Calibration and Data Analysis of the MC–130 Air Balance

D. Booth
Triumph Aerospace, Force Measurement Systems, San Diego, California 92121

N. Ulbrich
Jacobs Technology Inc., Moffett Field, California 94035–1000

Design, calibration, calibration analysis, and intended use of the MC–130 air balance are discussed. The MC–130 balance is an 8.0 inch diameter force balance that has two separate internal air flow systems and one external bellows system. The manual calibration of the balance consisted of a total of 1854 data points with both unpressurized and pressurized air flowing through the balance. A subset of 1160 data points was chosen for the calibration data analysis. The regression analysis of the subset was performed using two fundamentally different analysis approaches. First, the data analysis was performed using a recently developed extension of the Iterative Method. This approach fits gage outputs as a function of both applied balance loads and bellows pressures while still allowing the application of the iteration scheme that is used with the Iterative Method. Then, for comparison, the axial force was also analyzed using the Non–Iterative Method. This alternate approach directly fits loads as a function of measured gage outputs and bellows pressures and does not require a load iteration. The regression models used by both the extended Iterative and Non–Iterative Method were constructed such that they met a set of widely accepted statistical quality requirements. These requirements lead to reliable regression models and prevent overfitting of data because they ensure that no hidden near–linear dependencies between regression model terms exist and that only statistically significant terms are included. Finally, a comparison of the axial force residuals was performed. Overall, axial force estimates obtained from both methods show excellent agreement as the differences of the standard deviation of the axial force residuals are on the order of 0.001 % of the axial force capacity.

Nomenclature

\( AF \) = axial force component, [lbs]
\( B_1 \) = square matrix; used by iteration equation of alternate load iteration method
\( B_2 \) = square matrix; used by iteration equation of alternate load iteration method
\( C_1 \) = square matrix; used by iteration equation of primary load iteration method
\( C_2 \) = rectangular matrix; used by both primary and alternate load iteration method
\( F_i \) = balance load
\( F \) = part of matrix \( G \) that contains loads
\( G \) = load matrix
\( H \) = part of matrix \( G \) that contains absolute value and non–linear terms
\( k \) = dependent variable index
\( m \) = number of applied loads –or– strain–gage outputs
\( n \) = total number of possible regression model terms
\( NF \) = normal force, [lbs]
\( N1 \) = normal force component at forward gage, [lbs]
\( N2 \) = normal force component at aft gage, [lbs]
\( p \) = distance of normal force gages, [in]
\[ PM = \text{pitching moment, [in–lbs]} \]
\[ PR_1 = \text{first bellows pressure, [psia]} \]
\[ PR_2 = \text{second bellows pressure, [psia]} \]
\[ q = \text{distance of side force gages, [in]} \]
\[ R_i = \text{electrical output of a strain–gage} \]
\[ RM = \text{rolling moment component, [in–lbs]} \]
\[ R_1, R_2, \ldots, R_6 = \text{electrical outputs of the strain–gages of the balance, [microV/V]} \]
\[ s = \text{total number of independent variables (factors)} \]
\[ SF = \text{side force, [lbs]} \]
\[ S_1 = \text{side force component at forward gage, [lbs]} \]
\[ S_2 = \text{side force component at aft gage, [lbs]} \]
\[ YM = \text{yawing moment, [in–lbs]} \]
\[ \Delta AF = \text{residual of axial force, i.e., difference between measured and fitted value} \]
\[ \Delta R = \text{delta gage output vector or matrix} \]
\[ \zeta_1, \zeta_2, \ldots = \text{coefficient used in regression model of strain–gage outputs (Iterative Method)} \]
\[ \zeta_0 = \text{intercept term} \]
\[ \eta_1, \eta_2, \ldots = \text{coefficient used in regression model of loads (Non–Iterative Method)} \]
\[ \eta_0 = \text{intercept term} \]
\[ \xi = \text{load iteration step index} \]

**I. Introduction**

The MC–130 air balance was developed and manually calibrated at Triumph Aerospace (Force Measurement Systems) in San Diego. The balance itself is a six–component force balance that has two separate internal air flow systems and one external bellows system. It is designed such that the pressurized external bellows will primarily influence the electrical outputs of the axial force gage. The calibration of the balance consisted of a total of 1854 data points. A subset of 1160 data points was selected for the calibration data analysis because the remaining data points were not available when the analysis was performed. About 50% of those data points was recorded when the balance was pressurized.

