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EXPECTED NAVIGATION FLIGHT PERFORMANCE FOR THE MAGNETOSPHERIC MULTISCALE (MMS) MISSION

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The Magnetospheric Multiscale (MMS) mission consists of four formation-flying spacecraft placed in highly eccentric elliptical orbits about the Earth. The primary scientific mission objective is to study magnetic reconnection within the Earth's magnetosphere. The baseline navigation concept is the independent estimation of each spacecraft state using GPS pseudorange measurements (referenced to an onboard Ultra Stable Oscillator) and accelerometer measurements during maneuvers. State estimation for the MMS spacecraft is performed onboard each vehicle using the Goddard Enhanced Onboard Navigation System, which is embedded in the Navigator GPS receiver. This paper describes the latest efforts to characterize expected navigation flight performance using upgraded simulation models derived from recent analyses.

INTRODUCTION

The primary purpose of the Magnetospheric Multiscale (MMS) mission is to study the phenomenon of magnetic reconnection, a process by which magnetic energy from colliding magnetic field lines is converted to both kinetic energy and heat in the form of charged-particle accelerations and large-scale flows of matter. The ideal environment in which to study this effect is in Earth orbit, in regions on both the day and the night sides of the Earth where the solar winds and magnetic field lines interact with the Earth's magnetosphere. Studying this effect *in situ* is expected to dramatically advance understanding of events driven by magnetic reconnection, such as solar flares and polar lights known as auroras on Earth and other planets in the solar system.

The Earth-orbiting, formation-flying MMS mission consists of four spacecraft with identical instrument suites. The spacecraft are located in highly eccentric orbits about the Earth, with eccentricity values over 0.8. Within the science regions of interest, the spacecraft form a tetrahedral formation to collect measurements of the magnetic reconnection effects. Maintaining the tetrahedral formation is critical for this task because simultaneous measurements are needed at several locations within the science region of interest; in a sense, the formation itself is a scientific instrument. Relative navigation of the MMS spacecraft is challenging because the relative separations within the formation must be known with high accuracy.

The four spacecraft are scheduled to be launched directly into highly eccentric orbits about the Earth in the Fall of 2014. Shortly after launch, perigee raising maneuvers are performed to place all four spacecraft into orbits characterized by a 1.2 Earth Radii (Re) perigee and 12 Re apogee radius. After these maneuvers are complete, the commissioning phase commences, during which spacecraft operators will perform a check out of all spacecraft subsystems. The commissioning phase is also known as Phase 0.

Science operations for the MMS mission are conducted in two distinct Phases, known as Phase 1 and Phase 2. Phase 1 consists of Phases 1a, 1x, and 1b. Phase 1a begins when the orbit apogee crosses from the Earth nightside (i.e., in opposition to the Sun), to the dayside (i.e., in inferior conjunction with the Sun). In Phase 1, the distances between the spacecraft (also known as the formation sizes) within the science region of interest is adjusted in three stages from 160 kilometers to 10 kilometers. This variation in the formation size allows the formation to be used in the study of reconnection on the dayside boundary in the "bow-shock"

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region, which is located between the Earth's magnetosphere and the solar system magnetic field, generated primarily by the Sun. The duration of Phase 1a is approximately 6 months. The following 6 months, when the apogee is located on the night side of the Earth, constitute Phase 1x. Phase 1b occurs during the 6 months after the conclusion of Phase 1x, when the apogee once again is located in the dayside region and employs an "optimum" separation chosen by mission scientists after assessing science data from Phase 1a.

Phase 2 consists of Phase 2a and Phase 2b. During Phase 2a, a series of apogee raising maneuvers are executed to incrementally increase the orbital apogee of each spacecraft from 12 Re to 25 Re. Phase 2b commences after the apogee raising of all four spacecraft is complete. During Phase 2b the formation separations vary from 400 kilometers to 30 kilometers, followed by an "optimum" separation chosen by the scientists as they study the night-side reconnection events that occur within the Earth's magnetotail. After Phase 2b, the spacecraft are de-commissioned. A more detailed description of the mission concept appears in Reference 1.

The navigation concept for MMS is based on an independent estimation of each spacecraft's position, velocity, onboard clock bias, onboard clock bias drift rate, and onboard clock bias drift acceleration. These quantities are estimated onboard by an Extended Kalman Filter (EKF) that processes Global Positioning System (GPS) pseudorange measurements referenced to an Ultra Stable Oscillator (USO). This EKF is part of the Goddard Enhanced Onboard Navigation System (GEONS), which is embedded in the Navigator GPS receiver developed at Goddard Space Flight Center. The Navigator GPS receiver employs a weak signal tracking technology that significantly improves reception of GPS signals when the spacecraft is above 10 Re. Thrust acceleration measurements from an accelerometer within the Attitude Control System are recorded during maneuvers and incorporated into the dynamics modeling in the EKF. The estimated states are periodically downlinked and used in the MMS Flight Dynamics Operations Area (FDOA) to generate definitive and predictive products for support of mission and science operations. "Definitive" refers to the portion of the ephemeris that is produced by EKF estimates using measurements. "Predicted" ephemerides are propagated from the last point of the definitive ephemeris. This high-level navigation concept, along with each of the spacecraft's navigation and attitude control elements, appears in Figure 1.

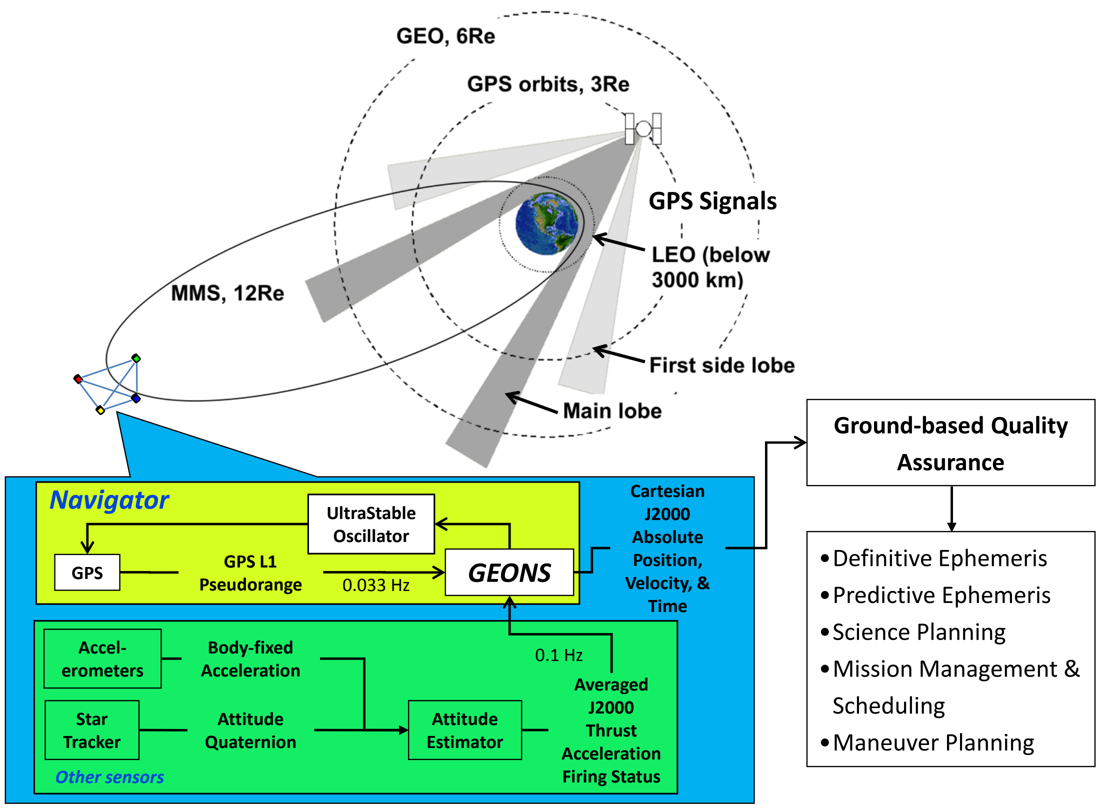

Figure 1. Navigation Concept and Onboard Configuration

Several challenging navigation requirements result from the fact that the majority of the MMS mission occurs above the GPS constellation, where GPS signal acquisition is sparse. Critical navigation requirements include

- A maximum mean semi-major axis (SMA) error of 100 meters above 3 Re within the definitive ephemeris; necessary to meet formation maintenance (FM) maneuver planning requirements
- A maximum relative definitive position error of one percent of the separation distance between spacecraft or 100 meters, whichever is larger; necessary to meet science requirements
- A maximum relative predicted position error growth of 200 meters per day; necessary to meet collision avoidance requirements
- A maximum USO clock bias error of 325 microseconds; necessary to meet a 1 millisecond maximum relative clock bias error science requirement

The "errors" referenced in the requirements above are navigation knowledge errors. In pre-launch analysis these errors are quantified by comparing estimated states to a set of known truth states of the spacecraft trajectory. For MMS, navigation knowledge errors arise primarily from

- The truncation of the onboard gravity model to a degree and order of 13x13
- Solar radiation pressure (SRP) modeling errors
- GPS pseudorange measurement noise
- GPS ephemeris and clock errors
- Receiver clock modeling errors
- Thrust acceleration knowledge errors

Note that onboard processing limitations restrict the number of terms used in the gravity model.

