Lattice Thermal Conductivity from Atomistic Simulations: ZrB$_2$ and HfB$_2$

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NASA Innovative Partners Program (IPP)
Overview

- Motivation and applications
- Multiscale materials modeling
- Atomic structure
- Interatomic potentials
- Simulations of lattice thermal conductivity for ZrB$_2$ and HfB$_2$
- Comparison to experiments
**UHTC for Sharp Leading Edges**

*Sharp leading edge for hypersonic aircraft*
- Enhances vehicle performance
- Improves safety

*Higher temperature requirements*
- Shuttle RCC leading edge: T~1650°C
- Sharp leading edged vehicles: T>2000°C

*UHTC advantages for sharp leading edges*
- Good mechanical properties
- Oxidation resistance
- *High thermal conductivity*
  - Effective thermal radiation
  - Thermal shock resistance

Multiscale Modeling of UHTC

• Framework integrates three methods
• Multiscale framework for ZrB$_2$ and HfB$_2$:
  – Ab initio – fundamental chemistry, electronic structure impact on basic material properties
  – Atomistic – thermal/mechanical properties, adhesion and thermal resistance of grain boundaries, fracture
  – Continuum – macro properties, thermal/mechanical analysis of microstructure
• This talk focuses on atomistic methods
  – Development of interatomic potentials
  – Lattice thermal conductivity simulations
  – Other topics presented elsewhere

JL, Daw, Squire and Bauschlicher, (2012), submitted
Atomic Structure: ZrB$_2$ and HfB$_2$

Alternating layers of Zr/Hf (red) and Boron (gray)

Graphitic Boron layers with Zr/Hf over each ring
Fundamental Properties: ZrB$_2$ & HfB$_2$

**Electronic Spectra**

- Total
- Zr d
- B p
- B s

**Vibrational Spectra**

- Zr/Hf modes
- B modes

Electronic properties essentially identical.

Acoustic modes carry heat.

Optical modes are resistive.

Tersoff Bond Order Potential

- Two body terms \((A, \lambda, B, \mu)\) energy

\[
E = \sum_{i \neq j} \left[ f_R^{ij}(d_{ij}) + b_{ij} f_A^{ij}(d_{ij}) \right]
\]

\[
f_R^{ij}(d) = A_{ij} \exp(-\lambda_{ij}d)
\]

\[
f_A^{ij}(d) = -B_{ij} \exp(-\mu_{ij}d)
\]

- Bond order \((\beta, \lambda_3, n, m)\)

\[
b_{ij} = (1 + \beta_i^{n_i} \zeta_{ij}^{n_i})^{-\frac{1}{2n_i}}
\]

\[
\zeta_{ij} = \sum_{k \neq i, j} f_C^{ij}(r_{ik}) \gamma_{ijk} g_i(\theta_{ijk}) \exp[\lambda_{3i}(d_{ij} - d_{ik})^m]
\]

- Angular function \((c, d, h)\)

\[
g_i(\theta) = 1 + c_i^2 / d_i^2 - c_i^2 / [d_i^2 + (h_i - \cos \theta)^2]
\]

Daw, JL and Bauschlicher, Comp. Mat. Sci., (2011)
First Step: Zr Potential

- Zr potential exists
- Developed new Zr potential
- Fit to *ab initio* database of crystal structures

<table>
<thead>
<tr>
<th>Property(units)</th>
<th>Target</th>
<th>New</th>
<th>WM2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$(FCC) (Å)</td>
<td>4.530</td>
<td>4.510</td>
<td>4.532</td>
</tr>
<tr>
<td>$E_0$(FCC) (eV)</td>
<td>-6.160</td>
<td>-6.159</td>
<td>-6.127</td>
</tr>
<tr>
<td>B(FCC) (eV/Å$^3$)</td>
<td>0.578</td>
<td>0.5899</td>
<td>0.6011</td>
</tr>
<tr>
<td>B'(FCC)(eV/Å$^4$)</td>
<td>-0.8160</td>
<td>-1.635</td>
<td>-1.948</td>
</tr>
<tr>
<td>C$_{11}$(FCC)(eV/Å$^3$)</td>
<td>0.7740</td>
<td>0.6885</td>
<td>0.7404</td>
</tr>
<tr>
<td>C$_{12}$(FCC)(eV/Å$^3$)</td>
<td>0.4810</td>
<td>0.5405</td>
<td>0.5314</td>
</tr>
<tr>
<td>C$_{44}$(FCC)(eV/Å$^3$)</td>
<td>0.3560</td>
<td>0.5307</td>
<td>1.395</td>
</tr>
<tr>
<td>$E_{\text{vac}}$(FCC)(eV)</td>
<td>2.500</td>
<td>6.072</td>
<td>8.338</td>
</tr>
<tr>
<td>$a_0$(HCP) (Å)</td>
<td>3.230</td>
<td>3.159</td>
<td>3.231</td>
</tr>
<tr>
<td>$E_0$(HCP) (eV)</td>
<td>-6.180</td>
<td>-6.242</td>
<td>-5.826</td>
</tr>
<tr>
<td>$E_0$(BCC) (eV)</td>
<td>-6.050</td>
<td>-6.159</td>
<td>-5.960</td>
</tr>
</tbody>
</table>