The calibration data was analyzed at the NASA Ames Balance Calibration Laboratory using first a recently developed extension of the Iterative Method (see Ref. [1] for a description of this extension). The extension was specifically designed such that the basic math term order and load iteration scheme of the Iterative Method would remain unchanged (see, e.g., Ref. [2] for a general description of the Iterative Method). Then, in order to compare calibration data analysis results for different analysis approaches, it was decided to also analyze the axial force of the MC–130 using the Non–Iterative Method (see Ref. [3] for a description of this alternate analysis approach). The data analysis for both the extended Iterative Method and the Non–Iterative Method itself was performed using optimized regression models. They were obtained by applying a regression model optimization process to the calibration data that is implemented in NASA Ames’ regression analysis tool BALFIT (see Refs. [4] and [5]).

In the next part of the paper the design and manual calibration of the MC–130 at Triumph Aerospace are discussed in more detail. Then, results of the regression analysis of the calibration data are presented.

**II. Balance Design and Calibration**

**A. History**

The design of air balances has evolved over the years. Many improvements were made as the result of wind tunnel test experiences and changing customer requirements and expectations. The early balances had internal bellows integrated with a basic force type balance in a standard balance envelope. These early balances used convoluted bellows which were found to be rigid and caused higher levels of hysteresis in the strain–gage outputs due to their non–repeatable characteristics during pressurization. The non–repeatability prevented the balance / air system assembly from achieving the high level of accuracy that users of a balance system require. Through testing of different types of bellows and attachments, the welded bellows with a
precision flange attachment were found to have a much higher level of repeatability, flexibility, and a lower hysteresis than other types of bellows.

The resulting balance system was also tested for momentum effects of the air flowing at pressure. It was found that the implementation of the opposing welded bellows and the routing path of the air through several very carefully oriented 90 degree turns as the flow crossed from the non-metric to the metric side of the balance helped reduce momentum effects. Testing confirmed the momentum effects were minimized and also determined it to be a minimal factor in the use of the balance system. The overall performance of the air balance system, with a well developed calibration method, was able to achieve acceptable results. The air balance design concept was expanded to include numerous variations of the air balance.

B. Design

In principle, the design of the MC–130 air balance integrates a force type of balance (standard, flexured or high capacity) with an air flow distribution system that attaches to the non–metric side and passes air to the metric side. The balance performance is subject to slight compromises in order to achieve the overall objectives of the total system. The balance design was started with the basic understanding of (i) how the air system has to bridge the balance either internally or externally, (ii) the amount of area required to achieve the mass flow and pressure required to supply the volume of air needed at the model, and (iii) the wind tunnel model geometry constraints. The challenge during the design of the MC–130 balance was to include test specific needs, which included the standard requirement of load capacity, resolution, rigidity, performance, and a myriad of smaller issues that had to be considered and accommodated. The addition of the air system to the balance created a challenge of “package” versus “performance” which, through many iterations and trade-offs, resulted in an integrated system design that best met all of the requirements and minimized any negative characteristics. A force type of balance like the MC–130 is an extremely stiff platform with very low deflections, which helps minimize negative affects of the attached air system. The balance design criteria are similar to those of the standard balance as load and gage output requirements, geometry, constraints, deflection limits, and other issues have to be considered. The design challenges, however, are increased with the addition of integrating the air system and model interfaces with the balance while maintaining performance standards.

C. Air Passage Design

The design of the air flow through the MC–130 required a significant level of effort. It is a challenging task to (i) maintain the mass flow requirements, (ii) minimize impact to the balance performance, (iii) implement an efficient air flow interface for the non–metric and metric side of the balance, and (iv) create a dependable long lasting system while trying to simplify manufacturing.

The concentric design air tube configuration in the MC–130 was very successful in allowing multiple air pressure systems within a single center passage. This approach makes it possible for each of the systems to operate independently of each other while maintaining the integrity of each air system and the performance of the balance.

The air system / bellows do have performance characteristics that need to be fully defined. The movement of the bellows during pressurization generates repeatable, though sometimes complex, output on the balances components. There are multiple processes to reduce the effects of the pressurization on the balance. Figure 1, for example, shows a 2.5 inch balance and air system prior to assembly. Figures 2 and 3 depict bellows both in a test fixture and installed on an air tube. The design solution chosen for the MC–130 balance is shown in Figs. 4 to 7.

D. Balance Fabrication and Instrumentation

The MC–130 balance and air systems were originally built in the early 1990’s for a test program prior to the one it is currently being used for. At the time the standard balance fabrication processes and procedures were completed to meet the exacting tolerances to maintain fits and functionality required for a complex assembly. In addition, the balance air system was fabricated (i) to meet the tolerance requirements and (ii) it was pressure tested to insure proper operation of the air passages prior to assembly.

The instrumentation of the MC–130 balance was completed per Triumph’s standard instrumentation process. The balance was prepared for the installation of the strain–gages, terminals, interconnected wires and cables. The balance gages are connected into 4 or 8 gage Wheatstone bridges. Figure 8 shows the
strain–gages and wiring on an air balance.

