Abstract

Pad B Liquid Hydrogen Storage Tank
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Kennedy Space Center is home to two liquid hydrogen storage tanks, one at each launch pad of Launch Complex 39. The liquid hydrogen storage tank at Launch Pad B has a significantly higher boil off rate that the liquid hydrogen storage tank at Launch Pad A. This research looks at various calculations concerning the at Launch Pad B in an attempt to develop a solution to the excess boil off rate. We will look at Perlite levels inside the tank, Boil off rates, conductive heat transfer, and radiant heat transfer through the tank. As a conclusion to the research, we will model the effects of placing an external insulation to the tank in order to reduce the boil off rate and increase the economic efficiency of the liquid hydrogen storage tanks.
Pad B Liquid Hydrogen Storage Tank

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Reviewed by NASA-USRP Mentor
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The Liquid Hydrogen storage tank at Launch Pad B has a significantly higher boil off rate than the Liquid Hydrogen storage tank at Launch Pad A. Both tanks were built in 1965 by the Chicago Bridge and Iron Company of Salt Lake City, Utah\(^1\); however, the tank at Pad B has consistently had a boil off rate more than double that of the tank at Pad A. The tanks are constructed to keep the liquid hydrogen extremely cold; consequently, there is insulation between the inner tank and outer tank. The insulation that was used for the construction of the tanks is Perlite, which has a powdered texture. The outside of the tank at Pad B has also developed a cold spot where we believe there may be a Perlite void. We will look at several calculations in order to better understand the make up of the tank, its dimensions, and to explain why there is such a significant difference in the boil off rates between the two tanks. We believe that refurbishing the tank is not the only solution to the existing boil off problem; consequently, this research will attempt determine if coating the cold spot on the outside of Tank B with Spray on Foam Insulation will reduce the boil off rate and increase the economic efficiency of the tank.

The tanks are constructed so that there is an inner tank and an outer tank. The inner tank, made of stainless steel, sits suspended on supports inside of the outer tank, made of carbon steel. Between these two tanks, the powdered insulation, Perlite, is pumped in and then a vacuum is created. On the outside of Tank B, there is a cold spot which tells us that there is heat escaping the tank from that area at a higher rate than at any other area on the tank. It is our belief that there exists a Perlite void at that location between the inner tank and the outer tank and that a possible solution is to use the Spray on Foam Insulation to lower the temperature on the outside of the tank thus reducing the temperature flow inside the tank leading to a lowered boil off rate.

As a beginning to this research, we will develop a formula that will allow us to convert the liquid volume readings to liquid level inside the tank in meters. This will allow us to better understand how the boil off rate relates to the liquid levels in the tank. This calculation applies to both tanks and therefore can be used for further analysis of related topics. The following

\(^{1}\text{Berg, Mark. Heat Transfer Analysis of the LC-39 Liquid Hydrogen Storage Tanks. Powerpoint Presentation pg. 4} \)
equation allows us to go from volume to the height in the tank:

\[ \pi h^3(r - \frac{h}{3}) = V \]

Where:

- \( h \) = The height from the top of the liquid to the top of the tank in meters.
- \( r \) = The Radius of the tank.

See Appendix A for the derivation of this formula. We can now fill in the value of the radius \(^2\) and use the resulting formula to solve for various heights given specific volumes inside the tank with a mathematical computer program:

\[ \pi h^3(9.35 - \frac{h}{3}) = V \]

After the tanks were constructed, the inner tank cooled as the liquid hydrogen was added to the tank. During this cooling process, the inner tank contracted which caused a decrease in the volume of the inner tank. The Perlite between the tanks dropped during this process relative to the change in volume. In order to determine the amount the Perlite may drop due to the change in volume of the inner tank during cooling, it is necessary to calculate the change in volume of the inner tank due to thermal expansion. One assumption that we make with this calculation is that the Perlite acts as a liquid between the tanks. Given this assumption, the value that we get will be a slight overestimate. First, we start with the volume change of the inner tank. The following equation is the thermal expansion equation for stainless steel \(^3\):

\[ \frac{L_T - L_{293}}{L_{293}} = (a + bT + cT^2 + dT^3 + eT^4) \cdot 10^{-5} \]