Williame and Massobrio, PRB 43 (1991), 11653
Second Step: Boron Potential

- No published Boron potentials
- Boron is *electron “deficient”*
- Boron may be “frustrated”
- Fit to simple structures

<table>
<thead>
<tr>
<th>Structure</th>
<th>Property</th>
<th>Target</th>
<th>Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hex sheet</td>
<td>$a_0$</td>
<td>2.91</td>
<td>2.89</td>
</tr>
<tr>
<td></td>
<td>$E_0$</td>
<td>-5.15</td>
<td>-5.08</td>
</tr>
<tr>
<td></td>
<td>$E_0''$</td>
<td>11.35</td>
<td>7.98</td>
</tr>
<tr>
<td>Tri sheet</td>
<td>$a_0$</td>
<td>1.70</td>
<td>1.81</td>
</tr>
<tr>
<td></td>
<td>$E_0$</td>
<td>-5.71</td>
<td>-5.75</td>
</tr>
<tr>
<td></td>
<td>$E_0''$</td>
<td>21.73</td>
<td>27.06</td>
</tr>
<tr>
<td>SC</td>
<td>$a_0$</td>
<td>1.88</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>$E_0$</td>
<td>-5.33</td>
<td>-5.21</td>
</tr>
<tr>
<td></td>
<td>$E_0''$</td>
<td>24.50</td>
<td>24.51</td>
</tr>
<tr>
<td>FCC</td>
<td>$a_0$</td>
<td>2.86</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>$E_0$</td>
<td>-5.07</td>
<td>-5.22</td>
</tr>
<tr>
<td></td>
<td>$E_0''$</td>
<td>21.85</td>
<td>12.28</td>
</tr>
</tbody>
</table>

Ogitsu et al, JACS 131 (2009) 1903
Third Step: ZrB$_2$ Potentials

- Zr-Zr parameters fixed
- B-B parameters fixed
- Zr-B fit to small database
- Pot A = “new Zr” + B
- Pot B = WM2 + B
- *Will Boron planes stay flat?*

### Fitting Results

<table>
<thead>
<tr>
<th>Property</th>
<th>Target</th>
<th>Pot A</th>
<th>Pot B</th>
</tr>
</thead>
<tbody>
<tr>
<td>a$_0$(A)</td>
<td>3.170</td>
<td>3.143</td>
<td>3.140</td>
</tr>
<tr>
<td>c$_0$(A)</td>
<td>3.550</td>
<td>3.547</td>
<td>3.547</td>
</tr>
<tr>
<td>E$_0$(eV)</td>
<td>-21.70</td>
<td>-21.29</td>
<td>-21.55</td>
</tr>
</tbody>
</table>

Stable, multilayered system with **flat, hexagonal** Boron sheets!
Lattice Thermal Conductivity

- Green-Kubo thermal conductivity tensor

\[ \kappa_{\mu\nu} = \frac{1}{V k_B T^2} \int_0^\infty \langle J_\mu(\tau) J_\nu(0) \rangle d\tau \]

- Heat current \( J(x_i,v_i) \), energy \( e_i \), stress-tensor \( S_i \)

\[ J = \frac{1}{V} \left[ \sum_i e_i v_i - \sum_i S_i v_i \right] \]

\[ J = \frac{1}{V} \left[ \sum_i e_i v_i + \frac{1}{2} \sum_{i<j} \left( f_{ij} \cdot (v_i + v_j) \right) \cdot x_{ij} \right] \]

Heat Current Correlation Function

- Monoatomic systems (e.g. Si) have monoatomic decay
- ZrB$_2$ has longer period than HfB$_2$ at T=300K
- ZrB$_2$ at T=1000K has longer period than T=300K
Correlation Function Power Spectra

- Correlations oscillates with metal-B optical modes
- $C_{xx}$ and $C_{yy}$ oscillate with in-plane mode frequency
- $C_{zz}$ oscillates with out-of-plane mode frequency
Lattice Thermal Conductivity: ZrB$_2$

