Modeling Humans as Reinforcement Learners: How to Predict Human Behavior in Multi-Stage Games

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Abstract

This paper introduces a novel framework for modeling interacting humans in a multi-stage game environment by combining concepts from game theory and reinforcement learning. The proposed model has the following desirable characteristics: (1) Bounded rational players, (2) strategic (i.e., players account for one another’s reward functions), and (3) is computationally feasible even on moderately large real-world systems. To do this we extend level-K reasoning to policy space to, for the first time, be able to handle multiple time steps. This allows us to decompose the problem into a series of smaller ones where we can apply standard reinforcement learning algorithms. We investigate these ideas in a cyber-battle scenario over a smart power grid and discuss the relationship between the behavior predicted by our model and what one might expect of real human defenders and attackers.

1 Introduction

We present a model of interacting human beings that advances the literature by combining concepts from game theory and computer science in a novel way. In particular, we introduce the first time-extended level-K game theory model [1, 2]. This allows us to use reinforcement learning (RL) algorithms to learn each player’s optimal policy against the level $K - 1$ policies of the other players. However, rather than formulating policies as mappings from belief states to actions, as in partially observable Markov decision processes (POMDPs), we formulate policies more generally as mappings from a player’s observations and memory to actions. Here, memory refers to all of a player’s past observations.

This model is the first to combine all of the following characteristics. First, players are strategic in the sense that their policy choices depend on the reward functions of the other players. This is in
contrast to learning-in-games models whereby players do not use their opponents’ reward information to predict their opponents’ decisions and to choose their own actions. Second, this approach is computationally feasible even on real-world problems. This is in contrast to equilibrium models such as subgame perfect equilibrium and quantal response equilibrium [3]. This is also in contrast to POMDP models (e.g. I-POMDP) in which players are required to maintain a belief state over spaces that quickly explode. Third, with this general formulation of the policy mapping, it is straightforward to introduce experimentally motivated behavioral features such as noisy, sampled or bounded memory. Another source of realism is that, with the level-K model instead of an equilibrium model, we avoid the awkward assumption that players’ predictions about each other are always correct.

We investigate all this for modeling a cyber-battle over a smart power grid. We discuss the relationship between the behavior predicted by our model and what one might expect of real human defenders and attackers.

2 Game Representation and Solution Concept

In this paper, the players will be interacting in an iterated semi net-form game. To explain an iterated semi net-form game, we will begin by describing a semi Bayes net. A semi Bayes net is a Bayes net with the conditional distributions of some nodes left unspecified. A pictorial example of a semi Bayes net is given in Figure 1. Like a standard Bayes net, a semi Bayes net consist of a set of nodes and directed edges. The oval nodes labeled “S” have specified conditional distributions with the directed edges showing the dependencies among the nodes. Unlike a standard Bayes net, there are also rectangular nodes labeled “U” that have unspecified conditional dependencies. In this paper, the unspecified distributions will be set by the interacting human players. A semi net-form game, as described in [4], consists of a semi Bayes net plus a reward function mapping the outcome of the semi Bayes net to rewards for the players.

An iterated semi Bayes net is a Bayes net which has been time extended. It comprises of a semi Bayes net (such as the one in Figure 1), which is replicated T times. Figure 2 shows the semi Bayes net replicated three times. A set of directed edges L sets the dependencies between two successive iterations of the semi Bayes net. Each edge in L connects a node in stage t − 1 with a node in stage t as is shown by the dashed edges in Figure 2. This set of L nodes is the same between every two successive stages. An iterated semi net-form game comprises of two parts: an iterated semi Bayes net and a set of reward functions which map the results of each step of the semi Bayes net into an incremental reward for each player. In Figure 2, the unspecified nodes have been labeled “UA” and “UB” to specify which player sets which nodes.

Having described above our model of the strategic scenario in the language of iterated semi net-form games, we now describe our solution concept. Our solution concept is a combination of the level-K model, described below, and reinforcement learning (RL). The level-K model is a game theoretic
solution concept used to predict the outcome of human-human interactions. A number of studies [1, 2] have shown promising results predicting experimental data in games using this method. The solution to the level-K model is defined recursively as follows. A level K player plays as though all other players are playing at level $K - 1$, who, in turn, play as though all other players are playing at level $K - 2$, etc. The process continues until level 0 is reached, where the level 0 player plays according to a prespecified prior distribution. Notice that running this process for a player at $K \geq 2$ results in ricocheting between players. For example, if player A is a level 2 player, he plays as though player B is a level 1 player, who in turn plays as though player A is a level 0 player playing according to the prior distribution. Note that player B in this example may not actually be a level 1 player in reality – only that player A assumes him to be during his reasoning process.