Where:

\[ a = -2.9546 \times 10^2 \]
\[ b = -4.0518 \times 10^{-1} \]

\(^2\)Berg, Mark. Heat Transfer Analysis of the LC-39 Liquid Hydrogen Storage Tanks. Powerpoint Presentation, pg. 21

\(^3\)Cryogenic Material Properties Database; E.D. Marquardt, J.P.Le, and Ray Radebaugh; National Institute of Standards and Technology; June 2000
\[ c = 9.4010 \times 10^{-3} \]
\[ d = -2.10989 \times 10^{-5} \]
\[ e = 1.8780 \times 10^{-8} \]
\[ T = \text{Temperature, (K)} \]
\[ L = \text{Length at a given temperature in meters.} \]

The tank cools down from room temperature (293K) to 20K as it is filled with the liquid Hydrogen. Therefore, the thermal expansion formula yields:

\[ \frac{L_T - L_{293}}{L_{293}} = -3.00 \times 10^{-3} \text{m}. \]

The radius of the inner tank at room temperature \(^4\) is given as: \( L_{293} = 9.373 \text{m} \). Using the previous result to solve for \( L_{20} \) yields a new radius of 9.345m.

The volume of the inner tank at room temperature (\( V_0_i \)) is:

\[ V = \frac{4}{3} \pi r^3 \]
\[ V_0_i = \frac{4}{3} \pi (9.373m)^3 \]
\[ V_0_i = 3449.25 \text{m}^3 \]

After the tank is cooled to 20K, the new volume of the tank (\( V_{1i} \)) is:

\[ V_{1i} = \frac{4}{3} \pi (9.345m)^3 \]
\[ V_{1i} = 3418.43 \text{m}^3 \]

Consequently, the change in volume due to cooling of the tank (\( \Delta V \)):

\[ \Delta V = V_{0i} - V_{1i} \]

\(^4\)Berg, Mark. Heat Transfer Analysis of the LC-39 Liquid Hydrogen Storage Tanks. Powerpoint Presentation pg. 21
\[ \Delta V = 30.82\,m^3 \]

\( \Delta V \) can also be calculated using the volume between the tanks:

\[ V_{0o} = \frac{4}{3} \pi (10.65\,m)^3 \]

\[ V_{0o} = 5059.85\,m^3 \]

Where \( V_{0o} \) is equal to the volume of the outer tank.\(^5\) Then the volume between the tanks at 293K is equal to:

\[ V_{ob} = V_{0o} - V_{0i} \]

\[ V_{ob} = 1610.6\,m^3 \]

and

\[ V_{1b} = V_{0o} - V_{1i} \]

\[ V_{1b} = 1641.42\,m^3 \]

Where \( V_{ob} \) is equal to the initial volume between the tanks before cooling and \( V_{1b} \) is equal to the volume between the tanks after cooling.

So the change in volume between the tanks (\( \Delta V \)) is:

\[ \Delta V = V_{1b} - V_{0b} \]

\[ \Delta V = 30.82\,m^3 \]

Which is the same \( \Delta V \) as the previous calculations.

Now that we know how much the volume changed due to the cooling of the inner tank, we can use the same formula that we used for the volume to liquid level conversion to determine if the perlite dropped below the top of the inner tank. It should be noted that this formula only works for this calculation if the Perlite drops an amount less than the distance between the

\(^5\)Berg, Mark. Heat Transfer Analysis of the LC-39 Liquid Hydrogen Storage Tanks. Powerpoint Presentation pg. 21
tanks. This calculation also applies to the tanks at both Pad A and Pad B. Beginning with the conversion equation:

\[ \pi h^3 (r - \frac{h}{3}) = V \]

we can use the dimensions for the outer tank and volume change of the inner tank to determine if the Perlite drops below the top of the inner tank. We let \( r = 10.64 \text{m} \), the radius of the outer tank, and we know the change in Volume is \( 30.82 \text{m}^3 \). This yields the following equation:

\[ \pi h^3 (10.64 \text{m} - \frac{h}{3}) = 30.82 \text{m}^3 \]

Now, using mathematical computer software such as Mathematica, we solve for \( h \) and find that if the Perlite acts as a liquid, it would drop an amount equal to 1.0m. The distance between the two tanks is equal to 1.3m; therefore, the top of the inner tank was not exposed due to the contraction of the inner tank. Since we found that the Perlite dropped less than the distance between the tanks, using this method works for this case.