The flexure size often allows for extra strain gages to be installed, making it possible for a redundant “backup” bridge to be on the balance for some of the load components. The overall life cycle for a balance using current materials and standards is estimated at 30+ years. With moderate use and a good stable storage environment, the life cycle is not based on age but performance. Fortunately, the MC–130 balance showed no performance deterioration due to age.

E. Combined Balance System Description

The MC–130 balance has two separate internal air systems and, due to wind tunnel model requirements, an additional external air supply system. The position of the external bellows, which were a required part of the external air supply system, resulted in a large area that generated an additional force on the axial force component when the bellows were pressurized. The large axial force was in excess of the original design loads of the balance. Therefore, the balance load envelope had to be redefined to accommodate the 140 percent increase in the axial force component. The increase in the axial force limit was achieved by lowering the maximum allowed loads on the other components while still maintaining the overall balance stress levels.

In addition, the other 5 components loads were tailored to the anticipated loads for the wind tunnel testing. The “hybrid” force type balance, such as the MC–130, permits a user to redefine the overall allowable load envelope of the balance. Triumph reworked the stress analysis and calculated the stress levels of the force measuring flexures to accommodate large changes in the operating envelope. Therefore, it was possible to extend the axial load range from the original design load of 500 pounds to the redefined load of 1200 pounds. Figure 9 shows the redefined load range for the MC–130–8.0 balance.

One of the MC–130s internal air system supplied air flow to propulsion simulator units that were to be used during the wind tunnel test (see Fig. 10 for a summary of the air flow data of the MC–130 balance). The air system was routed to separate plenums with onboard controllers that metered the flow as required to the two propulsion simulators and for the various air pathways on the wings of the wind tunnel test model. The other internal air system was not used.

F. Calibration Hardware

The calibration of the MC–130 air balance and the attached external air passage as a single system required the fabrication of a calibration fixture that would allow precision application of loads to fully characterize the balance system. The balance capacity allowed a large heavy box of steel plates to be designed that would fully enclose the balance and allow for single and multi–component loads to be applied. Load points were located on the centerline of all 6 faces of the box. Additional load points were located off the centerline to establish a grid pattern on the top and bottom surfaces. Precision locations were established by mapping the calibration equipment X, Y, and Z locations for each load point. Figure 11 shows the balance assembly with installed calibration hardware.

G. Calibration Process

In general, the calibration of the 6–component air balance differs from that of a conventional 6–component balance. The pressurized bellows systems typically causes changes of the strain–gage bridge outputs and also modifies the stiffness of the balance. Therefore, the primary sensitivity terms and some of the interaction terms will be influenced noticeably. These effects are functions of the level of pressure in the bellows system. Consequently, the bellows pressures must be considered as calibration variables in addition to the traditionally used six load components. In other words, the pressure systems transformed a 6–component balance into one having 8–components. This change considerably increases the complexity of the calibration loading design necessary to achieve an adequate characterization of the balance.

In theory, strain–gage outputs of an air balance may also depend on the momentum flux (mass flow) of the air supply systems. Fortunately, the influence of the momentum flux on the gage outputs of the MC–130 was small enough to be effectively ignored. This simplification did not negatively influence the stated load prediction accuracy goals.

The balance assembly was calibrated with the application of manual dead weight loads to the load point locations that were discussed in the previous section. This allowed the global characterization of the balance both with and without pressure. At the beginning of the calibration pressure tests verified that the balances bellows and air system were in good working order. Then, the load calibration began with the single
application of loads and pressures to establish a baseline of performance of the balance assembly. This was followed by the application of combinations of two, three, or more loads being applied to several load locations simultaneously. Then, pressures and loads were simultaneously applied in varying combinations to complete a matrix of load and pressure combinations that populated the cross plots of each two components versus each other. This method does not necessarily identify near–linear dependencies (co–linearity) between regression model terms of the calibration data but does help to exercise the various load and pressure combinations that the balance will likely encounter during the wind tunnel test. Figures 12a and 12b show plots of all loads and bellows pressures that were applied during the calibration of the balance.

III. Calibration Data Analysis

A. General Remarks

The regression analysis of the calibration data of the MC–130 was performed using both the extended Iterative and the Non–Iterative Method. The complete calibration data set consists of a total of 1854 data points. Only 1160 of these data points were selected for the current analysis because the remaining 694 data points were not available when the analysis was performed.

Six applied load components, two bellows pressures, and six measured strain–gage outputs were supplied for each data point. The two bellows pressures have a noticeable influence on the outputs of the axial gage. Therefore, their presence added a certain degree of complexity to the regression analysis problem that made it possible to evaluate a recently developed extension of the Iterative Method.