The navigation analysis discussed in this paper addresses two types of maneuver sequences: formation maintenance (FM) maneuver sequences, which are designed to maintain the quality of the formation, and formation resize (FR) maneuver sequences, which are designed to change the formation size. Both types of maneuver sequences consist of two maneuvers. The first maneuver occurs immediately after exiting the science Region-of-Interest (ROI), which is defined as the portion of the orbit above 9 Re for Phase 1 and above 15 Re for Phase 2b. The second maneuver is performed near the entrance to the science ROI, following perigee. A representative simulation timeline for Phase 2b showing the durations of each event and including these two maneuvers appears in Figure 2. Note that the Phase 2b orbital period is 2.78 days.

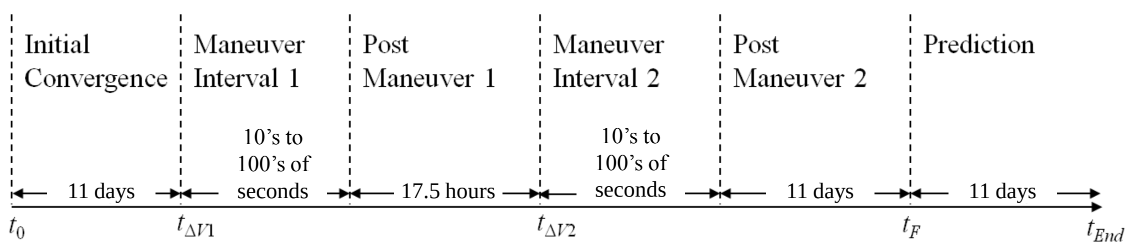

Figure 2. Phase 2b Simulation Timeline

This paper first describes the methods employed to conduct navigation simulations, as well as the process for generating truth trajectories and maneuvers needed for the navigation analysis. Descriptions of the models used in the overall navigation analysis including trajectory models, the measurements model, filter settings, the acceleration knowledge error model, and the variations performed within the Monte Carlo (MC) simulations, follow. These descriptions address the analysis and sensitivity studies conducted since the MMS Mission Critical Design Review (CDR)[*] and the 2011 Space Flight Mechanics conference paper by Olson, et al., which led to some of the final values used in the models and the updated operations concept.[2] The effects of these changes on the navigation results are presented.

[*]The MMS Mission CDR occurred in August of 2010.

SIMULATION METHOD

Single-run and MC navigation simulations are conducted as part of the navigation analysis. A high-level flow of the simulation steps used in both a single-run navigation simulation and in MC analysis appears in Figure 3. The FreeFlyer® mission analysis software is used to generate the truth trajectories and finite maneuver acceleration profiles that are employed within the simulation process. The steps that involve variations during a MC simulation are indicated within the dashed box. The truth trajectories are ingested into the DatSim program, which generates realistic GPS L1 pseudorange (PR) measurements. These measurements include errors that are varied as part of the MC process.[3] The acceleration profiles are also modified to include appropriate knowledge errors. Next, the measurements, the truth trajectories, and the modified acceleration profiles are used in the GEONS flight software (FSW) to produce navigation solutions, or estimates of the spacecraft state. GEONS also produces filter covariance values for each of the estimated states.[4]

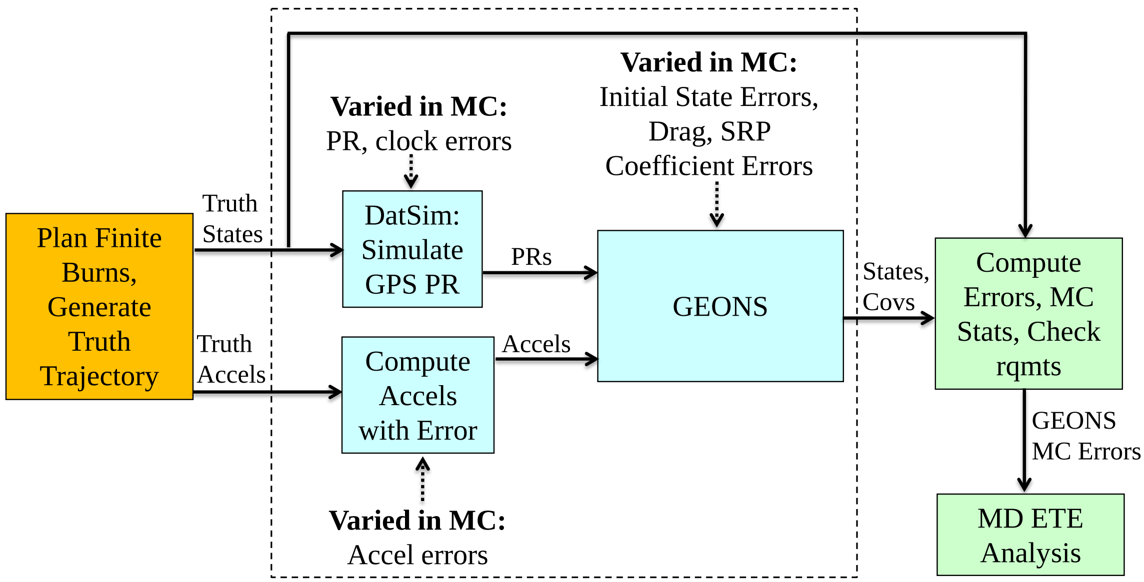

Figure 3. Monte Carlo Simulation Method

These state and covariance values, along with the truth ephemeris, are post-processed to compute arrays of state errors and to verify requirements. These arrays consist of nine state errors (position, velocity, and the three clock errors) for each of the individual MC runs at three epochs of interest. The arrays are inputs to the Mission Design (MD) End-To-End (ETE) simulation, which models the MMS mission from commissioning to the end of Phase 2b and whose purpose is to verify all MD and Conjunction Assessment (CA) requirements. The ETE uses the arrays of state errors to generate simulated empirical navigation covariance matrices, which are then sampled to generate navigation errors at the same epochs within the ETE simulation.

TRUTH TRAJECTORY GENERATION PROCEDURE

The first step in the navigation analysis simulation method described above is generating the truth trajectory and associated maneuver accelerations. The procedure for this step is illustrated in Figure 4. For this procedure, the states are reported every 10 seconds for the truth trajectories and the accelerations for the maneuvers are averaged over the previous 10 seconds based on simulated acceleration values at each 10-second marker. The truth trajectories and maneuvers are propagated at 1-second intervals, as explicitly propagating at 10-second intervals does not generate averaged truth accelerations with sufficient accuracy for GEONS to match the truth maneuvers when integrating the estimated trajectory.

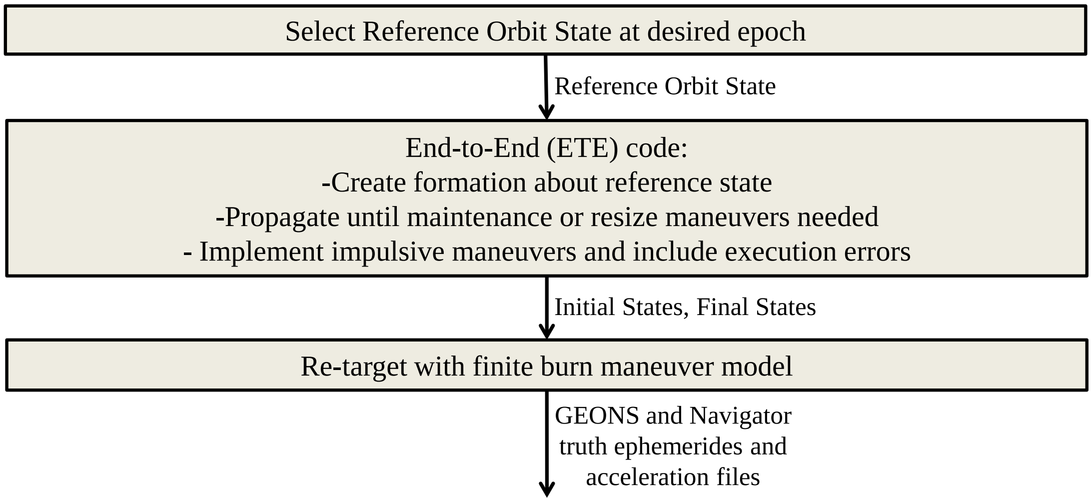

Figure 4. Truth Trajectory Generation Procedure

The particular maneuvers for this analysis are representative large and small maneuvers based on histograms of impulsive maneuver ΔV values from ETE MC runs within each phase. The maneuver magnitudes for the scenario with FM maneuvers in Phase 2b are approximately 0.48 and 0.60 meters per second, and for the scenario with FR maneuvers the magnitudes are approximately 4.51 and 6.41 meters per second.

MODELS, MODIFIED OPERATIONS CONCEPTS, AND SENSITIVITIES

This section contains a description of how modeling values and operations concepts employed the navigation simulations have been modified since the MMS Mission CDR as well as papers published in 2009 and 2011.[2,5,6] The changes were made such that the current models reflect the expected flight performance. The section also describes the sensitivity of the navigation performance to these modifications.

Trajectory Models

The trajectory model employed by GEONS for pre-launch analysis is consistent with the expected flight configuration, which is tailored to execute within the limited resources of the Navigator receiver's flight processor. Table 1 lists the primary models and associated values used to propagate the truth and filtered trajectory of each MMS spacecraft. Many differences exist, but because no model perfectly reflects reality, these differences also serve to exercise the robustness of the GEONS propagation. The remainder of this section provides details of analyses performed to evaluate trajectory model changes. The results of these analyses have led to an increase in the fidelity of the onboard gravity model, a reduction in the planetary position update interval, and a confirmation that the navigation performance has negligible sensitivity to using the DE405 planetary ephemeris instead of a trigonometric series fit to the DE404 planetary ephemeris.