An essential assumption made for this research is that there is a Perlite void between the tanks at the area of the cold spot. In order to demonstrate the validity of this assumption, we looked at the excess boil off rates of Pad B when compared to Pad A. Building on the research of Mark Berg, we take the excess boil off rate and compare it to the formulas for conductive heat transfer and radiant heat transfer. With this data, we can produce valid evidence that supports the claim that the cause of the higher boil off rated is due to radiant heat transfer. First, we tried to produce a constant, \( k \), that would explain the heat transfer for both cases. However, we found that the constant does not fit the model if there is solely conductive heat transfer through the tank. This comparison explains why the darker grey tank had a much higher boil off rate than the white tank. The following chart was produced that demonstrates the difference in the conductive heat transfer and the radiant heat transfer:

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6Berg, Mark. Heat Transfer Analysis of the LC-39 Liquid Hydrogen Storage Tanks. Powerpoint Presentation, pg. 21
Given these temperatures, it is not possible to produce a constant that explains the higher boil off rate for the tank at Pad B using only the conductive heat transfer.

Further evidence that there is a Perlite void at the Pad B tank can be seen in inferred images. Appendix B is a thermal image of the LH2 tank at Pad B. This image can be compared to Appendix C which is an inferred image taken of scale models of the liquid hydrogen tanks where Perlite voids have been created. They appear to have similar qualities the lead us to believe there is a Perlite void at Pad B.

In order to demonstrate the differences between the insulation provided by Perlite and that of a Blackbody, we will first exam the effects of placing SOFI on the outside of the tank if there exists Perlite between the outer and inner tanks. We will later compare this result to the effects of placing SOFI on the tank where there exists a void. By calculating the amount of heat transfer through the Perlite, we can demonstrate how adding varying thicknesses of SOFI to the tank will affect the overall heat transfer through the tank. We will use the following simplified model of the tank with Perlite:

<table>
<thead>
<tr>
<th>Heat Transfer Type</th>
<th>Conductive</th>
<th>Radiative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$k(To-T_i)=618.31$</td>
</tr>
<tr>
<td></td>
<td>$k(300\times20)=618.31$</td>
<td>$k(333\times20)=940.27$</td>
</tr>
<tr>
<td></td>
<td>$k=2.21$</td>
<td>$k=7.634E-8$</td>
</tr>
</tbody>
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<td></td>
<td>$k=2.21$</td>
<td>$k=3.004$</td>
</tr>
</tbody>
</table>

Given these temperatures, it is not possible to produce a constant that explains the higher boil off rate for the tank at Pad B using only the conductive heat transfer.
In this model, the outer insulation is Spray On Foam Insulation (SOFI). The temperature of the middle of the two insulations varies with the thickness of the SOFI. Therefore, the thickness of the SOFI is used as the variable \( L \). Where:

\[
T_o = \text{temperature on the outside of the SOFI (300K)}
\]

\[
T_m = \text{temperature(K) between the two insulations, variable temperature}
\]

\[
T_i = \text{The temperature on the inside of the inner tank (20.28K)}
\]

\[
x_2 - x_1 = \text{The thickness(m) of the Perlite between the tanks (1.308m)}
\]

\[
x_3 - x_2 = L = \text{The thickness(m) of the SOFI, variable distance}
\]

The equation 7 for the energy transfer for a homogeneous substance is given as:

\[
q = -kA\nabla T \cdot \hat{n}
\]

Where:

\[
q = \text{energy transfer rate}
\]

\[
\nabla T \cdot \hat{n} = \text{temperature gradient in the direction normal to the area, A}
\]

\[
k = \text{thermal conductivity of insulation}
\]