The applied balance loads were originally provided in direct–read format. The MC–130, however, is a force balance. Therefore, four of the six gage outputs are no longer proportional to the corresponding balance load. This disadvantage of expressing loads of a force balance in direct–read format is depicted in Figs. 13a and 13b using the normal force gage outputs of the balance as an example. The two figures show the forward and aft normal force gage outputs $R_{1}$ and $R_{2}$ plotted versus the normal force $NF$ and pitching moment $PM$. It can clearly be seen that the loads and gage outputs are no longer located along a straight line which makes the interpretation of the calibration data more difficult. In addition, the primary gage sensitivities of the normal force and side force gages cannot be defined unless the normal force, side force, and related moments are transformed from direct–read format to force balance format (see Ref. [6] for a discussion of balance load and gage output formats).

The original load set in direct–read format, i.e., $NF$, $PM$, $SF$, $YM$, $RM$, $AF$, was transformed to force balance format, i.e., $N_{1}$, $N_{2}$, $S_{1}$, $S_{2}$, $RM$, $AF$, using the following equations (from Ref. [7]):

\[
N_1 = \frac{NF}{2} + \frac{PM}{p} \tag{1a}
\]
\[
N_2 = \frac{NF}{2} - \frac{PM}{p} \tag{1b}
\]
\[
S_1 = \frac{SF}{2} + \frac{YM}{q} \tag{1c}
\]
\[
S_2 = \frac{SF}{2} - \frac{YM}{q} \tag{1d}
\]

$RM$ \\
$AF \Rightarrow$ not transformed \tag{1e}

where the distances $p$ and $q$ between the forward and aft gages of the normal and side force components of the MC–130 balance are given as

\[
p = q = 16.5 \text{ [in]} \tag{2}
\]

Figures 14a and 14b show the forward and aft normal force gage outputs $R_{1}$ and $R_{2}$ plotted versus the transformed normal force components $N_{1}$ and $N_{2}$ at the forward and aft gages. Now, loads and gage outputs are again located along a straight line. Consequently, the primary gage sensitivities of the normal force and side force gages exist.
The balance is designed such that the two bellows pressures have primarily an impact on the electrical outputs of the axial force gage. This behavior can easily be understood if the axial gage output \( R_6 \) is first plotted versus the axial force \( AF \) for the subset of the calibration data that was obtained for the unpressurized balance (see Fig. 15a). Then, the axial gage output \( R_6 \) is plotted versus the axial force \( AF \) for the entire calibration data set that has data points for both the pressurized and unpressurized balance (see Fig. 15b). It can clearly be seen, after comparing Fig. 15a with Fig. 15b, that the data points of the unpressurized balance are located on a straight line through the origin of the coordinate system. The data points of the pressurized balance, on the other hand, are located in a region that is highlighted by a red oval in Fig. 15b.

It will be shown in the next section how the extended version of the Iterative Method can be applied to the calibration data of the MC–130. Then, for comparison, the Non–Iterative Method is used to develop a regression model for the axial force so that analysis results for the extended version of the Iterative Method can be compared with corresponding results for the Non–Iterative Method.

**B. Iterative Method**

This section describes the calibration data analysis using a recently developed extension of the Iterative Method. This extension is discussed in great detail in Ref. [1]. Therefore, only an abbreviated description of the method will be given in this section.

In principle, the Iterative Method fits gage outputs as a function of applied calibration loads. The extension of the Iterative Method simply treats additional calibration variables like pressures or temperatures exactly like balance loads. Therefore, the generic regression model of the gage outputs of the MC–130 has the following form after the two bellows pressures are included as independent calibration variables:

\[
R_k = \zeta_0(k) + \zeta_1(k) \cdot F_{t} + \cdots + \zeta_m(k) \cdot F_{t} + \zeta_{m+1}(k) \cdot P_{R1} + \zeta_{m+2}(k) \cdot P_{R2} + \cdots \tag{3}
\]

The MC–130 balance is a six–component air balance with two bellows pressures. Consequently, each gage output depends on a total of eight independent variables (factors). The total number of possible regression model terms for each gage output can easily be computed considering the ten main term choices defined in Ref. [2]. The following formula may be used for that purpose

\[
N = 1 + 2 \cdot s \cdot ( s + 2 ) \tag{4}
\]

where \( s \) is the number of independent variables. Therefore, as the MC–130 data set has eight independent variables, a total of 161 regression model term choices are possible.

Balance loads need to be computed iteratively after the completion of the regression analysis of the gage outputs. Therefore, the extension of the Iterative Method introduces the bellows pressures as both independent and dependent variables so that the initial guess of the load iteration process can be computed as usual by multiplying the inverse of a linear coefficient matrix with the measured gage output differences. This idea has the advantage that the load iteration process can be applied to an air balance without any modifications.