Increasing Onboard Gravity Model Fidelity A study to determine the sensitivity of navigation performance to the degree and order of the onboard gravity model employed by GEONS indicates that an 8x8 gravity model results in unacceptable navigation errors. In this study, a launch date of November 1, 2014, a nominal FM sequence for Phase 2b, and a formation size of 30 kilometers are employed. The definitive time span, defined as the interval in which measurements are filtered by the GEONS EKF to produce state estimates, starts at day 0 and ends at day 20. All measurement data corresponding to epochs when the spacecraft is located above 15 Re are removed. After the definitive time span, the predictive time span begins and lasts until the end of the simulation. During the predictive time span, GEONS propagates the spacecraft states using a gravity model size set to 21x21, since this propagation will be performed by the ground system using a similar high-fidelity gravity model.

The first scenario in this study is a simulation with the onboard gravity field degree and order both set to 8 during the definitive time span. Figure 5 contains the Root Sum Square (RSS) total position error and filter

Table 1. Trajectory Propagation Models

Simulation Parameter	FreeFlyer® Truth	GEONS Filter
Nonspherical Earth Gravity Model	21x21 Earth Gravitational Model 1996	13x13 Joint Gravity Model-2 (onboard), 21x21 Joint Gravity Model-2 (ground)
Point Mass Gravity	Sun, Moon using DE 405 ephemeris	Sun, Moon using analytical fit to DE 404 ephemeris, with 30 sec min lunar update interval
Atmospheric Drag	Jacchia Roberts, Schatten +2 sigma prediction solar flux, C_D of 2.2, Drag area of 7.1 m^2	Analytical fit to Harris Priester model, C_D of 2.2, Drag area of 7.1 m^2
Solar Radiation Pressure	Spherical model, C_R of 1.8, SRP area of 2.026712 m^2	Spherical model, C_R of 1.8, SRP area of 2.02 m^2
Integrator	8(9) Variable Step Runge-Kutta	4^{th} Order Fixed Step Runge-Kutta
Integration Stepsize	1 second	10 seconds
Precession/Nutation Update Interval	1 second	10 seconds
Maneuver Model	Finite burns	Accelerometer measurements averaged over 10 seconds, including acceleration knowledge errors

root-variance for the MMS2 spacecraft. The root-variance is defined as the square root of the formal diagonal covariance terms. A large spike appears following FM maneuver 2 because the acceleration knowledge error model assumes acceleration knowledge errors later determined to be unrealistically large. Additionally, no measurements are included when the MMS spacecraft is located above 15 Re. Strategies to improve this behavior and mitigate the spike are discussed in the results section. However, in order to determine the sensitivity to the gravity model size, preliminary analysis with the older acceleration knowledge error model was used.

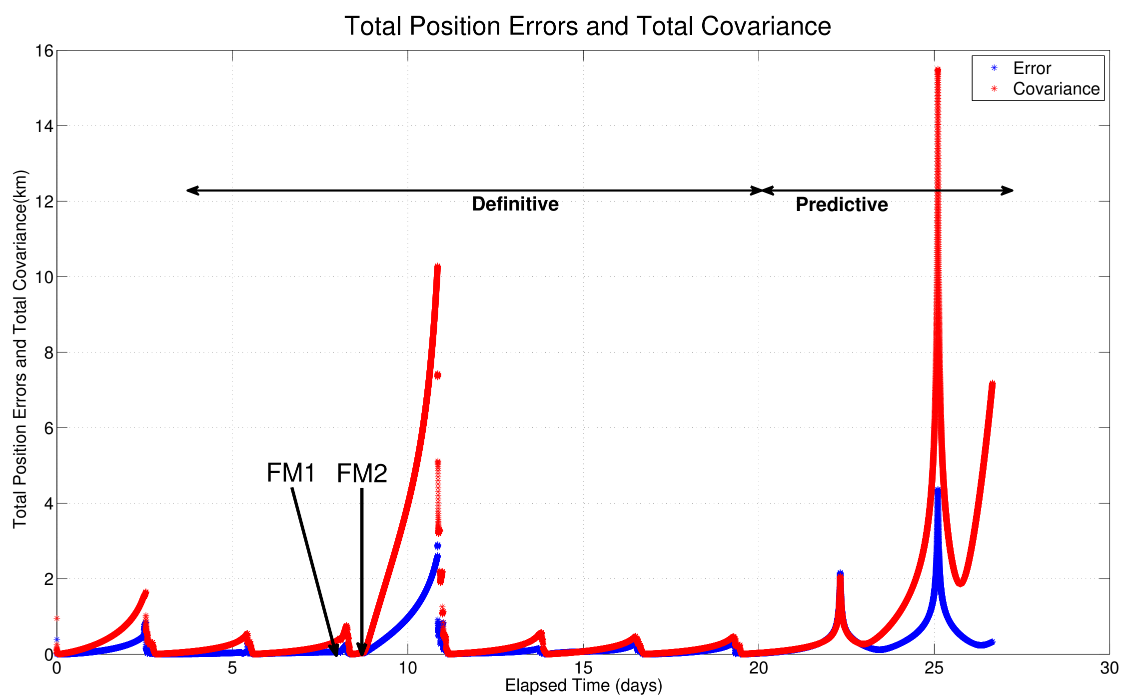

Figure 5. RSS Position Error and Root-Variance with 8x8 Onboard Gravity Field

When using an onboard gravity model of 8x8 degree and order, it is clear that a large amount of predictive error growth results. To test how much prediction error is introduced from the truncation of the gravity field, the onboard model is modified to a 21x21 degree and order, which is the same degree and order that was used to generate the truth trajectory. The RSS position errors and filter root-variance for this 21x21 onboard gravity model scenario for the MMS2 spacecraft appear in Figure 6.

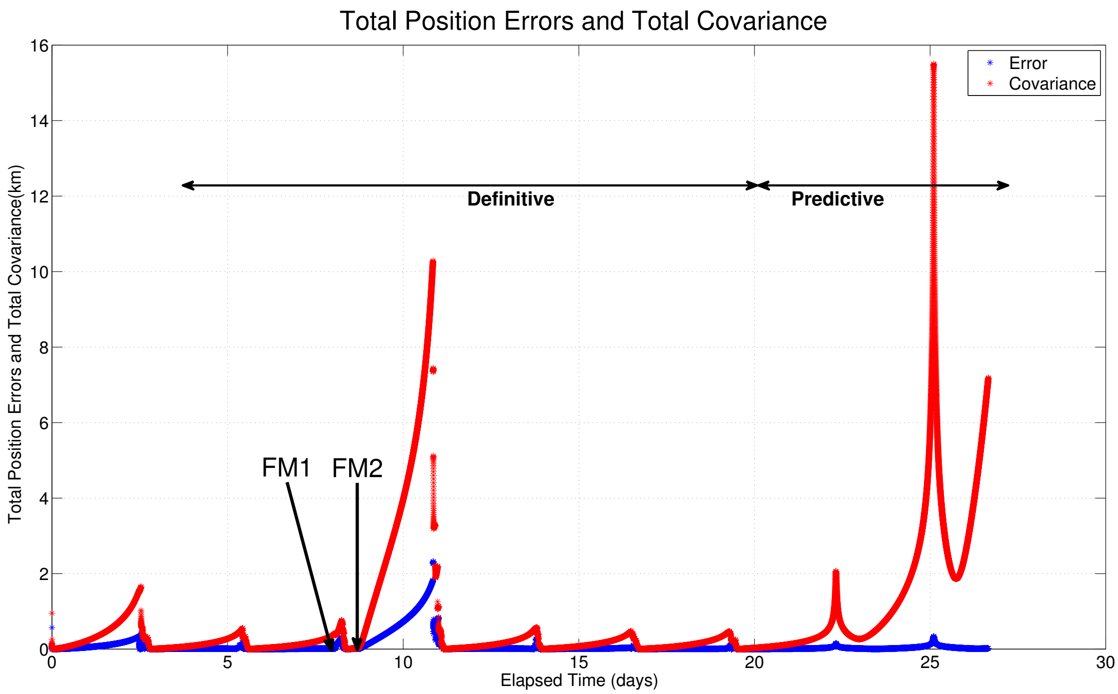

Figure 6. RSS Position Error and Root-Variance with 21x21 Onboard Gravity Field

Note the difference in the predictive errors, especially near day 25, in Figure 5 and Figure 6. Using a 21x21 gravity field during the definitive time span, instead of an 8x8 gravity field, results in slightly smaller errors during the definitive time span and significantly smaller errors during the predictive time span. These lower predictive errors are primarily due to significantly lower velocity errors at the end of the definitive time span.

Unfortunately, using a 21x21 gravity model during the mission requires too much processing power of the CPU for the Navigator receiver. Recent analysis of the processing capacity available on the spacecraft, or "CPU margin," reveals that the Navigator GPS receiver can run with a gravity model as high as 13x13, a significant increase in terms of processing capability over the previous 8x8 level. A degree and order of 13 is employed in a simulation to determine the effect this increase has on the navigation performance. The resulting performance is nearly identical to the simulation using 21x21 in both the definitive time span and predictive time span. This result indicates that nearly all of the error introduced through gravity model truncation is removed by increasing the degree and order to 13. Performance is on par with the simulation that uses a 21x21 gravity field, which is used to generate the truth trajectory, but with far lower computational processing cost. Therefore, a 13x13 gravity model is appropriate for onboard use during the mission.