This work extends the standard level-K model to time-extended strategic scenarios, such as iterated semi net-form games. In particular, each Undetermined node associated with player $i$ in the iterated semi net-form game represents an action choice by player $i$ at some time $t$. We model player $i$’s action choices using the policy function, $\pi(i)$, which takes an element of the Cartesian product of the spaces given by the parent nodes of $i$’s Undetermined node to an action for player $i$. Note that this definition requires a special type of iterated semi-Bayes net in which the spaces of the parents of each of $i$’s action nodes must be identical. This requirement ensures that the policy function is always well-defined and acts on the same domain at every step in the iterated semi net-form game. We calculate policies using reinforcement learning (RL) algorithms. That is, we first define a level 0 policy for each player, $\pi(0)$. We then use RL to find player $i$’s level 1 policy, $\pi(1)$, given the level 0 policies of the other players, $\pi(0)$, and the iterated semi net-form game. We do this for each player $i$ and each level $K$.

3 Application: Cybersecurity of a Smart Power Network

In order to test our iterated semi net-form game modeling concept, we adopt a model for analyzing the behavior of intruders into cyber-physical systems. In particular, we consider Supervisory Control and Data Acquisition (SCADA) systems [5], which are used to monitor and control many types of critical infrastructure. A SCADA system consists of cyber-communication infrastructure that transmits data from and sends control commands to physical devices, e.g., circuit breakers in the electrical grid. SCADA systems are partly automated and partly human-operated. Increasing connection to other cyber systems creating vulnerabilities to SCADA cyber attackers [6].

Figure 3 shows a single, radial distribution circuit [7] from the transformer at a substation (node 1) serving two load nodes. Node 2 is an aggregate of small consumer loads distributed along the circuit, and node 3 is a relatively large distributed generator located near the end of the circuit. In this figure, $V_i$, $P_i$, and $Q_i$ are the voltage, real power, and reactive power at node $i$. $P_i$, $Q_i$, $r_i$, and $x_i$ are the real power, reactive power, resistance and reactance of circuit segment $i$. Together, these values represent the following physical system [7], where all terms are normalized by the nominal system voltage.

$$\begin{align*}
P_2 &= -q_3, & Q_2 &= -q_3, & P_1 &= P_2 + p_2, & Q_1 &= Q_2 + q_2 \\
V_2 &= V_1 - (r_1 P_1 + x_1 Q_1), & V_3 &= V_2 - (r_2 P_2 + x_2 Q_2)
\end{align*}$$

In this model, $r$, $x$, and $p_3$ are static parameters, $q_2$ and $p_2$ are drawn from a random distribution at each step of the game, $V_1$ is the decision variable of the defender, $q_3$ is the decision variable of the attacker, and $V_2$ and $V_3$ are determined by the equations above. The injection of real power $p_3$ and reactive power $q_3$ can modify the $P_1$ and $Q_1$ causing the voltage $V_2$ to deviate from 1.0. Excessive deviation of $V_2$ or $V_3$ can damage customer equipment or even initiate a cascading failure beyond the circuit in question. In this example, the SCADA operator’s (defender’s) control over $q_3$ is compromised by an attacker who seeks to create deviations of $V_2$ causing damage to the system.

In this model, the defender has direct control over $V_1$ via a variable-tap transformer. The hardware of the transformer limits the defenders actions at time $t$ to the following domain

$$D_D(t) = \{ \min(v_{max}, V_{1,t-1} + v), V_{1,t-1}, \max(v_{min}, V_{1,t-1} - v) \}$$

1Although this work uses level-K and RL exclusively, we are by no means wedded to this solution concept. Previous work on semi net-form games used a method known as Level-K Best-of-M [8] instead of RL to determine actions. This was not used in this paper because the possible action space is so large.
where $v$ is the voltage step size for the transformer, and $v_{\text{min}}$ and $v_{\text{max}}$ represent the absolute minimum and maximum voltage the transformer can produce. Similarly, the attacker has taken control of $q_3$ and its actions are limited by its capacity to produce real power, $p_{3,\text{max}}$ as represented by the following domain.

$$D_A(t) = \{-p_{3,\text{max}}, \ldots, 0, \ldots, p_{3,\text{max}}\}$$

Via the SCADA system and the attacker’s control of node 3, the observation spaces of the two players includes

$$\Omega_D = \{V_1, V_2, V_3, P_1, Q_1, M_D\}, \quad \Omega_A = \{V_2, V_3, p_3, q_3, M_A\}$$

where $M_D$ and $M_A$ are used to denote each real number that represents the respective player’s memory of the past events in the game. Both the defender and attacker manipulate their controls in a way to increase their own rewards. The defender desires to maintain a high quality of service by maintaining the voltages $V_2$ and $V_3$ near the desired normalized voltage of one while the attacker wishes to damage equipment at node 2 by forcing $V_2$ beyond operating limits, i.e.

$$R_D = -\left(\frac{V_2 - 1}{\epsilon}\right)^2 - \left(\frac{V_3 - 1}{\epsilon}\right)^2, \quad R_A = \Theta|V_2 - (1 + \epsilon)| + \Theta|(1 - \epsilon) - V_2|$$

Here, $\epsilon \approx 0.05$ for most distribution system under consideration, $\Theta$ is a Heaviside step function.