In principle, one of two iteration equation options may be used in combination with the load iteration process that the extended Iterative Method performs. These options are defined as follows (see also Ref. [6], Eqs. (1) and (2)):

\[
\text{Primary Load Iteration Method} \implies F_{\xi} = \underbrace{C_1^{-1} \Delta R}_{\text{constant}} - \left[ C_1^{-1} \cdot C_2 \right] \cdot H_{t-1} \tag{5a}
\]

\[
\text{Alternate Load Iteration Method} \implies F_{\xi} = \underbrace{B_1^{-1} \Delta R}_{\text{constant}} - \left[ B_1^{-1} \cdot B_2 \right] \cdot F_{\xi-1} - \left[ B_1^{-1} \cdot C_2 \right] \cdot H_{t-1} \tag{5b}
\]
The calibration data was analyzed using the Iterative Method in combination with both Eq. (5a) and Eq. (5b). In addition, the regression model optimization process outlined in Refs. [4] and [5] was applied. Both iteration equations independently lead (i) to a convergence of the iteration process and (ii) to identical load predictions. This observation is expected as (i) all matrices used by both load iteration equations, i.e., $C_1$, $C_2$, $B_1$, and $B_2$, are derived from the same regression model coefficients of the gage outputs and (ii) the balance loads are given in force balance format, i.e., in the design format of the balance. The regression models of the six gage outputs and two bellows pressures are shown in Fig. 16a (not all regression model terms are depicted because of space limitations).

For simplicity, only the regression analysis result for the axial gage output $R6$ is discussed in more detail in the paper. Figure 16b shows the Analysis of Variance (ANOVA) result for the optimized 18–term regression model of the axial gage output (see Refs. [8] and [9] for more details on both calculation and interpretation of the statistical metrics that are displayed on the ANOVA page). Three important observations can be made after inspecting the ANOVA results. (1) The coefficients of the regression model are all statistically significant because the p–value of each coefficient is less than the threshold 0.0001. (2) The regression model of the axial gage output does not have any near–linear dependencies because all primary Variance Inflation Factor (VIF) values are less than the threshold of 10. (3) The terms $AF$ and $PR2(i)$ have by far the largest $t$–statistic (+3628 and −835). Therefore, those two terms have the greatest influence on the axial gage outputs. This observation agrees with intuitive expectations that an analyst may have who is familiar with the design of the balance.

The data reduction matrix of the balance calibration data was assembled after the completion of the regression analysis of the gage outputs. Then, coefficients of the data reduction matrix were combined with the iteration equation defined in Eq. (5a) and the load iteration process to get a set of fitted loads that could be compared with the original set of applied balance loads. The load iterations converged after only 5 steps (see Fig. 16c). Finally, Fig. 16d shows the load residuals for the axial force. Overall, the accuracy of the axial force prediction is excellent as the standard deviation of the load residuals is only 0.1114 % of the axial force capacity.

C. Non–Iterative Method

The balance calibration data of the MC–130 air balance was also analyzed using the Non–Iterative Method so that residuals of the axial force prediction could be compared with corresponding residuals for the extended version of the Iterative Method. Differences between the Non–Iterative Method and the Iterative Method are discussed in great detail in Ref. [3]. Therefore, only a few important characteristics of the Non–Iterative Method are reviewed in this section.

In principle, the Non–Iterative Method exchanges the independent and dependent variables that Iterative Method uses. Now, strain–gage outputs and bellows pressures become independent variables. The balance loads, on the other hand, are the dependent variables for the regression analysis of the balance calibration data. In other words, the balance loads of the MC–130 air balance are fitted as a function of the measured strain–gage outputs and bellows pressures using the following generic regression model:

$$ F_{k,\text{load}} = \eta_0(k) + \eta_1(k) \cdot R_1_{\text{output}} + \cdots + \eta_m(k) \cdot R_m_{\text{output}} + \zeta_{m+1}(k) \cdot PR1_{\text{pressure}} + \zeta_{m+2}(k) \cdot PR2_{\text{pressure}} + \cdots $$

The Non–Iterative Method has the advantage that it is a one–step method. No iteration is needed to compute loads from measured strain–gage outputs and bellows pressures during a wind tunnel test. An analyst, however, must not forget that the method ignores the fact that the balance loads and bellows pressures are the “true” independent variables of the calibration experiment as loads and bellows pressures are “applied” and strain–gage outputs are “measured” during the calibration of an air balance. Therefore, the success of the Non–Iterative Method hinges on the fundamental assumption that an exchange of the independent and dependent variables of the calibration data set does not negatively influence the mathematical description of the “true” physical behavior of the balance. In addition, the robustness and reliability of the regression model of each balance load depends on the fact that (i) the model does not have near–linear dependencies between terms and that (ii) it only consists of statistically significant terms (see again Refs. [8] and [9] for a discussion of these issues).

