An Improved Model of Cryogenic Propellant Stratification in a Rotating, Reduced Gravity Environment

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OVERVIEW: UPPER STAGE MODELING

Lockheed Martin
Atlas V 401

Boeing Delta IV Heavy


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OVERVIEW: WHAT CAN HAPPEN INSIDE TANKS?

- Stage exposed to solar heating
- Propellants (LH$_2$ and LOX) may thermally stratify
- Propellants may boil
- Slosh events during maneuvers
- Upper stage must re-start at conclusion of coast phase for insertion

XSS-10 view of Delta II rocket: An Air Force Research Laboratory XSS-10 micro-satellite uses its onboard camera system to view the second stage of the Boeing Delta II rocket during mission operations Jan. 30. (Photo courtesy of Boeing.), http://www.globalsecurity.org/space/systems/xss.htm
OVERVIEW: WHAT CAN HAPPEN INSIDE TANKS?

- Propellant T&P must be within specified range for turbomachinery operation
  - If propellants outside specified T&P box engine may not restart
  - Orbit cannot be circularized
MOTIVATION

- Rotation present during missions to evenly heat spacecraft
- Effect rotation has on propellant thermal properties unknown
- Upgrade current analytical/numerical stratification models to include rotation
MISSION PARAMETER RANGES

- Tank Dimensions:
  - Square 3 m diameter tanks
- Cryogenics: LH₂, LOX
  - \( T_{\text{bulk}} \) LH₂: 16 K, 28.8 °R, -430.9 °F
  - \( T_{\text{bulk}} \) LOX: 91 K, 163.8 °R, -295.9 °F
- Tank Pressure (All Cryogenics): 30 psi
- Initial Fill Levels: 10, 20, 30%

- Heating Conditions:
  - Constant wall temperature: \( \theta = T_{\text{wall}} - T_{\text{bulk}} \): \( \Delta T = 0.1, 0.5, 1.0 \) K
  - Heat flux to fluid: 5-100 W/m²

- Reduced Gravity Environment: \( g/g_0 = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 1 \)
- Rotation rates: \( \omega = 0.1, 1, 5 \) °/sec
- Orbital Transfer Time (Simulation Time): 2 – 4 HR
MASTER MODEL: BASIC FRONT END OPTIONS

1. Tank geometry
   - Tank diameter, height
   - Square bottom

2. Boundary layer nature and heat transfer coefficient selection
   - Free convection
     • Laminar/Turbulent (w/ & w/out switching)

3. Wall temperature settings
   - Constant inner wall temperature
   - Constant inner wall heat flux

4. Rotation rate

5. Gravity level
GENERAL MODELING PHILOSOPHY

- Stratum growth $\Delta(t)$
  - $u(y)$ depends on if heating is constant wall temperature or constant heat flux, $q$
  - $u(y)$ depends on nature of boundary layer
  - Provides differential equation for $\Delta(t)$

- Stratum temperature, $T_s(t)$
  - Heat entering side wall into boundary layer is used to increase stratum temperature
  - Energy exchange with ullage negligible
  - $T_s$ assumed uniform

\[ m_{bl} = 2\pi R \rho \int_0^\delta u(y) dy = \rho \pi R^2 \frac{d\Delta}{dt} \]

\[ \dot{q} 2\pi RH = \rho \pi R^2 \Delta c_p \frac{dT}{dt} \]
# RELEVANT NON-DIMENSIONAL NUMBERS

- Grashof number, $Gr$, governs heat transfer regime for constant wall temperature
  - Ratio of buoyancy to viscous forces
  - $\beta$, Volumetric thermal expansion coefficient
  - $\theta$, Wall to Bulk temperature difference

$$Gr = \frac{g\beta\theta L^3}{\nu^2}$$

- Rayleigh number, $Ra$, is product of Grashof and usual Prandtl number, $Pr$

$$Ra = Gr \cdot Pr$$

- Prediction of boundary layer transition
  - If $Ra < 10^9 \rightarrow$ Laminar
  - If $Ra > 10^9 \rightarrow$ Turbulent

$$Gr^* = \frac{g\beta q_w L^4}{kv^2}$$

- Modified Grashoff number, $Gr^*$, governs heat transfer regime for uniform heat flux, $q_w$

$$Ra^* = Gr^* \cdot Pr$$

- Modified Rayleigh number, $Ra^*$, for uniform heat flux

- Others: Reynolds Number, $Re$ (momentum to viscous), Weber number, $We$ (inertial to capillary), Froude number, $Fr$ (inertial to body), and Bond number $Bo$ (body to capillary)
Ra and Ra* vs. g/g₀ MAPS for LH₂ and LOX

- Maps laminar or turbulent boundary layers possible for typical mission profiles (NIST data)

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WHAT IS IMPACT OF BBQ ROLL ROTATION:

Typical rotation rate, $\omega \sim 1^\circ/\text{sec}$

Shape at $g/g_0=1$, $\omega \sim 850^\circ/\text{sec}$

- Does $1^\circ/\text{sec}$ matter?
- Not at $g/g_0=1$ but in coast phase $g/g_0 \sim 10^{-4}$
  $\rightarrow$ significant dishing effect

Key Question: How does rotation impact results?
- Assume liquid is in solid body rotation (transients can also be treated)
- Model extra height that liquid gains along wall as a longer interfacial heat transfer length
- Center point in radial direction of tank is taken to be point where percent of bulk remaining is referenced $\rightarrow$ worst case scenario
- Trade off between heated area and surface area to distribute warm stratum
\[ h = \frac{(\omega R)^2}{2g} \]

Spin Rate, \( \omega = 1^\circ/\text{sec} \)

- \( g/g_0 < 10^{-3} \)
- \( g/g_0 = 10^{-4} \)
- \( g/g_0 = 10^{-5} \)

Note: Parabola bottoms out
ROTATIONAL CASES

Re-examine boundary layer / stratum mass balance

\[
\dot{m}_{bl} = \pi R^2 \rho \left( \frac{d\Delta}{dt} \right) = S_{paraboloid} \rho \left( \frac{d\Delta}{dt} \right) = \frac{\pi R}{6h^2} \left[ \left( R^2 + 4h^2 \right)^{3/2} - R^3 \right] \rho \left( \frac{d\Delta}{dt} \right)
\]

Turbulent

\[
\frac{\Delta(t)}{H_\omega} = 1 - \left[ 1 + 0.082 \frac{H_\omega \pi R}{S_p} \left( Gr^* \right)^{2/7} \left( \frac{1}{Pr} \phi \right) \right]^{-7}
\]

Laminar

\[
\frac{\Delta(t)}{H_\omega} = 1 - \left[ 1 - 0.616 \frac{H_\omega \pi R}{S_p} \left( Gr^* \right)^{1/5} \left( \frac{4}{5 + Pr} \right) \right]^{5} \frac{1}{Pr^{3/5} \phi}
\]

\[
Gr^* = \frac{g \beta q_w (H + \frac{h}{2})^4}{k \nu^2} = \frac{g \beta q_w (H_\omega)^4}{k \nu^2}
\]

\[
\phi = \frac{\nu t}{H_\omega^2}
\]

Re-derive energy balance to take into account additional heating area

\[
\dot{q} 2\pi R \left( H + \frac{h}{2} \right) = \dot{q} 2\pi R H_\omega = \rho \pi R^2 \Delta c_p \frac{dT}{dt}
\]
COMBINED ROTATION / STRATIFICATION MODEL: LH$_2$ and LOX

For q=10 W/m$^2$, L=3, R=1.5, 20% fill level, H/R=0.4 and $\omega=1^\circ$/sec at g/g$_0$=10$^{-4}$:

- Rotation decreases time to stratification time by $\sim$ 15%
- Rotation increases stratification temperature by $\sim$ 1.0 K
EFFECTS OF ROTATION AND TRADEOFFS

- Increased boundary layer running length ($H \rightarrow H_\omega$)
  - more heated area
  - larger Grashof number

- Larger surface area at bulk-stratum interface ($S \rightarrow S_{\text{paraboloid}}$)
  - increased mass flow rate into stratum layer
  - more area to spread mass flow
EFFECTS OF ROTATION

- Spinning always increases stratification
- Stratum temperature affected by spin rate; especially at low gravity levels
- LOX cases shown with heat flux of 5 W/m² after 2 hour mission
EFFECTS OF ROTATION

- $\omega_{\text{critical}} \rightarrow$ spin rate to minimize stratum temperature
- $\omega_{\text{critical}}$ needed for large $g/g_o$ impractical
- $\omega_{\text{critical}}$ needed for typical mission profiles very practical ($\omega < 1.5 \, \text{deg/s}$)
- LOX results discussed previously shown

![Graph showing effects of rotation with $g/g_o = 10^{-3}$ and $g/g_o = 10^{-4}$]
SUMMARY/ CONCLUDING REMARKS

• Thermal stratification impacts T&P at conclusion of coast phase

• Rotation (creeping of fluid up side walls) has large effect for $\omega=1^\circ/s$ and $g/g_0=10^{-4}$
  – ‘Classical’ literature model upgraded to include rotation effects
  – Can decrease time to stratify by 30-60 minutes during 4 hour coast
  – Larger heating area and lower liquid level above sump inlet
  – For various missions stratum temperature may increases or decrease relative to no-spin case
  – Mixed tank temperatures always larger because $\Delta$ increased with rotation

• Future work
  – Comparison with CFD studies
SELECTED REFERENCES

Literature Review References:

Web-based References for Graphics:
• http://www.skyrocket.de/space/index_frame.htm
• http://www.boeing.com/defense-space/space/delta/delta4/d4h_demo/book01.html

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