- 8 independent, 10 ns simulations, $T=300K$
- 8x8x16 unit cell, 12,255 atoms
- $\kappa_{xx} = 60 \text{ W/(m.K)}$, $\kappa_{zz} = 40 \text{ W/(m.K)}$
Lattice Thermal Conductivity: HfB$_2$

- 8 independent, 10 ns simulations
- 8x8x16 unit cell (12 atoms) = 12,255 atoms
- $\kappa_{xx} = 76 \text{ W/(m.K)}$, $\kappa_{zz} = 65 \text{ W/(m.K)}$
Thermal Conductivity vs Temperature

- 8 independent, 10 ns simulations for each point
- Data fit to $1/T$ curves
Experimental Data Comparison

• Polycrystalline ZrB$_2$
  • $\kappa_e = 33$ W/mK, $\kappa_{lat} = 22$ W/mK
  • $\kappa_{lat} \approx 0.3 \kappa_{tot}$
• Single crystal ZrB$_2$
  • $\kappa_{xx} = 140$ W/mK, $\kappa_{zz} = 100$ W/mK
  • 1 sample, 1 measurement
  • defects uncharacterized
  • $\kappa_{xx} = 45$ W/mK , $\kappa_{zz} = 30$ W/mK
• Data needed for ZrB$_2$ and HfB$_2$
• *Simulation data reasonable at 300K but too low for higher T*

$K = K_e + K_{lat}$

Conclusions

- Atomistic simulations for ZrB$_2$ and HfB$_2$:  
  - Developed first interatomic potentials for UTHC  
  - Lattice thermal conductivity using Green-Kubo formalism  
  - Heat current correlation function oscillations  
  - Thermal conductivity versus temperature  
  - Reasonable agreement with experiment

- Modeling unanswered questions:  
  - Interatomic potential fidelity  
  - Lattice TC without potentials (ab initio, Boltzmann,...)  
  - Conducting versus resistive vibrational modes  
  - Isotope and defect effects  
  - Interface thermal resistance: grain boundaries *

- Experimental unanswered questions:  
  - Single crystal characterization and thermal conductivity  
  - Electronic versus lattice thermal conductivity

* JL, Daw, Squire and Bauschlicher, (2012), submitted
Extra Slides
ZrB$_2$ Potential Curves

**Test Results**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Ab Initio</th>
<th>Pot A</th>
<th>Pot B</th>
</tr>
</thead>
<tbody>
<tr>
<td>C$_{11}$</td>
<td>556</td>
<td>365</td>
<td>422</td>
</tr>
<tr>
<td>C$_{12}$</td>
<td>57</td>
<td>156</td>
<td>156</td>
</tr>
<tr>
<td>C$_{13}$</td>
<td>113</td>
<td>173</td>
<td>171</td>
</tr>
<tr>
<td>C$_{33}$</td>
<td>419</td>
<td>307</td>
<td>320</td>
</tr>
<tr>
<td>C$_{44}$</td>
<td>234</td>
<td>106</td>
<td>119</td>
</tr>
<tr>
<td>B</td>
<td>233</td>
<td>227</td>
<td>240</td>
</tr>
<tr>
<td>G</td>
<td>226</td>
<td>98</td>
<td>118</td>
</tr>
<tr>
<td>A(=C$<em>{33}$/C$</em>{11}$)</td>
<td>0.75</td>
<td>0.84</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Properties not included in fit

VASP = *ab initio* code
Physics of Lattice Thermal Conductivity

- $\kappa = \rho C_v \nu l_{mfp}$
  - scattering restricts $l_{mfp}$
- **Region I**: $\kappa \sim T^3$
  - dilute phonons
  - boundary scattering
  - quantum statistics
- **Region II**: $\kappa_{\text{max}}$
- **Region III**: $\kappa \sim 1/T$
  - high phonon density
  - phonon, pt. defect scattering
- **Region IV**: $\kappa_{\text{min}}$, $l_{mfp} = "a"$

Summary

• No atomistic simulations for $\text{ZrB}_2$ due to lack interatomic potentials
• Potentials are prerequisite for atomistic simulations of *mechanical* and *thermal* properties
• We developed such potentials for ZrB$_2$
• ZrB$_2$ potentials give stable structures with flat, hexagonal B planes
• We performed the first atomistic simulations for these materials
• Lattice thermal conductivity was evaluated for single crystals
• Reasonable agreement with experiments

• **Future/current work:**
  • Grain boundaries: energetics and thermal interface resistance
  • Integration into multiscale framework
  • Potentials and applications for Hf and HfB$_2$

JL, Daw, Squire and Bauschlicher, (2012), submitted