Only the regression analysis of the axial force using the Non–Iterative Method is discussed in the paper so that a comparison with results of the Iterative Method can be performed. Again, the regression model
optimization process outlined in Refs. [4] and [5] was applied to the regression analysis problem that is defined by Eq. (6). The final optimized regression model of the axial force component is shown in Fig. 17a.

Figure 17b shows the Analysis of Variance (ANOVA) result for the optimized 17-term regression model of the axial force. Again, three important observations can be made after inspecting the ANOVA results. (1) The coefficients of the regression model are all statistically significant because the p-value of each coefficient is less than the threshold 0.0001. (2) The regression model of the axial force does not have any near-linear dependencies because all primary Variance Inflation Factor (VIF) values are less than the threshold of 10. (3) As expected, the terms $R_6$, i.e., the axial gage output, and $PR_2(i)$, i.e., the second bellows pressure, have by far the largest t-statistic (+3768 and +821). Therefore, those two terms have the greatest influence on the prediction of the axial force which agrees with an analyst’s intuitive expectations.

Finally, the regression model terms and coefficients listed in the second and third column of Fig. 17b were used to compute both the fitted axial force and the corresponding axial force residuals. Figure 17c shows load residuals for the axial force. Overall, the accuracy of the axial force prediction is excellent as the standard deviation of the load residuals is only 0.1098% of the axial force capacity. This standard deviation compares very well with the standard deviation of the load residuals that was obtained for the Iterative Method (see Fig. 16d). Therefore, it can be concluded that the recently developed extension of the Iterative Method is working as intended. It predicted the axial loads as accurate as the Non–Iterative Method.

IV. Summary and Conclusions

Design, calibration, and calibration data analysis of the MC–130 air balance were discussed. First, the design of the complete balance system was reviewed. Then, specifically designed calibration hardware and the calibration load schedule were described. This part also included a discussion of both the design and calibration challenges that an air balances poses. Finally, results of the regression analysis of the balance calibration data were presented.

The regression analysis of the balance calibration data was performed using two fundamentally different approaches. First, a recently developed extension of the Iterative Method was applied. Then, the Non–Iterative Method was used to process the calibration data. In addition, the original balance loads were converted from direct-read to force balance format in order to make it easier to interpret the regression analysis result.

Analysis of variance results of the regression models of the axial gage outputs and loads were investigated in more detail for both analysis approaches to gain confidence in the regression analysis result. The most significant terms were identified using the $t$-statistic of the regression coefficient. They agree with the choice that an analyst would intuitively make based on an understanding of the design characteristics of the balance. Overall, standard deviations of the load residuals of the axial force for both analysis approaches show excellent agreement validating the recently developed extension of the Iterative Method.

Acknowledgements

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References

Iterative/Non–Iterative Method:


Regression Model Term Selection:


Fig. 1 A 2.5 [inch] balance and air system prior to assembly.

Fig. 2 Example of bellows in test fixtures.

Fig. 3 Bellows installed on an air tube.
Fig. 4 MC–130: The 8.0 [inch] balance prior to assembly with external bellows and model hardware.

Fig. 5 MC–130: The balance with external bellows attached to each side of the model block.

Fig. 6 MC–130: Installation of calibration plates in progress.
**Fig. 7** MC-130: The balance system after installation of all calibration plates.

**Fig. 8** Strain gages and wiring on an air balance.

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<td>13,000</td>
<td>107,250</td>
<td>3,000</td>
<td>18,000</td>
<td>32,000</td>
<td>500</td>
</tr>
<tr>
<td>MC-130-8.0**</td>
<td>8.00</td>
<td>5,000</td>
<td>25,250</td>
<td>2,000</td>
<td>10,500</td>
<td>19,500</td>
<td>1200</td>
</tr>
<tr>
<td>MC-30-8.80</td>
<td>9.50</td>
<td>3,000</td>
<td>18,000</td>
<td>3,000</td>
<td>18,000</td>
<td>4,000</td>
<td>400</td>
</tr>
</tbody>
</table>

* Dual ranges show the balance maximums / test article load range at 48 inches from balance center
** Redefined Load Envelope for current test requirements

