EMPIRICAL TESTS OF ACCEPTANCE SAMPLING PLANS

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Abstract

Acceptance sampling is a quality control procedure applied as an alternative to 100% inspection. A random sample of items is drawn from a lot to determine the fraction of items which have a required quality characteristic. Both the number of items to be inspected and the criterion for determining conformance of the lot to the requirement are given by an appropriate sampling plan with specified risks of Type I and Type II sampling errors. In this paper, we present the results of empirical tests of the accuracy of selected sampling plans reported in the literature. These plans are for measureable quality characteristics which are known have either binomial, exponential, normal, gamma, Weibull, inverse Gaussian, or Poisson distributions. In the main, results support the accepted wisdom that variables acceptance plans are superior to attributes (binomial) acceptance plans, in the sense that these provide comparable protection against risks at reduced sampling cost. For the Gaussian and Weibull plans, however, there are ranges of the shape parameters for which the required sample sizes are in fact *larger* than the corresponding attributes plans, dramatically so for instances of large skew. Tests further confirm that the published inverse-Gaussian (IG) plan is flawed, as reported by White and Johnson (2011).

Keywords: Quality and reliability, inspection, sampling, requirements

Introduction

Acceptance sampling by attributes (ASA) assesses the quality of a lot based on the number of nonconforming items discovered in a random sample drawn for inspection. Inspection requires only a pass/fail determination for each item, where the characteristic is necessarily binomially distributed. Because it is conceptually straightforward, easily to implement, and can be applied to qualitative as well as quantitative performance measures, ASA is the first choice for sampling inspection.

For quantitative performance measures, however, a pass/fail determination typically is accomplished by comparing the measured value to a limiting value, without regard to magnitude of conformance or nonconformance for each item tested. It seems reasonable that this additional information might be exploited to decrease the number of items that need to be inspected. This is the rationale behind acceptance sampling by *variables* (ASV). If there is adequate information to posit a distribution for the measure, then in many instances the ASV alternative translates into significantly smaller samples to achieve the same operating characteristic. While far more restrictive in its assumptions, ASV should be considered when larger samples required by ASA are an issue.

The objective of this paper is to provide an independent assessment of the accuracy of variables plans reported in the literature. Note that, with the exception of normal plans, our search for off-the-shelf ASV plan calculators was essentially fruitless. This scarcity strongly suggests that non-normal ASV largely has been limited to an academic audience and not fully vetted in practice. The need to implement and test plans reported in the literature is especially important for those plans based on approximations. We report here on the results of empirical tests conducted using spreadsheet implementation of calculators developed for this purpose.

1. Test protocol

The following sampling plans were implemented as spreadsheet calculators and tested empirically using Monte Carlo simulation: binomial (White *et al.*, 2009), exponential (Guenther, 1977), normal (Bowker and H. P. Goode, H.P., 1952; Guenther, 1977; Kao, 1971; G. J. Lieberman and G.J Resnikoff; and Montgomery, 2005; among others), gamma (Takagi,1972), Weibull(Takagi,1972), inverse Gaussian Aminzadeh (1996), and Poisson (Guenther, 1977). The test protocol enforced a limit standard (I, ρ, β) with (1) a specification limit on the measured variable X , either $x_{\min}$ or $x_{\max}$, as the performance indicator I, (2) minimum reliabilityρ=0.005, and (3) maximum consumer's risk β =0.100 (the risk of accepting a nonconforming lot). Additionally, maximum producer's risk of α=0.200 (the risk of rejecting a conforming lot) was enforced. We chose these particular test conditions as representative of certain high-level requirements in the design of spacecraft (White, *et al.*, 2009).

For each test, values were specified for the limit and for any distribution parameters assumed to be known or estimated. The null and alternative means μ_0 and μ_1 were determined such that, for a lower limit, $F(x_{min};\mu_0)$=0.001 and $F(x_{min};\mu_1)$=0.005. For an upper limit, 1-$F(x_{max};\mu_0)$=0.999 and 1-$F(x_{max};\mu_1)$=0.995 . The corresponding (n,κ) sampling plan was then determined from the appropriate calculator.

100,000 Monte Carlo trials were run for both the null and alternative distributions, each run comprising n observations as determined by the sampling plan. The proportions $\hat{\alpha}$ and $\hat{\beta}$ were estimated from the sampling distribution of the acceptance limit $A(n,k)$. These estimates were compared to the specified operating characteristic to assess the accuracy of the plan. The

efficiency of the variable sampling plan was determined by comparing the required sample size to that for the closest attributes sampling plan.

2. Test results

Test results are summarized in Tables 1-3. The last column in each table is the ratio of the sample size for the variables plan to the sample size for the corresponding attributes (binomial) plan.

Table 1. Summary of test results for continuous variables given lower limit x_{min}=1000 and nominal OC (p_0,α)=(0.001,0.2) and (p_1,β)=(0.005,0.1).

Variable	***n***	***k***	$\hat{\alpha}$	$\hat{\beta}$	$\boldsymbol{n_v/n_a}$
Exponential(μ)	2	$2.43\text{x}10^{-3}$	0.200	0.082	0.003
Normal(μ,σ=100)	18	2.886	0.191	0.097	0.023
Normal($\mu,\hat{\sigma}$)	88	2.886	0.191	0.097	0.099
Gamma($\hat{\nu}=10$, $\hat{\lambda}=338,\delta$)	206	2.131	0.193	0.096	0.224
Weibull($\hat{\nu}=10$, $\hat{\lambda}=1995,\delta$)	91	3.623	0.189	0.079	0.117
IG($\hat{\mu}$=1502, $\hat{\lambda}$=100,000,δ)	18	2.886	0.173	0.382	unusable

Table 2. Summary of results for continuous variables given upper limit x_{max}=10,000 and nominal OC (p_0,α)=(0.001,0.2) and (p_1,β)=(0.005,0.1).

Variable	***n***	***k***	$\hat{\alpha}$	$\hat{\beta}$	$\boldsymbol{n_v/n_a}$
Exponential(μ)	66	6.26922	0.200	0.082	0.085
Gamma($\hat{\nu}=10$, $\hat{\lambda}=441,\delta$)	77	3.667	0.189	0.104	0.099
Weibull($\hat{\nu}=10$, $\hat{\lambda}=3800,\delta$)	156	3.623	0.188	0.081	0.201

Table 3. Summary of results for Discrete Variables

Variable	***n***	***c***	$\hat{\alpha}$	$\hat{\beta}$	$\boldsymbol{n_v/n_a}$
Binomial(n,p)	777	1	0.188	0.100	1
Poisson(n,p)	21	88	0.191	0.097	0.035

Conclusions

In this paper we report the results of empirical tests designed to provide an independent assessment of the validity and accuracy of six published ASV procedures. Overall, the plans are shown to provide adequate or superior protection against producer's and consumer's risk for samples substantially smaller than those required for the corresponding ASA plans. The exception is the IG plan, which was previously shown to be in error.

While this overall conclusion supports the assertion in the literature that ASV plans require smaller samples than ASA plans, we also discovered that this assertion does not hold absolutely. In particular, for gamma and Weibull variables with small shape parameters, the ASV plans are in fact larger than the corresponding ASV plans. We believe that this discovery is original and needs to be assimilated into the literature on acceptance sampling.

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