Solar/Lunar Update Interval Sensitivity To reduce onboard processing requirements and to free up CPU margin for running a 13x13 onboard gravity field, mission analysts investigated the effect on the navigation performance of reducing the frequency of solar and lunar position updates during spacecraft propagation. Nominally, GEONS is configured to compute the positions of the Sun and Moon, as well as the rotation matrices involved in the propagation of the spacecraft state, at each stage of the fixed-step fourth order Runge Kutta integration. This update normally occurs multiple times within each 10-second simulation state output interval. To reduce computational load, the GEONS FSW is configured such that these positions and rotation matrices are computed only at the beginning of each propagation interval, that is, only once every 10 seconds.

The same Phase 2b 30 kilometer scenario described previously is used to compare the nominal update interval case and the modified reduced update interval case. The navigation error during the definitive timespan for the case with reduced solar and lunar positions update intervals is nearly identical to the corresponding error in the original case, and navigation error during the predictive span of the reduced case is slightly worse than the corresponding navigation error in the original case. The maximum difference in RSS position error is approximately 48 meters, as shown in Figure 7.

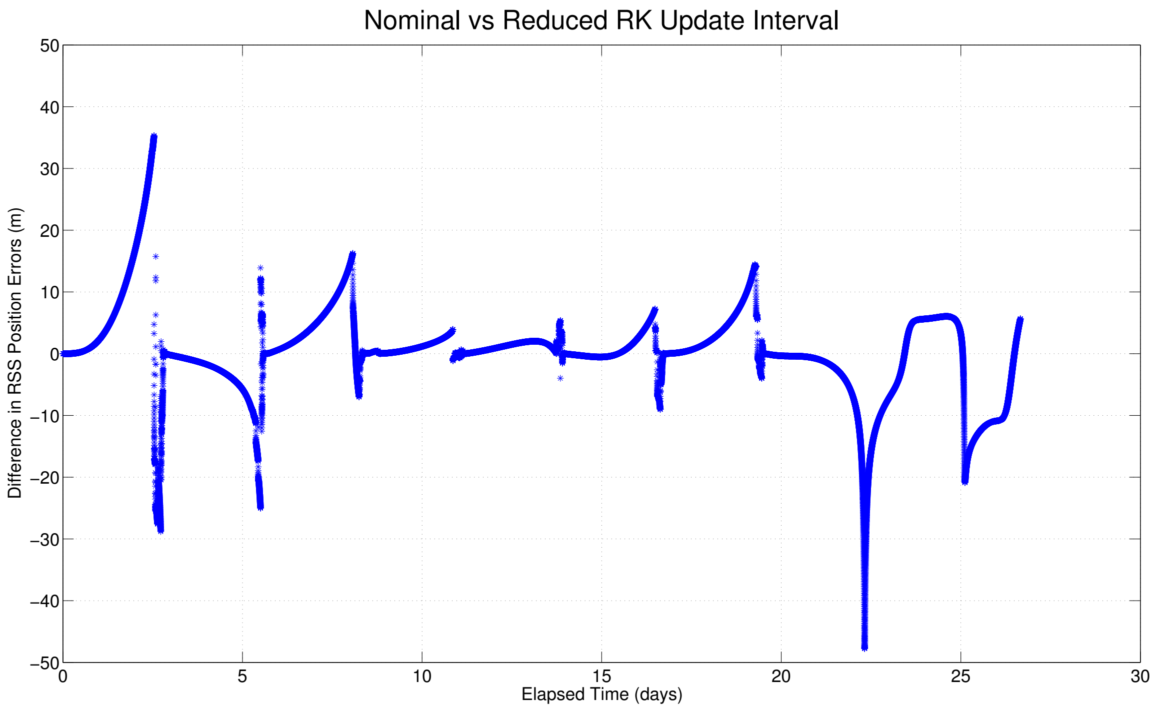

Figure 7. Difference in RSS Position Error for Nominal versus Reduced Runge Kutta Solar and Lunar Positions Update Interval

An additional test investigates the effects of increasing the lunar position update interval to 30 seconds. The difference in the RSS position error between the nominal case (plotted in Figure 5) and the 30 second lunar position and 10 second solar position update interval case appears in Figure 8. This small difference in error values (at most 126 meters) due to the increased lunar position update interval is not significant relative to the navigation error requirements of 100 kilometers absolute and one percent relative error.

DE405 versus DE404 for Planetary Positions - Sensitivity Unlike changes to the onboard gravity model and planetary ephemeris update intervals, the type of planetary ephemeris used onboard in the GEONS FSW cannot be modified due to memory constraints. However, mission analysts on the ground may find navigation performance sensitivity to this parameter valuable as they select the planetary ephemeris to use for ground analysis.

The GEONS FSW is configured to compute the planetary positions using a trigonometric series fit to the DE404 ephemeris. However, the more common and more accurate DE405 ephemeris is used in the generation of the truth trajectory. A test where the GEONS propagator employs the DE405 ephemeris is performed to determine if this difference has a significant impact on the results. Using the same nominal case as in the Increasing Onboard Gravity Model Fidelity subsection above, the maximum difference in the spacecraft trajectory when using these two planetary ephemerides is approximately 2 meters, with most differences less than 0.1 meters in the definitive region. Thus, using a DE405 planetary ephemeris instead of the default DE404 planetary ephemeris produces no significant difference in the results. Therefore, using the more accurate DE405 instead of the trigonometric series fit to the DE404 to propagate the spacecraft trajectories in ground operations is less critical than previously assumed.

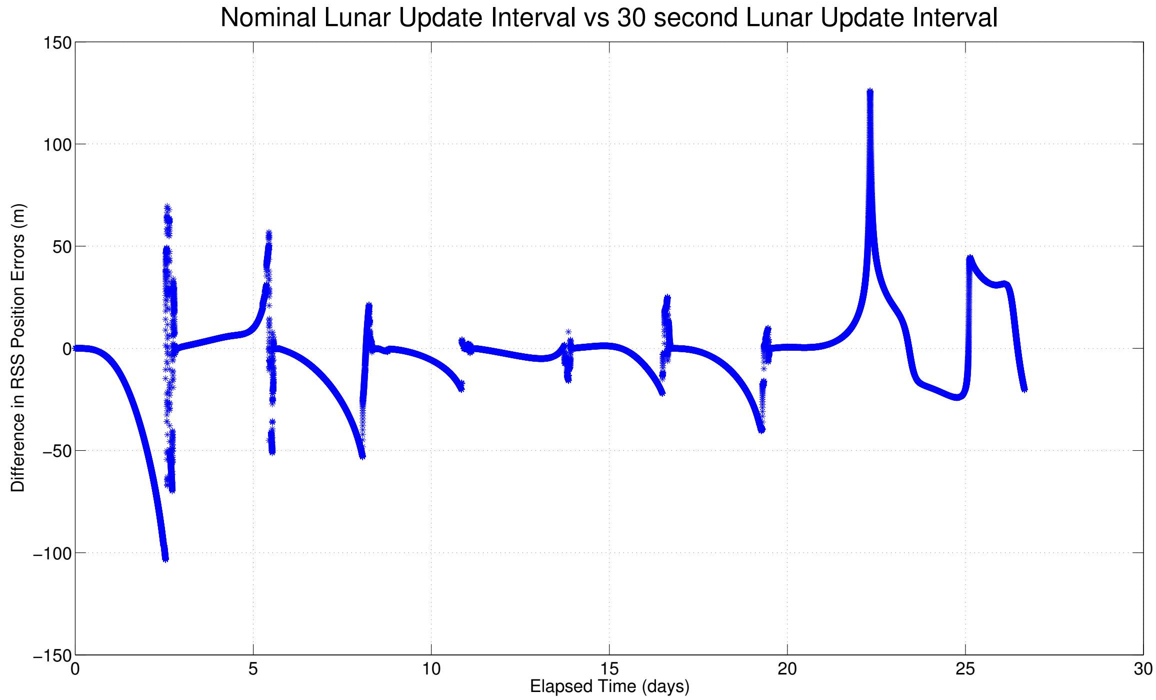

Figure 8. Difference in RSS Position Error for Nominal versus 30 second Lunar Update Interval

Measurement Models

In addition to changes in the trajectory models described above, significant changes have been made to the measurement simulation parameters to reflect recent Navigator GPS receiver hardware performance analysis. These updates are used in the latest analysis and are listed in Table 2. Some of the changes include

- A change in the definition of "strong signals," which now applies to those signals with signal-to-noise ratios (SNR) greater than 40 dB-Hz to those signals with SNR greater than 45 dB-Hz. All signals with SNR below 45 dB-Hz are considered "weak signals".
- A reduction in the minimum signal acquisition delay from 600 seconds to 300 seconds.
- A change in the minimum signal strength for tracking, from 28 dB-Hz SNR, which remains the minimum acquisition signal strength, to 25 dB-Hz SNR.
- An increase in the 1σ GPS psuedorange (PR) noise for strong signals from 4 meters to 5 meters.
- Modifications of the initial clock bias, clock bias rate, and clock acceleration values, with significant increases in the clock bias and clock bias rates consistent with USO specifications.
- Modifications of the clock aging parameters to reflect the USO specification values.
- A modification of the periodic variation in the clock acceleration to reflect the latest understanding of the expected eclipses on the spacecraft temperature.
- A more realistic Navigator receiving antenna model, which is accompanied by an increase in the receiver system noise loss term from 4 dB to 7.16 dB, and a reduction in the receiving antenna noise temperature from 190 K to 90 K.