**Fig. 9** Comparison of allowable forces and moments for various balances.
### Table 1

<table>
<thead>
<tr>
<th>Balance Identification</th>
<th>Number Air Supplies</th>
<th>Mass Flow (lb/sec)</th>
<th>Max. Pressure (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC-24-1.50</td>
<td>1</td>
<td>2.0</td>
<td>1000</td>
</tr>
<tr>
<td>MC-15-2.5</td>
<td>2</td>
<td>4.0, 1.25</td>
<td>500</td>
</tr>
<tr>
<td>MC-16-3.0</td>
<td>1</td>
<td>1.5</td>
<td>500</td>
</tr>
<tr>
<td>MC-10-3.75</td>
<td>3</td>
<td>7.5, 4.0, .2</td>
<td>500</td>
</tr>
<tr>
<td>MC-59-3.5</td>
<td>1</td>
<td>15.2</td>
<td>600</td>
</tr>
<tr>
<td>MC-60-7.5</td>
<td>3</td>
<td>10.0, 3.5, 1.0</td>
<td>500</td>
</tr>
<tr>
<td><strong>MC-130-8.0</strong></td>
<td>2</td>
<td>13.0, 5.0</td>
<td>600 / 210</td>
</tr>
<tr>
<td><strong>MC-30-8.80</strong></td>
<td>2</td>
<td>9.9, 5.0</td>
<td>500 / 20</td>
</tr>
</tbody>
</table>

**Fig. 10** Comparison of air flow data for various air balances.

**Fig. 11** MC–130: Balance assembly with calibration hardware installed.

**Fig. 12a** MC–130: Applied bellows pressures (all 1854 calibration points shown).
Fig. 12b MC–130: Applied balance loads (all 1854 calibration points shown).
Fig. 13a Direct–Read Format: Strain–gage output $R_1$ versus normal force $NF$.

Fig. 13b Direct–Read Format: Strain–gage output $R_2$ versus pitching moment $PM$. 

8th International Symposium on Strain–Gauge Balances
Fig. 14a Force Balance Format: Strain–gage output $R_1$ versus normal force component $N_1$.

Fig. 14b Force Balance Format: Strain–gage output $R_2$ versus normal force component $N_2$. 

8th International Symposium on Strain–Gauge Balances
Fig. 15a Force Balance Format: Strain-gage output $R_6$ versus axial force $AF$ for unpresurized balance.

Fig. 15b Force Balance Format: Strain-gage output $R_6$ versus axial force $AF$ for pressurized balance.
Fig. 16a Extended Iterative Method: Optimized regression models of all gage outputs and bellows pressures. (column = model of gage output or bellows pressure; black box = selected coefficient)
### ANOVA RESULTS FOR REGRESSION MODEL OF AXIAL GAGE OUTPUT R6

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>Degrees of Freedom</th>
<th>F-Value of Regression</th>
<th>P-Value of Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1.3368e+09</td>
<td>1.3368e+09</td>
<td>114</td>
<td>2.0036e+05</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Residual</td>
<td>4.584-4055</td>
<td>4.586918</td>
<td>114</td>
<td>2.581476</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Total</td>
<td>1.3368e+09</td>
<td>1.3368e+09</td>
<td>115</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

R-Square: 0.999667
Adj. R-Square: 0.999667
PRESS R-Square: 0.999666

Math term contributes significantly if the t-statistic of its coefficient is greater than 1.962 (Significance = 2.50 %, Degrees of Freedom of Residual = 114).

Diagnostic describing near-linear dependencies (multicollinearity) between Math model terms:
Maximum of Variance Inflation Factor (Primary Value) = 9.72 < Selected Limit of 10.