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\(^7\) Pitts, Donald. *Heat Transfer*. McGraw-Hill Book Company. 1977. pg. 1
Since this is a one dimensional case, $\nabla T \cdot \hat{n}$ becomes $\frac{\Delta T}{\Delta x}$. Therefore, assuming a linear model for the heat transfer allows the following substitution:

$$\frac{\Delta T}{\Delta x} = \frac{T_2 - T_1}{x_2 - x_1}$$

This produces the new equation:

$$q = -kA\left(\frac{T_2 - T_1}{x_2 - x_1}\right)$$

With the simplified model, the energy transfer rate from the outside of the SOFI to the middle and the energy transfer rate from the middle to the inner side of the Perlite, must be equal. Consequently, in order to solve for the varying temperature and thickness of SOFI, it is permissable to set the equations for the two types of insulation equal to each other. This yields:

$$-k_{sofi}A\left(\frac{T_o - T_m}{x_3 - x_2}\right) = -k_{perl}A\left(\frac{T_m - T_1}{x_2 - x_1}\right)$$

Where:

$$k_{sofi} = 2.4 \times 10^{-2} \frac{w}{mK}$$

$$k_{perl} = 1.5 \times 10^{-3} \frac{w}{mK}$$

Filling in all the values:

$$-2.4 \times 10^{-2}\left(\frac{300 - T_m}{L}\right) = -1.5 \times 10^{-3}\left(\frac{T_m - 20.28}{4.29}\right)$$

$$T_m = \frac{7.09 \times 10^{-3} + \frac{7.2}{L}}{3.50 \times 10^{-4} + \frac{2.4 \times 10^{-2}}{L}}$$

The following graph shows the thickness of SOFI ranging from 0-10m. and temperature ranging from 0-300K.
It is evident that the temperature changes slowly with the thickness of the SOFI when there is Perlite between the tanks.

Now we will develop a new model, removing the Perlite from the model and replacing it with a blackbody state. Since our previous calculations suggest that this is a more probable explanation of the heat transfer through the cold spot, we can examine the effects of adding SOFI to a tank with these conditions.

For these calculations, we will use the formula for heat transfer in a blackbody. The derivation of this formula can be found in Appendix D. In order to examine the heat transfer through the tank caused by radiant heat, we will assume that the outer tank has an emissivity of 0.2 and the inner tank has an emissivity of 0.8. Using the heat transfer formula in Appendix D, the net emissivity of the tank is 0.19. Now, as with the previous calculations, we
set the two heat transfer formulas equal to each other to determine the effect of the SOFI on the heat transfer for the tank:

\[ k_{sofi} A \frac{T_o - T_m}{x_3 - x_2} = \frac{\varepsilon_1 \varepsilon_2 (\sigma T_m^4 - \sigma T_i^4)}{\varepsilon_2 + \varepsilon_1 (1 - \varepsilon_2)} \]

By filling in the values that we know, we get the following equations:

\[ 2.4 \times 10^{-2} \left( \frac{300 - T_m}{L} \right) = .19 \times (5.67 \times 10^{-8})(T_m^4 - 20^4), \]

Now we can solve this equation for \( L \) and produce plots that will relate the thickness of the SOFI on the outside of the tank to the temperature of the outer tank \( (T_m) \).

\[ L = \frac{2.4 \times 10^{12} - 8 \times 10^9 T_m}{-5.7456 \times 10^8 + 3591 T_m^4} \]

In order to compare the effect of adding SOFI to the outside of a tank that has Perlite to the effect that it has on a tank with a Perlite void, we plot the two graphs together:

![Graph showing the effect of SOFI on tank temperature](image)

It appears evident from this plot that adding SOFI to the outside of the tank would reduce the temperature of the outer tank by a greater amount.
if there exists a Perlite void. This would also lower the amount of heat that is transferred through the outer tank to the inner parts of the tank. The plot also demonstrates that SOFI would not have as much of an effect on areas of the tank where the perlite is in place.