Another significant update to the measurements models is the transition from a GPS IIA transmitting antenna pattern to a new GPS IIRM transmitting antenna pattern, which also has a slightly lower effective isotropic radiated power (EIRP). The new IIRM antenna pattern and the associated loss of approximately 1 dB in the EIRP results in about 17% fewer measurements, but the navigation performance is nearly identical. Thus, the measurements that are no longer received have little effect on the navigation solution.

Changes in the simulation parameters and models are accompanied by a significant modification of the operations concept that leads to many more measurements and a significant improvement in navigation performance. The change to the operations concept is to leave the Navigator GPS receiver fully on when the

Table 2. Measurement Simulation Models

Simulation Parameter	Nominal Values
Measurement Type	GPS Pseudoranges (PR)
Navigator Receiving Antenna	Composite toroidal model for spinning spacecraft with peak gain normal to the spin axis
GPS PR Measurement Rate	1 Measurement set every 30 seconds for each formation member, with measurements from a maximum of 12 visible GPS SVs when below 15Re, no measurements above 15Re in Phase 2b except for apogees immediately before and after maneuvers
GPS Acquisition Threshold (Based on Navigator Performance)	C/N0 $\geq$ 45 dB-Hz: 95% probability of acquisition, minimum acquisition delay of 300 sec C/N0 = 28 to 45 dB-Hz: 75% probability of acquisition, minimum acquisition delay of 300 sec C/N0 $<$ 28 dB-Hz: 0% probability of acquisition C/N0 $\geq$ 25 dB-Hz: weak signal tracking
GPS Antenna Model	GPS IIRM with Transmitting Antenna Effective Isotropic Radiated Power (SVEIRP) = 12.41 dB-Watts
GPS Constellation	21+3 GPS Space Vehicles
GPS Space Vehicle (SV) Selection	Min to max Transmitter ID values
1σ GPS PR Noise (Based on Navigator Specifications)	5 meters for C/N0 $\geq$ 45 dB-Hz (strong signal) 10 meters for C/N0 $<$ 45 dB-Hz (weak signal)
GPS Ephemeris and Clock Errors	Sinusoidal ephemeris model with 2 meter amplitudes for each component
Ionospheric Delay Model	GPS Ionospheric Model based on GPS broadcast coefficients

spacecraft is above 15 Re during the apogee before and after all maneuvers during Phase 2b. In the previous operations concept, Navigator is always placed into an ultra-low power mode with only the USO left on when the spacecraft is above 15 Re in Phase 2b. Recall that this altitude range defines the science region of interest for Phase 2b, and thus this concept arose due to power sharing concerns with the scientific instruments. A plot of how many GPS space vehicles (SV) are tracked at each time in the simulation for spacecraft MMS2 in the Phase 2b 40km formation scenario with the new operations concept appears in Figure 9. The plot illustrates how many measurements are now included above 15 Re for the orbit before and the orbit after the maneuver sequence. These measurements are selected for inclusion in the filtering process within GEONS by a decision engine that is designed to implement this new operations concept. Note that the maximum number of GPS SVs that can be tracked at any time is 12 because the Navigator GPS receiver possesses 12 channels. Figure 10 shows the MMS2 spacecraft orbit radius for the same scenario. Comparing Figures 9 and 10 shows that the large variation in the number of visible GPS satellites is a function of satellite radial distance. When compared with plots of estimation error results provided in the Results section, the relationship between the number of GPS satellites tracked by the receiver and the estimation state error is apparent.

Inclusion of these additional measurements leads to slightly improved navigation solutions before the first maneuver is executed and dramatically improves navigation solutions after the second maneuver is executed. The additional measurements significantly reduce the error introduced by the acceleration knowledge error in the second maneuver, despite the fact that only sparse measurement are available when the spacecraft is above 15 Re. To see the improvement in the navigation solutions, a comparison between two different scenarios is needed. The scenario from the *Increasing Onboard Gravity Model Fidelity* section above serves as the baseline with representative performance during the definitive time span, and the errors for this scenario appear in Figure 11. Recall that the scenario uses an older acceleration knowledge error model that introduces much larger acceleration errors than the current model does, and all measurements above 15 Re are removed. The new scenario includes measurements above 15 Re in the orbit before and after the maneuvers, and the navigation error in the definitive time span from this scenario appears in Figure 12. It is clear from the figures

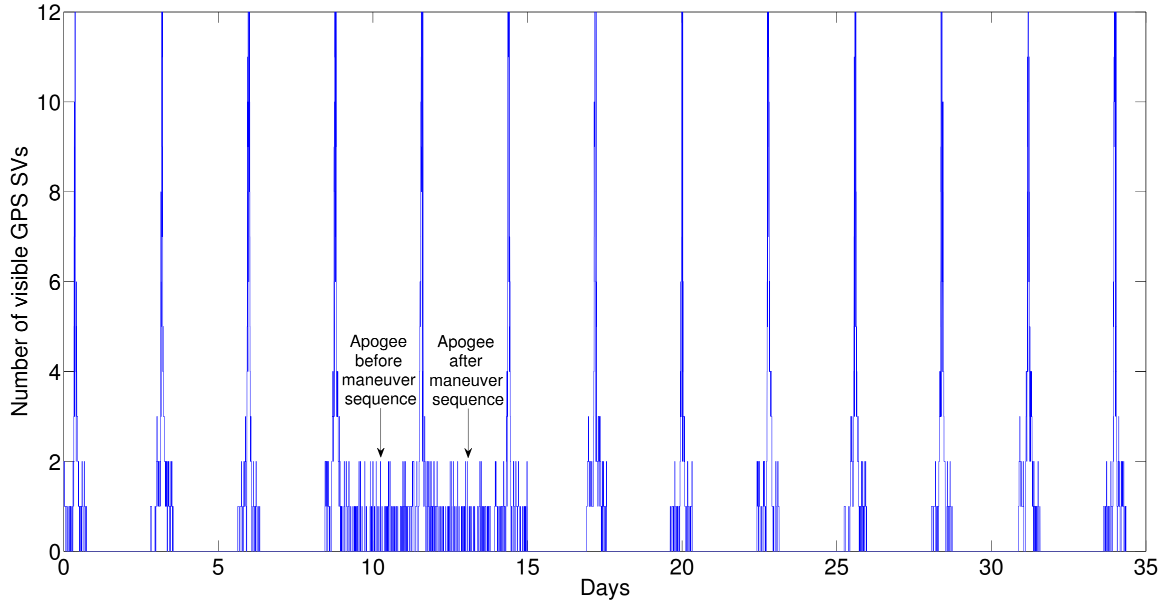

Figure 9. GPS Acquisition versus Time for MMS2 of Phase 2b 40km formation scenario

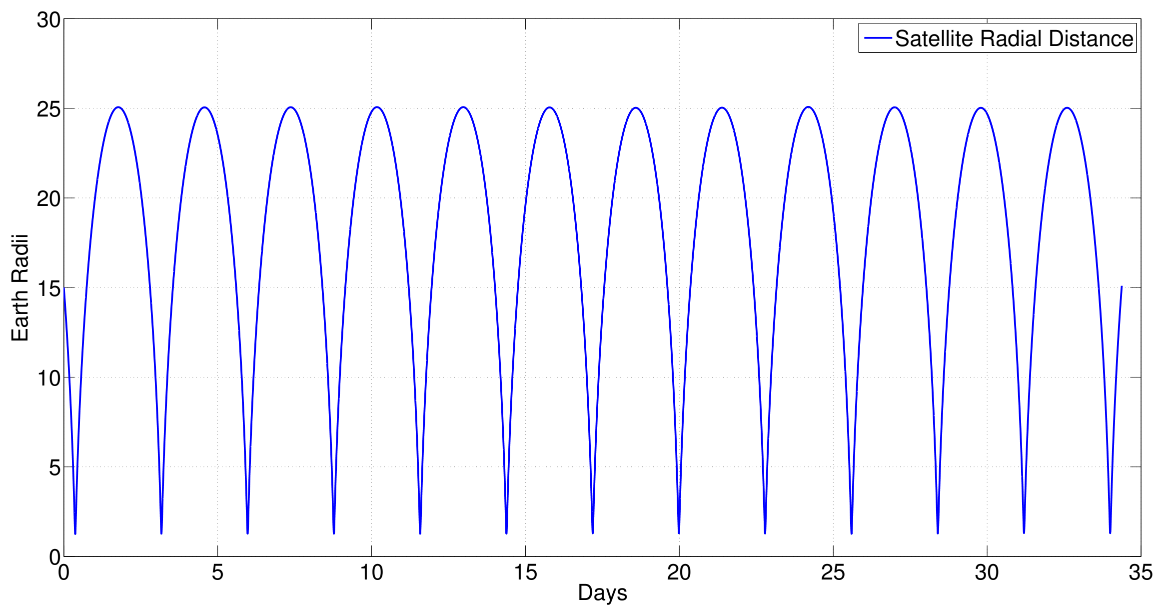

Figure 10. Orbit Radius versus Time for MMS2 of Phase 2b 40km formation scenario

that leaving the receiver card on above 15 Re for those apogees is beneficial, particularly because improved navigation accuracy reduces the risk of spacecraft collisions. Despite this large improvement, Phase 2b remains the most challenging phase of the mission for navigation of the spacecraft due to the length of time spent above the GPS constellation.

