = Haversine(t1(0), t1(1), t0(0), t0(1))
    If ((dl + d2) < (da)) Then
        isInside = "Yes"
    Else
        isInside = "No"
    End If
End Property

Private Sub setT()
    Dim thisT As Double, minDist As Double, thisDist As Double, negDist As Double, posDist As Double, returnT As Double, step As Double
    Dim pointLat As Double, pointLon As Double
    returnT = -pie
    point = getEPoint(returnT)
    minDist = Haversine(p_poi_geo(0), p_poi_geo(1), point(0), point(1))

    For thisT = -pie To pie Step pie / 4
        point = getEPoint(thisT)
        thisDist = Haversine(p_poi_geo(0), p_poi_geo(1), point(0), point(1))
        If (thisDist < minDist) Then
            minDist = thisDist
        End If
    Next
End Sub
returnT = thisT
End If
Next thisT
step = pie / 4
While step > pie / (2 ^ 16)
  point = getEPoint(returnT - step)
  negDist = Haversine(p_poi_geo(0), p_poi_geo(1), point(0), point(1))
  point = getEPoint(returnT + step)
  posDist = Haversine(p_poi_geo(0), p_poi_geo(1), point(0), point(1))
  If (negDist < minDist) Then
    returnT = returnT - step
    minDist = negDist
  ElseIf posDist < minDist Then
    returnT = returnT + step
    minDist = posDist
  End If
  step = step / 2
Wend
p_criticalT = returnT
End Sub

Private Function getEPoint(t As Double) As Variant
  Dim ret(1) As Double
  Dim point() As Double
  Dim diffs() As Double
  diffs = getEPointDiffs(p_strike_geo(O), p_strike_geo(1), p_a, p_b, p_phi, t)
  point = addPoints(p_strike_ecef, diffs)
  getEPoint = EceffoGeod(point(O), point(I), point(2))
End Function

Private Function getCPoint(t As Double) As Variant
  Dim ret(1) As Double
  Dim point() As Double
  Dim diffs() As Double
  diffs = getEPointDiffs(p_strike_geo(O), p_strike_geo(I), pJadius, pJadius, pyhi, t)
  point = addPoints(pyoi_ecef, diffs)
  getCPoint = EceffoGeod(point(O), point(I), point(2))
End Function

Public Sub setURL(nPoints As Integer, showPath As Boolean)
  Dim thisT As Double, tStep As Double
  Dim point() As Double
  tStep = 2 * pie / nPoints
  For i = 1 To nPoints
    thisT = -pie + i * tStep
    point = getEPoint(thisT)
    p_URL = p_URL & point(O), point(I), point(2))
Public Sub setProbability()
    On Error GoTo probErr
    'This program computes the probability of a lightning strike occurring within a specified distance of an asset of interest.
    'The cross-section is a circle.

    Dim alpha As Double, LonP As Double, LatP As Double, LonS As Double, LatS As Double
    Dim Xprime As Double, Zprime As Double, SigmaXp As Double, SigmaZp As Double, sigma As Double
    Dim X As Double, Z As Double, theta As Double
    Dim U As Double, V As Double
    Dim m As Integer, AspectRatio As Double

    p_prob_rician = 0
    p_prob_numeric = 0

    If (p_major = 0 Or p_minor = 0) Then
        Exit Sub
    End If

    LatP = toRad(p_poi_geo(0))    'Latitude of POI (radians)
LonP = toRad(p_poi_geo(1)) 'Longitude of POI (radians)
LatS = toRad(p_strike_geo(0)) 'Latitude of Strike Spot (radians)
LonS = toRad(p_strike_geo(1)) 'Longitude of Strike Spot (radians)
alpha = toRad(p PHI) 'Heading of Semi-Major Axis of 50% Confidence Ellipse (radians)

AspectRatio = p_major / p_minor

X = (LonP - LonS) * Cos(LatS)
Z = LatP - LatS

theta = alpha - ((pie / 2) - Atn2(X, Z))

Xprime = p_rangeCenter * Cos(theta)
Zprime = p_rangeCenter * Sin(theta)

SigmaXp = p_major / 1.177410023
SigmaZp = p_minor / 1.177410023

sigma = Sqr(SigmaXp * SigmaZp)

'Compute Rician Integral
'U = (p_radius / sigma) ^ 2
'V = Xprime ^ 2 / SigmaXp ^ 2 + Zprime ^ 2 / SigmaZp ^ 2
'm = Int(Sqr(U * V)) + 1
'p_prob_rician = RicianIntegral(U, V, m)

'skip numeric method if aspect ratio is low
If (m < 25 And AspectRatio < 10) Then
  Exit Sub
'End If

'Use numerical method for computing lightning strike probability:
'p_prob_numeric = numericIntegral(Xprime, Zprime, SigmaXp, SigmaZp, p_rangeCenter, p_radius, 1000)

  p_prob_simpson = simpsonIntegral(Xprime, Zprime, SigmaXp, SigmaZp, p_radius, 200)
Exit Sub

probErr:

End Sub
References