There is much more research to be conducted in this area. Using scale models of the LH2 tanks to test this hypothesis would be a good direction for future research. Another area that needs examining is a more complex analysis of the heat transfer taking into consideration the heat transfer through the steel and into the outer edges of the SOFI.
Appendix A:

To find an equation to go from the volume of a sphere to the liquid level inside the sphere, we begin with the general general equation of a circle:

\[ x^2 + y^2 = r^2 \]

From this, we solve for \( x^2 \):

\[ x^2 = r^2 - y^2 \]

Now, we can use an integral to represent the volume:

\[ \pi \int_{r-h}^{r} r^2 - y^2 dy = V \]

This becomes:

\[ \pi (r^2 y - \frac{y^3}{3})|_{r-h}^{r} = V \]

Now, filling in the limits of integration, yields:

\[ \pi [r^3 - \frac{r^3}{3} - (r^2 - (r - h) - \frac{(r - h)^3}{3})] = V \]

Simplifying:

\[ \pi [\frac{2}{3}r^3 - \frac{1}{3}(r - h)[3r^3 - (r - h)^2]] = V \]

Now, we can factor out \( \frac{1}{3} \):

\[ \frac{\pi}{3} [2r^3 - (r - h)[3r^3 - (r^2 - 2rh + h^2)]] = V \]

Which is equal to:

\[ \frac{\pi}{3} (2r^3 - 2r^3 - 2r^2h + rh^2 + 2r^2h + 2rh^2 - h^3) = V \]

Combining like terms yields:

\[ \frac{\pi}{3} (3rh^2 - h^3) = V \]

Factoring out the \( h^2 \):

\[ \frac{\pi h^2}{3} (3r - h) = V \]

And this simplifies to our equation for converting volume to height:

\[ \frac{\pi h^2}{3} \left( r - \frac{h}{3} \right) = V \]
Appendix B

IR Photograph of Pad B LH2 Dewar
Appendix C: IR Photographs of Test Dewars in Cryogenics Test Bed
Left: Void in Perlite, No Vacuum, >90% Liquid Level
Right: Bubbles Tank, Soft Vacuum (6 tor), 90% Liquid Level

Perlite Voids Near Top, No Vacuum 94% Liquid Level

Perlite Void(s)

Anti-Rotation Supports

07-03-05 11:20:38 e=0.84

Perlite Voids Near Top, No Vacuum 94% Liquid Level

Full Perlite Level, Hard Vacuum (3 micron), 93% Liquid Level

OK Reference image

07-02-06 08:46:14 e=0.84

Perlite Void(s)

07-03-05 13:34:17 e=0.84

Perlite Voids Near Top, No Vacuum 93% Liquid Level
Appendix D:

The following is the derivation of the heat transfer equation for a black-body:

The net energy lost from surface 1 ($E_{1_{\text{net}}}$) is given by the directly radiated energy minus the reabsorption of the reflected energy and the corresponding radiation from surface 2. $E$ is the radiant energy flux per unit area and is given by the Stephen-Boltzmann Law $E = \epsilon \sigma T^4$. Lower case epsilon, $\epsilon$ is the surface emissivity and $\sigma$ is the Stephen-Boltzmann constant of $5.67 \times 10^{-8} \frac{W}{m^2K^4}$. Rho, $\rho$ is the surface reflectivity. The relationship between $\rho$ and $\epsilon$ for an opaque surface is $\rho + \epsilon = 1$.

$$E_{1_{\text{net}}} = E_1 - E_1\epsilon_1\rho_2 - E_1\epsilon_1\rho_2\rho_1\rho_2 - E_1\epsilon_1\rho_2\rho_1\rho_2\rho_1 \ldots -(E_2\epsilon_1 + E_2\epsilon_1\rho_2\rho_1 + E_2\epsilon_1\rho_2\rho_1 \rho_2 \rho_1 \ldots)$$

Rewriting the formula in summation notation yields:

$$= [E_1(1 - \epsilon_1 \sum_{n=1}^{\infty} \rho_2^n \rho_1^{n-1})] - [E_2\epsilon_1(\sum_{n=0}^{\infty} \rho_2^n \rho_1^n)]$$