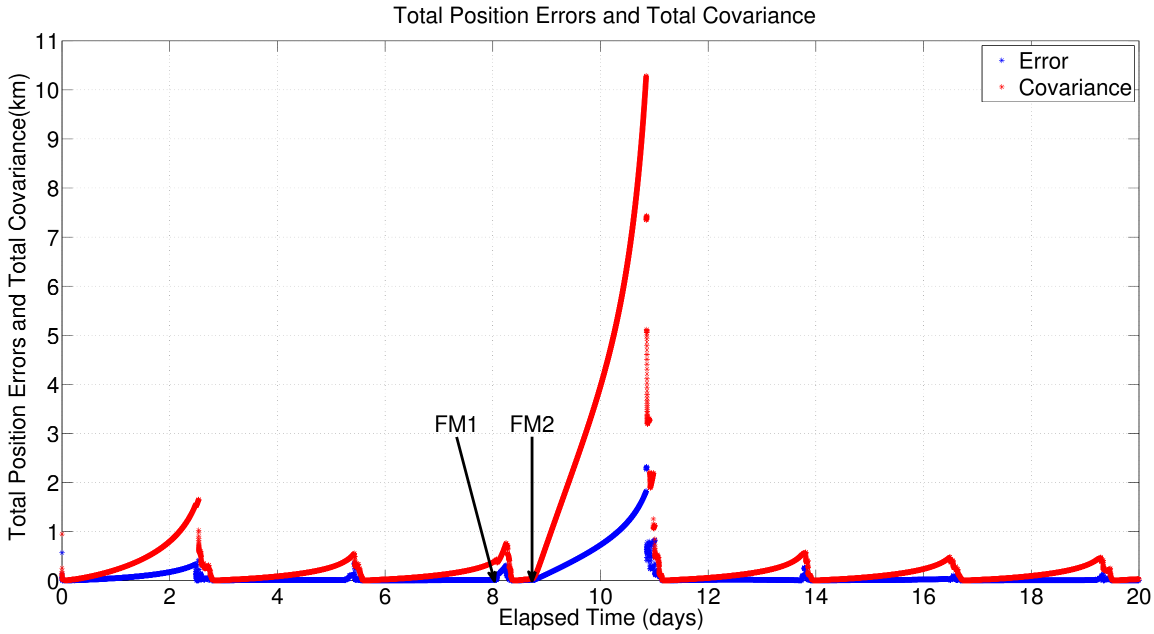

Figure 11. RSS Position Errors and Root-variance without measurements above 15 Re

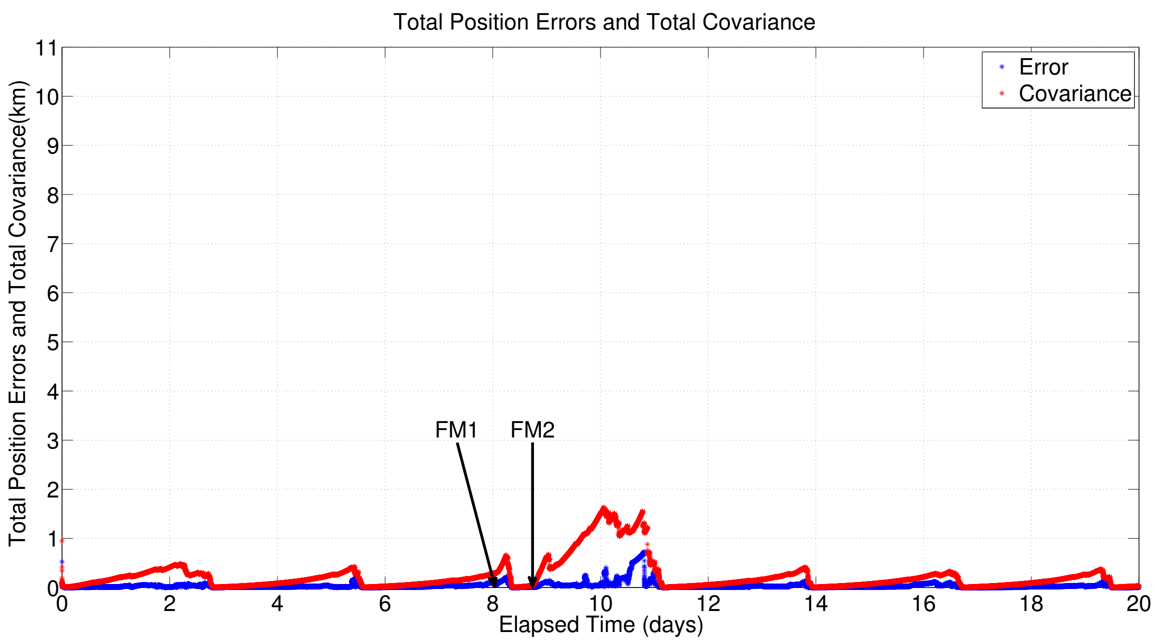

Figure 12. RSS Position Errors and Root-variance with measurements above 15 Re

Filter Settings

The simulation parameters for the EKF within GEONS are based on the new simulation truth trajectories, maneuvers, and GPS pseudorange measurements. These parameters are listed in Table 3. The initial clock acceleration error is listed as "Truth Value" because the error is the difference between the truth value generated by DatSim and the initial value of 0.0 set by GEONS.

Table 3. GEONS Filter Settings

Parameter	Nominal Values
Estimation State	Position, velocity, clock bias, clock bias rate, clock bias acceleration
Initial position and velocity state errors	100 m and 0.5 m/s per axis (1σ)
Initial clock bias, drift, and acceleration errors	100 m (0.3 μs) (1σ), 0.5 m/s (1σ), (Truth Value) m/s^2
GPS PR standard deviation	40 m
Minimum Height-of-Ray-Path Altitude	1000 km (eliminates measurements with largest ionospheric delays)

The steady state and maneuver process noise values used within the GEONS EKF are updated for all of the Launch Window Analysis navigation MC simulation scenarios, and some process noise values are adjusted to improve the navigation performance. This improvement leads to lower steady-state, post-maneuver, and predictive navigation errors.

Acceleration Knowledge Error Model

The acceleration knowledge error model is employed to add expected acceleration knowledge error to the simulated truth 10-second averaged acceleration data. A full revision of this model, based on extensive MC simulations performed by the MMS Attitude Control System (ACS) team, has been incorporated into the navigation analysis simulations.[2]

A description of how the acceleration data is created on the spacecraft is necessary to understand how the acceleration knowledge error model works. During the mission, the Guidance, Navigation and Control (GN&C) subsystem, which encompasses the ACS, converts the raw acceleration measurements, taken by the Acceleration Measurement System four times per second, in the body frame to the mean of J2000 inertial (MJ2000.0) frame using the spacecraft attitude. Then, the GN&C subsystem averages these values over the previous 10 seconds at each whole 10-second mark before delivering the data to the Navigator receiver. The modified thrust acceleration knowledge error model parameters, which include a 50% modeling uncertainty factor, appear in Table 4.

Table 4. Error Coefficients to be used in Acceleration Knowledge Error Model

	1-σ Value	**Fixed Value**
ΔV Knowledge Error **Drift Rate** - Axial Component - Radial Component	 1.5 μg 0.375 μg	 N/A N/A
ΔV Knowledge Error **Random Walk**	1.5 μm/s/s$^{1/2}$	N/A
Acceleration Error **Temperature Variation**	N/A	0.5°C variation with 3 μg/°C sensitivity
ΔV Knowledge Average **Scale Factor Error**	0.1289%	0.5°C variation with 3 ppm/°C sensitivity
ΔV Knowledge Error **Noise**	1.5 mm/s	N/A

The primary modification to the acceleration knowledge error model is that the error is now decomposed into radial and axial components, which are based on the attitude of the spacecraft, rather than magnitude and direction errors. Direction error is still introduced, however, by having different magnitude errors for the axial and radial directions. The axial direction is defined as the direction along the spin axis of the spacecraft, given in the inertial frame at each time of the maneuver acceleration profile. The radial direction is defined perpendicular to the axial direction, along the direction of the net thrust as the spacecraft spins, which is also given in the inertial frame at each time of the maneuver acceleration profile. The error in the averaged acceleration is applied at every acceleration time throughout the maneuver. Thus, the error components listed below are calculated for every time step during the maneuver.

Drift Rate The averaged acceleration knowledge error due to the axial drift rate is calculated with the equation

$$a_{err\_drift\_axial} = 1.5\mu g \; RV_{gauss\_axial}\left(t_0\right) \tag{1}$$

where $1.5\mu g$ is 1.5 * 9.81 * 10^{-6} m/s^2, t_0 is the first epoch in the acceleration profile, and $RV_{gauss\_axial}\left(t_0\right)$ is a random number value generated from a Gaussian distribution of mean equal to 0 and standard deviation equal to 1 and is constant throughout the maneuver. The averaged acceleration knowledge error due to the radial drift rate is calculated with the equation

$$a_{err\_drift\_radial} = 0.375\mu g \; RV_{gauss\_radial}\left(t_0\right) \tag{2}$$

where $RV_{gauss\_radial}\left(t_0\right)$ is constructed in the same way as $RV_{gauss\_axial}\left(t_0\right)$.

Random Walk The averaged acceleration error due to the integration of noise introduced throughout the system is modeled as a random walk. This error is computed with the equation

$$a_{err\_RW\_axial} = 37.2\frac{\mu m/s}{\sqrt{s}}\frac{\sqrt{t-t_{prev}}}{t-t_{prev}} \; RV_{gauss\_RW\_axial}\left(t\right) \tag{3}$$

where t is the current time within the acceleration profile as the algorithm builds the error values, t_{prev} is the previous time of the acceleration profile, and $RV_{gauss\_RW\_axial}\left(t\right)$ is a random number value generated at every acceleration time in the maneuver. For this analysis, $t-t_{prev}$ is always 10 seconds. The acceleration error due to random walk is analogously calculated for the inertial radial component, with the exception of incorporating a different random number at each time t in the acceleration profile, that is, $RV_{gauss\_RW\_radial}\left(t\right)$. The error covariance in the body frame is equivalent to the error covariance in the inertial frame, so it is acceptable to apply the random walk errors to the inertial components.