### PHYSICAL VARIABLES & UNITS — REGRESSION COEFFICIENT ESTIMATES AND STATISTICAL METRICS (R6)

<table>
<thead>
<tr>
<th>Term Index</th>
<th>Term Name</th>
<th>Coefficient Value</th>
<th>Standard Error</th>
<th>t-Statistic of Coefficient</th>
<th>P-Value of Coefficient</th>
<th>VIF (Primary)</th>
<th>VIF (Alternate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTERCEPT</td>
<td>-4.0107</td>
<td>4.0094</td>
<td>-1.0908</td>
<td>0.3320</td>
<td>4.0688</td>
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<tr>
<td>2</td>
<td>H1</td>
<td>-0.0058</td>
<td>0.0002</td>
<td>-34.5440</td>
<td>&lt; 0.0001</td>
<td>4.3320</td>
<td>4.3320</td>
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<tr>
<td>3</td>
<td>K2</td>
<td>-0.0070</td>
<td>0.0011</td>
<td>-5.9358</td>
<td>&lt; 0.0001</td>
<td>4.1915</td>
<td>4.1915</td>
</tr>
<tr>
<td>4</td>
<td>L1</td>
<td>-0.0248</td>
<td>0.0004</td>
<td>-64.9251</td>
<td>&lt; 0.0001</td>
<td>4.6913</td>
<td>4.6913</td>
</tr>
<tr>
<td>5</td>
<td>S2</td>
<td>-0.0044</td>
<td>0.0004</td>
<td>-1.0319</td>
<td>&lt; 0.0001</td>
<td>4.3356</td>
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<tr>
<td>6</td>
<td>K1</td>
<td>-0.0013</td>
<td>0.0005</td>
<td>12.0954</td>
<td>&lt; 0.0001</td>
<td>4.5632</td>
<td>4.5632</td>
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<tr>
<td>7</td>
<td>L2</td>
<td>-1.4163</td>
<td>0.0004</td>
<td>3628.0033</td>
<td>&lt; 0.0001</td>
<td>4.9224</td>
<td>4.9224</td>
</tr>
<tr>
<td>8</td>
<td>FR1(L)</td>
<td>-0.0704</td>
<td>0.0017</td>
<td>-50.5664</td>
<td>&lt; 0.0001</td>
<td>4.6206</td>
<td>4.6206</td>
</tr>
<tr>
<td>9</td>
<td>FR2(R)</td>
<td>0.0055</td>
<td>0.0002</td>
<td>4.9395</td>
<td>&lt; 0.0001</td>
<td>4.7161</td>
<td>4.7161</td>
</tr>
<tr>
<td>10</td>
<td>ST(L)</td>
<td>3.2416e-06</td>
<td>4.0236e-07</td>
<td>-0.8283</td>
<td>0.3931</td>
<td>4.2561</td>
<td>4.2561</td>
</tr>
<tr>
<td>11</td>
<td>SF(L)</td>
<td>-3.9456e-06</td>
<td>4.1631e-07</td>
<td>-7.3330</td>
<td>&lt; 0.0001</td>
<td>4.0856</td>
<td>4.0856</td>
</tr>
<tr>
<td>12</td>
<td>FR1(L)</td>
<td>3.0575e-09</td>
<td>7.526e-10</td>
<td>4.2725</td>
<td>&lt; 0.0001</td>
<td>4.8039</td>
<td>4.8039</td>
</tr>
<tr>
<td>13</td>
<td>SF(L)</td>
<td>-3.7762e-06</td>
<td>4.7717</td>
<td>-0.201</td>
<td>0.2088</td>
<td>4.2880</td>
<td>4.2880</td>
</tr>
<tr>
<td>14</td>
<td>FR1(L)</td>
<td>4.0004</td>
<td>0.0001</td>
<td>5.2724</td>
<td>&lt; 0.0001</td>
<td>4.0783</td>
<td>4.0783</td>
</tr>
<tr>
<td>15</td>
<td>FR2(R)</td>
<td>0.0057</td>
<td>0.0002</td>
<td>2.0445</td>
<td>&lt; 0.0001</td>
<td>4.0726</td>
<td>4.0726</td>
</tr>
<tr>
<td>16</td>
<td>SF(R)</td>
<td>-0.0011</td>
<td>0.0010</td>
<td>15.4874</td>
<td>&lt; 0.0001</td>
<td>4.6800</td>
<td>4.6800</td>
</tr>
<tr>
<td>17</td>
<td>FR1(L)</td>
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<td>0.0005</td>
<td>0.0005</td>
<td>1.0000</td>
<td>4.0334</td>
<td>4.0334</td>
</tr>
<tr>
<td>18</td>
<td>FR2(R)</td>
<td>0.0000</td>
<td>0.0002</td>
<td>-0.2102</td>
<td>0.2102</td>
<td>4.0334</td>
<td>4.0334</td>
</tr>
</tbody>
</table>

---

**Fig. 16b** Extended Iterative Method: Analysis of Variance results for the regression model of axial gage output R6.

**Fig. 16c** Extended Iterative Method: Load iteration history.
Fig. 16d Extended Iterative Method: Load residuals of the axial force $AF$.

Fig. 17a Non–Iterative Method: Optimized regression model of axial force $AF$. (column = model of axial force; black box = selected coefficient)
ANOVA RESULTS FOR REGRESSION MODEL OF AXIAL FORCE AF

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degr. of Freedom</th>
<th>Mean Square</th>
<th>F-value of Regression</th>
<th>P-value of Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>4.791×10^7</td>
<td>16</td>
<td>2.994×10^6</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>2.31×10^3</td>
<td>1159</td>
<td>1.96×10^-2</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4.791×10^7</td>
<td>1175</td>
<td>1.96×10^-2</td>
<td>1.0001</td>
<td></td>
</tr>
</tbody>
</table>

R-Square: 0.999995
Adj. R-Square: 0.999995
Press R-Square: 0.999995

**Fig. 17b** Non-Iterative Method: Analysis of Variance results for the regression model of the axial force AF.

**Fig. 17c** Non-Iterative Method: Load residuals of the axial force AF.