Changing the index on the first term and simplifying the second term:

$$= [E_1(1 - \epsilon_1 \sum_{n=0}^{\infty} \rho_2^{n+1} \rho_1^n)] - [E_2\epsilon_1(\sum_{n=0}^{\infty} (\rho_2\rho_1)^n)]$$
Simplifying the first term:

\[ [E_1(1 - \epsilon_1 \sum_{n=0}^{\infty} \rho_2^n \rho_1^n)] - [E_2 \epsilon_1 \sum_{n=0}^{\infty} (\rho_2 \rho_1)^n] \]

Pulling out the constant \( \rho_2 \):

\[ [E_1(1 - \epsilon_1 \rho_2 \sum_{n=0}^{\infty} (\rho_2 \rho_1)^n)] - [E_2 \epsilon_1 \sum_{n=0}^{\infty} (\rho_2 \rho_1)^n] \]

Using the identity \( 1 + x + x^2 + x^3 \ldots x^n = \frac{1}{1-x} \) for \( |x| < 1 \), the formula becomes:

\[ [E_1(1 - \epsilon_1 \rho_2 \left( \frac{1}{1 - \rho_2 \rho_1} \right))] - [E_2 \epsilon_1 \left( \frac{1}{1 - \rho_2 \rho_1} \right)] \]

Distributing:

\[ [E_1(1 - \epsilon_1 \rho_2 \left( \frac{1}{1 - \rho_2 \rho_1} \right))] - \frac{E_2 \epsilon_1}{1 - \rho_2 \rho_1} \]

Obtaining a common denominator for the first term:

\[ [E_1 \left( \frac{1 - \rho_2 \rho_1 - \epsilon_1 \rho_2}{1 - \rho_2 \rho_1} \right)] - \frac{E_2 \epsilon_1}{1 - \rho_2 \rho_1} \]

Distributing:

\[ \frac{E_1 - E_1 \epsilon_1 \rho_2 - E_1 \epsilon_1 \rho_2 - E_2 \epsilon_1}{1 - \rho_2 \rho_1} \]

Substituting \( \rho_1 = 1 - \epsilon_1 \) and \( \rho_2 = 1 - \epsilon_2 \):

\[ \frac{E_1 - E_1 (1 - \epsilon_2)(1 - \epsilon_1) - E_1 \epsilon_1 (1 - \epsilon_2) - E_2 \epsilon_1}{1 - [(1 - \epsilon_2)(1 - \epsilon_1)]} \]

Distributing:

\[ \frac{E_1 - E_1 + E_1 \epsilon_1 + E_1 \epsilon_2 - E_1 \epsilon_1 \epsilon_2 - E_1 \epsilon_1 + E_1 \epsilon_1 \epsilon_2 - E_2 \epsilon_1}{1 - (1 - \epsilon_1 - \epsilon_2 + \epsilon_2 \epsilon_1)} \]
Canceling terms yields:

\[ \frac{E_1 \varepsilon_2 - E_2 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \]

Simplifying the denominator:

\[ \frac{E_1 \varepsilon_2 - E_2 \varepsilon_1}{\varepsilon_2 + \varepsilon_1 (1 - \varepsilon_2)} \]

Substituting \( E_1 = \varepsilon_1 \sigma T_1^4 \) and \( E_2 = \varepsilon_2 \sigma T_2^4 \):

\[ \frac{\varepsilon_1 \sigma T_1^4 \varepsilon_2 - \varepsilon_2 \sigma T_2^4 \varepsilon_1}{\varepsilon_2 + \varepsilon_1 (1 - \varepsilon_2)} \]

Simplifying:

\[ \frac{\varepsilon_1 \varepsilon_2 (\sigma T_1^4 - \sigma T_2^4)}{\varepsilon_2 + \varepsilon_1 (1 - \varepsilon_2)} \]

This formula can be found in cryogenic texts such as Flynn, Thomas; Cryogenic Engineering pg. 363.