Temperature Variation The averaged acceleration error due to temperature variation that is directly applied to the acceleration is modeled as a sinusoid, with a 3 $\mu g/°$C sensitivity, a period of one hour, and a temperature variation of 0.5°C, as described by the equation

$$a_{err\_TempVar}(t) = \left(3\times 10^{-6}\frac{1}{°\mathrm{C}}\right)\left(9.81m/s^2\right)\left(0.5°\mathrm{C}\right)\sin\left(\frac{2\pi}{T}\left(t-t_0\right)\right) \tag{4}$$

where T is the period of the temperature sinusoidal variation (typically 60 minutes) that is derived from the fact that the temperature is expected to decrease by 1°C during a 30 minute eclipse, and $t-t_0$ is the time that has elapsed from the maneuver start time. This error is applied equally to the axial and radial directions.

Scale Factor The averaged acceleration error due to scale factor (SF) is determined as follows: the scale factor error varies with time as a sinusoid due to temperature variation, with a 3 ppm/°C sensitivity, a period of one hour, and a temperature variation of 0.5°C, as described by the equation

$$SF(t) = SF_{avg} + \left(3\times 10^{-6}\frac{1}{°\mathrm{C}}\right)\left(0.5°\mathrm{C}\right)\sin\left(\frac{2\pi}{T}\left(t-t_0\right)\right) \tag{5}$$

where SF_{avg} is the average scale factor error of 0.1289%. Using this value for the scale factor error, the averaged acceleration error due to scale factor on the inertial axial component of the imparted acceleration is given by

$$a_{err\_SF\_axial}(t) = \left[SF(t)\left\|\vec{a}_{axial}(t)\right\| - SF(t_{prev})\left\|\vec{a}_{axial}(t_{prev})\right\|\right] RV_{gauss\_SF\_axial}\left(t_0\right) \tag{6}$$

where $\vec{a}_{axial}(t)$ is the axial component of the true 10-second-averaged acceleration in the inertial frame corresponding to time t in the acceleration file. The random number $RV_{gauss\_SF\_axial}(t_0)$ is generated only once at the beginning of the maneuver and used for the duration of the maneuver. The error on the radial component of the inertial acceleration is similarly determined, with a different constant random variable and the scale factor multiplied by the acceleration in the radial inertial direction.

White Noise Finally, the averaged acceleration error due to white Gaussian noise in the axial direction is determined using the equation

$$a_{err\_noise\_axial}(t) = NoiseSigma\frac{[RV_{gauss\_noise\_axial}(t) - RV_{gauss\_noise\_axial}(t)]}{(t - t_{prev})} \tag{7}$$

where $NoiseSigma$ is the standard deviation of the noise to be added and $RV_{gauss\_noise\_axial}(t)$ is a random number value generated at every acceleration time in the maneuver.

Total Acceleration Knowledge Error The component errors from the above equations are summed to generate the total acceleration knowledge error, as described in the equation

$$\begin{aligned}
&\vec{a}_{err_total}(t) = \\
&a_{err_drift_axial} * \vec{u}_{axial}(t) + a_{err_drift_radial} * \vec{u}_{radial}(t) + \\
&a_{err_RW_axial} * \vec{u}_{axial}(t) + a_{err_RW_radial} * \vec{u}_{radial}(t) + \\
&a_{err_SF_axial} * \vec{u}_{axial}(t) + a_{err_SF_radial} * \vec{u}_{radial}(t) + \\
&a_{err_TempVar} * (\vec{u}_{axial}(t) + \vec{u}_{radial}(t)) + \\
&a_{err_noise_axial} * \vec{u}_{axial}(t) + a_{err_noise_radial} * \vec{u}_{radial}(t)
\end{aligned} \tag{8}$$

where $\vec{u}_{axial}(t)$ is the unit vector of the axial component of the acceleration in the inertial frame at each acceleration time and $\vec{u}_{radial}(t)$ is the unit vector of the radial component of the acceleration in the inertial frame at each acceleration time. The acceleration vector with error at a specified time in the maneuver is

$$\vec{a}_{with\_err}(t) = \vec{a}(t) + \vec{a}_{err\_total}(t) \tag{9}$$

Representative Results Using this updated model, new acceleration knowledge errors are generated. The RSS acceleration alone, the RSS acceleration with knowledge error, and the RSS knowledge error alone versus time for the first MMS2 FM maneuver for a Phase 1 25 kilometer separation case appear in Figure 13. The case corresponds to the first run of a MC simulation. The total RSS acceleration knowledge error in this case is approximately 0.74% of the total RSS acceleration, which is less than the 1% total control error requirement.

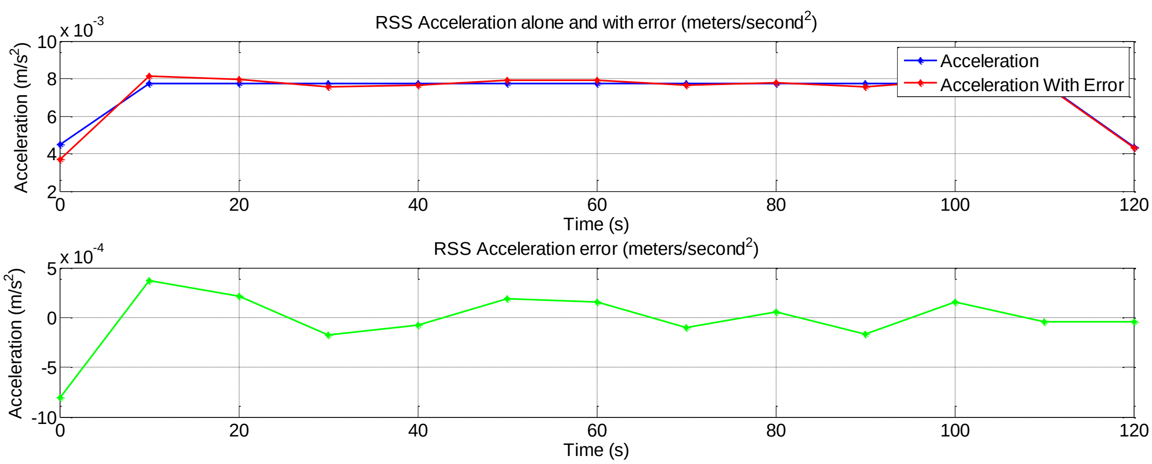

Figure 13. RSS Acceleration alone, with error, and error alone for FM Maneuver 1, Phase 1, 25km separation, MMS2, Run 1

Monte Carlo Variations

MC simulations are performed to investigate navigation performance, determine if navigation requirements are met, and generate error values for use in MD ETE simulations. The parameters varied and the nature of those variations are updated from previous MC analyses. These MC parameters reflect USO error specifications and initial errors expected when GEONS is initialized using a GPS single-point solution, which is computed near perigee when 12 GPS PR measurements are available. These new variations are listed in Table 5.

Table 5. Monte Carlo Variation Parameters for Latest Navigation Analysis

Variable	Varied In	Mean	Distribution	1σ Variation	Min/Max
Initial Time Bias	DatSim	0.0	Uniform	N/A	-0.5/0.5
Initial Time Bias Rate	DatSim	0.0	Gaussian	2×10^{-7} s/s	N/A
Initial Time Bias Acceleration	DatSim	0.0	Uniform	N/A	$0/1.2 \times 10^{-15}$ s/s^2
Clock Error Random Number Seed	DatSim	32767.5	Uniform	N/A	0/65535
Measurement Error Random Number Seed	DatSim	32767.5	Uniform	N/A	0/65535
Reference Solar Radiation Pressure Coefficient	GEONS	1.80	Gaussian	0.180 (10%)	N/A
Reference Drag Coefficient	GEONS	7.73	Gaussian	0.773 (10%)	N/A
Initial Position	GEONS	Truth Value	Gaussian	100 m	N/A
Initial Velocity	GEONS	Truth Value	Gaussian	0.5 m/s	N/A
Initial Time Bias	GEONS	Truth Value	Gaussian	100 m	N/A
Initial Time Bias Rate	GEONS	Truth Value	Gaussian	0.5 m/s	N/A
Thrust Acceleration Errors	MATLAB	Truth Value	Gaussian	See Table 4	N/A

RESULTS

These new models and operations concepts are used in a full navigation analysis conducted to support a Launch Window Analysis, analyzing the feasibility of a launch window in August of 2014. The results include two different scenarios: the first contains a Phase 2b FM maneuver sequence with a formation size of 40 kilometers, and the second contains a Phase 2b FR maneuver sequence that changes the formation size from 400 kilometers to 160 kilometers. Only Phase 2b results are given; recall that Phase 2b is the most challenging phase of the mission for navigation.[5] All results are for the MMS2 spacecraft and are representative of results associated with other formation members.