**Level 1 defender policy** The level 0 defender is modeled myopically and seeks to maximize his reward by following a policy that adjusts $V_1$ to move the average of $V_2$ and $V_3$ closer to one, i.e.

$$\pi_D(V_2, V_3) = \arg\min_{V_1 \in D_D(t)} \frac{(V_2 + V_3)}{2} - 1$$

**Level 1 attacker policy** The level 0 attacker adopts a *drift and strike* policy based on intimate knowledge of the system. If $V_2 < 1$, we propose that the attacker would decrease $q_3$ by lowering it by one step. This would cause $Q_1$ to increase and $V_2$ to fall even farther. This policy achieves success if the defender raises $V_1$ in order to keep $V_2$ and $V_3$ in the acceptable range. The attacker continues this strategy, pushing the defender towards $v_{\text{max}}$, until he can quickly raise $q_3$ to push $V_2$ above $1 + \epsilon$. If the defender has neared $v_{\text{max}}$, then a number of time steps will be required to for the defender to bring $V_2$ back in range. More formally this policy can be expressed as

```python
LEVEL0ATTACKER()
V+ = max_{q \in D_A(t)} |V_2 - 1|
if V+ > q_A
    then return arg max_{q \in D_A(t)} |V_2 - 1|
if V2 < 1
    then return q_{3,t-1} - 1
return q_{3,t-1} + 1;
```

where $q_A$ is a threshold parameter.

### 3.1 Reinforcement Learning Implementation

Using defined level 0 policies as the starting point, we now bootstrap up to higher levels by training each level $K$ policy against an opponent playing level $K - 1$ policy. To find policies that maximize reward, we can apply any algorithm from the reinforcement learning literature. In this paper, we use
an \( \epsilon \)-greedy policy parameterization (with \( \epsilon = 0.1 \)) and SARSA on-policy learning [8]. Training updates are performed epoch-wise to improve stability. Since the players’ input spaces contain continuous variables, we use a neural-network to approximate the Q-function [9]. We improve performance by scheduling the exploration parameter \( \epsilon \) in 3 segments during training: An \( \epsilon \) of near unity, followed by a linearly decreasing segment, then finally the desired \( \epsilon \).

### 3.2 Results and Discussion

We present results of the defender and attacker’s behavior at various level \( K \). We note that our scenario always had an attacker present, so the defender is trained to combat the attacker and has no training concerning how to detect an attack or how to behave if no attacker is present. Notationally, this is also true for the attacker’s training. However in real-life the attacker will likely know that there is someone trying to thwart this attack.

**Level 0 defender vs. level 0 attacker** The level 0 defender (see Figure 4(a)) tries to keep both \( V_2 \) and \( V_3 \) close to 1.0 to maximize his immediate reward. Because the defender makes steps in \( V_1 \) of 0.02, he does nothing for \( 30 < t < 60 \) because any such move would not increase his reward. For \( 30 < t < 60 \), the \( p_2, q_2 \) noise causes \( V_2 \) to fluctuate, and the attacker seems to randomly drift back and forth in response. At \( t = 60 \), the noise plus the attacker and defender actions breaks this “symmetry”, and the attacker increases his \( q_3 \) output causing \( V_2 \) and \( V_3 \) to rise. The defender responds by decreasing \( V_1 \), indicated by the abrupt drops in \( V_2 \) and \( V_3 \) that break up the relatively smooth upward ramp. Near \( t = 75 \), the accumulated drift of the level 0 attacker plus the response of the level 0 defender pushes the system to the edge. The attacker sees that a strike would be successful (i.e., post-strike \( V_2 < 1 - 6\epsilon \)), and the level 0 defender policy fails badly. The resulting \( V_2 \) and \( V_3 \) are quite low, and the defender ramps \( V_1 \) back up to compensate. Post strike \( (t > 75) \), the attackers threshold criterion tells him that an immediate second strike would not be successful, however, this shortcoming will be resolved via level 1 reinforcement learning. Overall, this is the behavior we have built into the level 0 players.

**Level 1 defender vs. level 0 attacker** During the level 1 training, the defender likely experiences the type of attack shown in Figure 4(a) and learns that keeping \( V_1 \) a step or two above 1.0 is a good way to keep the attacker from putting the system into a vulnerable state. As seen in Figure 4(b), the defender is never tricked into performing a sustained drift because the defender is willing to take a reduction to his reward by letting \( V_3 \) stay up near 1.05. For the most part, the level 1 defender’s reinforcement learning effectively counters the level 0 attacker drift-and-strike policy.

**Level 0 defender vs. level 1 attacker** The level 1 attacker learning sessions correct a shortcoming in the level 0 attacker. After a strike \( (V_2 < 0.95 \) in Figure 4(a)), the level 0 attacker drifts up from his largest negative \( q_3 \) output. In Figure 4(c), the level 1 attacker anticipates that the increase in \( V_2 \) when he moves from \( m = -5 \) to \( m = 5 \) will cause the level 0 defender to drop \( V_1 \) on the next move. After this drop, the level 1 attacker also drops from \( m = +5 \) to \(-5 \). In essence, the level 1 attacker is leveraging the anticipated moves of the level 0 defender to create oscillatory strikes that push \( V_2 \) below \( 1 - \epsilon \) nearly every cycle.

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### References


Figure 4: Voltages and attacker moves of various games.