The position errors for the MMS2 spacecraft for the Phase 2b FM maneuver scenario, which has a formation size of 40 kilometers and executes two FM maneuvers, appear in Figure 14. The minimum, maximum, mean, and mean $\pm$ 3-σ values are plotted for the simulation time span for an ensemble of 75 simulations. Note that the 3-σ values are based on the ensemble of errors rather than the formal covariance. The points labeled FM1 and FM2 are the times of the first and second FM maneuvers. Even with the measurements included above 15 Re after the second maneuver, the largest errors of the definitive time span occur during that time, following day 12. Before day 5 the filter is still converging on the navigation solution, and the predictive time span begins around day 24. At this time GEONS is switched from filtering to propagation mode to simulate trajectory prediction performed on the ground. The first FM maneuver is approximately 0.48 meters per second, and the second FM maneuver is approximately 0.60 meters per second. The steady state position error ranges from about 2 meters RSS following perigee to about 250 meters RSS approaching perigee.

The position errors for the MMS2 spacecraft for the Phase 2b FR maneuver scenario, which transitions from a 400 kilometer formation to a 160 kilometer formation as it executes two FR maneuvers, appear in Figure 15. The first FR maneuver is approximately 4.51 meters per second, and the second FR maneuver is approximately 6.41 meters per second. The errors observed in the FR maneuver scenario are larger than those in the FM maneuver scenario, which is expected due to the longer maneuvers taking more time to complete and the resulting introduction of additional acceleration knowledge error. The steady state position error range is approximately the same as in the FM maneuver scenario.

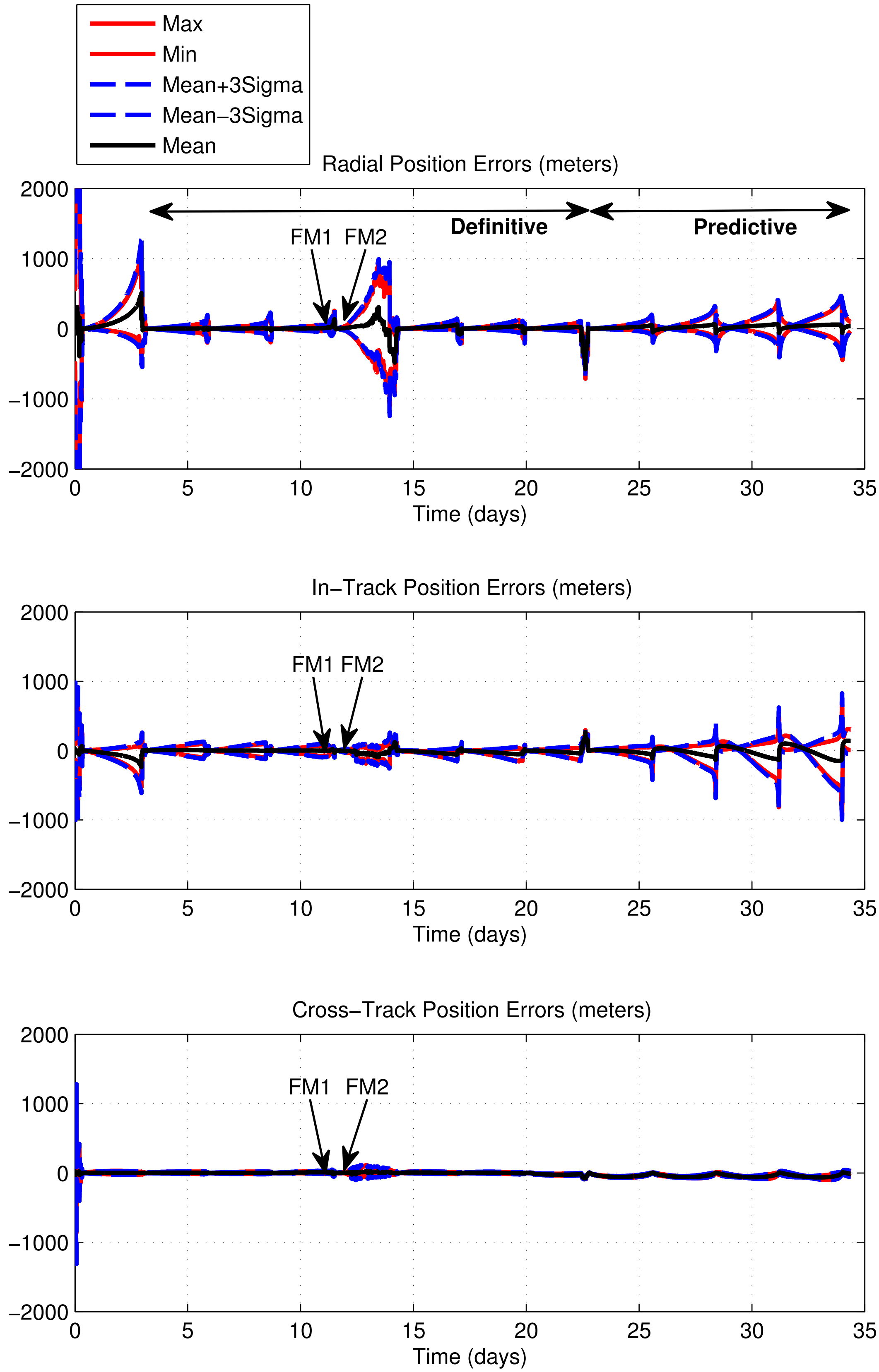

Figure 14. Position Component Errors, Phase 2b, 40km separation, MMS2, Two FM maneuvers

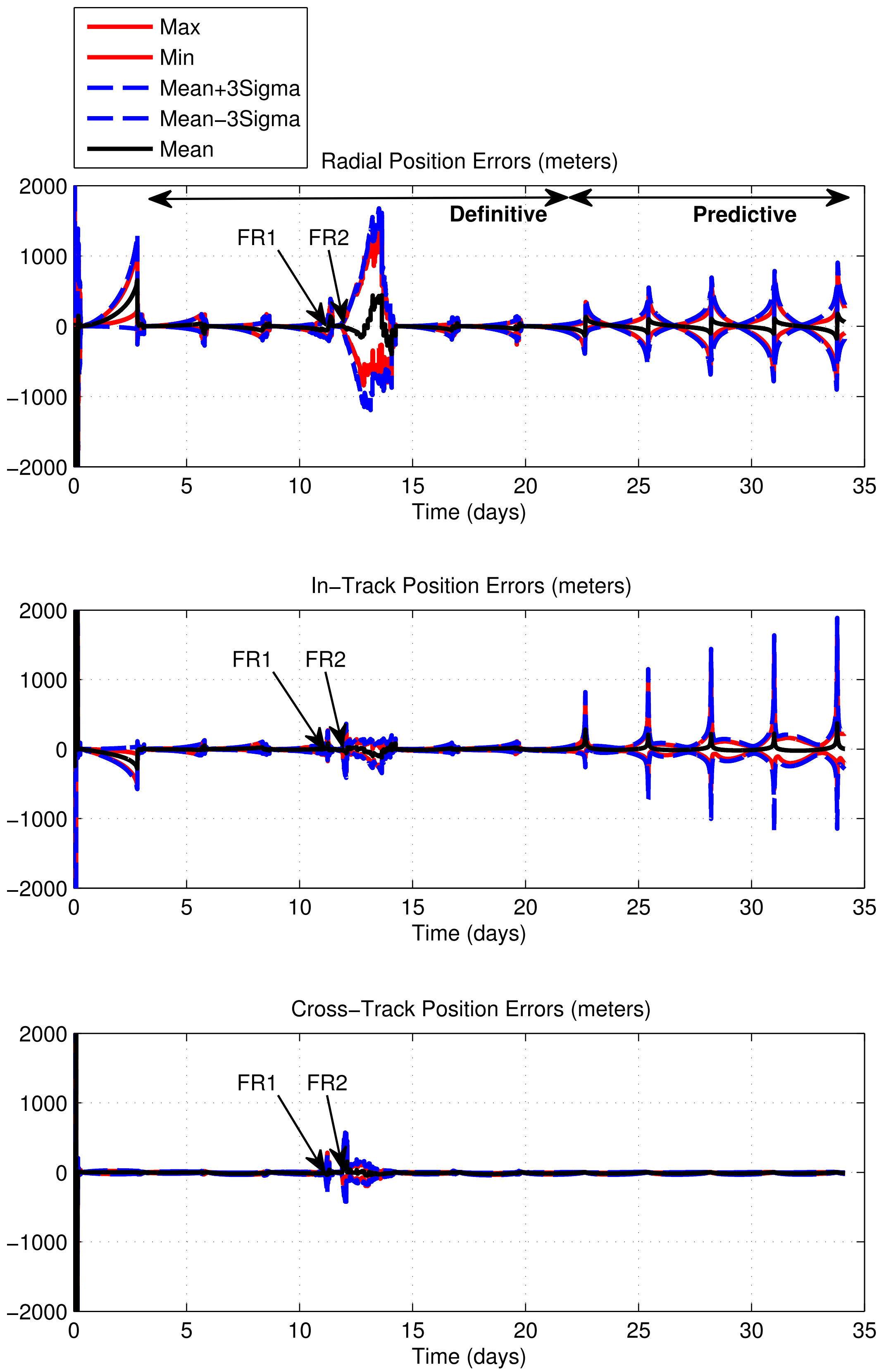

Figure 15. Position Component Errors, Phase 2b, 400km separation, MMS2, Two FR maneuvers

CONCLUSION

Overall, expected navigation performance for the MMS mission is significantly improved significantly due to modifications in the models flown onboard the spacecraft and the flight operations concept. The two primary updates that lead to the most improvement are the increase in fidelity of the gravity model that will be used onboard the spacecraft and the flight rules modification in Phase 2b to allow the Navigator GPS receiver card to remain on and processing measurements above 15 Re both before and after maneuver sequences. All simulations indicate that all navigation requirements are met, and with these improvements there is greater margin than ever before.

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