Phase Coupling in Langmuir Wave Interactions in Solar Type III Radio Bursts

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The four wave interaction process, known as the oscillating two stream instability (OTSI), is considered as one of the mechanisms responsible for stabilizing the electron beams associated with solar type III radio bursts. It has been reported that (1) an intense localized Langmuir wave packet associated with a type III burst contains the spectral characteristics of the OTSI: (a) a resonant peak at the local electron plasma frequency, \( f_{pe} \), (b) a Stokes peak at a frequency slightly lower than \( f_{pe} \), (c) anti-Stokes peak at a frequency slightly higher than \( f_{pe} \), and (d) a low frequency enhancement below a few hundred Hz, (2) the frequencies and wave numbers of these spectral components satisfy the resonance conditions of the OTSI, and (3) the peak intensity of the wave packet is well above the thresholds for the OTSI as well as spatial collapse of envelope solitons. Here, for the first time, applying the trispectral analysis on this wave packet, we show that the tricoherence, which measures the degree of coherent four-wave coupling amongst the observed spectral components exhibits a peak. This provides an additional evidence for the OTSI and related spatial collapse of Langmuir envelope solitons in type III burst sources.

1. Introduction

Solar type III radio bursts are characterized by very fast negative frequency drifts from hundreds of MHz to tens of kHz. Ginzburg and Zheleznyakov [1958] were the first to propose that the production of these bursts involves the excitation of high levels of Langmuir waves at local electron plasma frequency \( f_{pe} = 9n_e^{1/2} \), where \( n_e \) is the electron density in \( m^{-3} \) by flare accelerated electron beam through bump-on-tail instability [Bohm and Gross, 1949], and their subsequent conversion into radio emissions at \( f_{pe} \) and \( 2f_{pe} \), which has been confirmed by the in situ detection of Langmuir waves [Gurnett and Anderson, 1976, 1977] as well as electron beams [Lin, 1970; Lin et al., 1973, 1986]. According to Sturrock [1964], the excitation of Langmuir waves would extract all the streaming energy from the electron beam within 100 km or less, whereas, the bump-on-tail distributions of electron beams are detected [Lin et al., 1986] over the distances of 1 AU and more. It is now believed that some nonlinear process removes the Langmuir waves rapidly from the spectral regions of resonance with the beam, which leads to the beam stabilization. For example, the induced scattering of ion clouds, which is the electrostatic decay (ESD) of initial Langmuir wave into a daughter Langmuir wave and an ion sound wave when \( T_e > T_i \) [Barbash and Goldstein, 1976] is proposed as one of such mechanisms [Kaplan and Tsyganov, 1968], where, \( T_e \) and \( T_i \) are the electron and ion temperatures, respectively. Although the signatures of electrostatic decay are observed in the type III sources [Lin et al., 1986; Gurnett et al., 1993; Hospodarsky and Gurnett, 1995; Thejappa and MacDowall, 1998; Thejappa et al., 2003; Henri et al., 2009], their time scale appears to be too long to prevent the plateau formation [Zheleznyakov and Zaitsev, 1970].

The type III associated Langmuir waves are usually estimated to be very intense and therefore, the four-wave interaction called the oscillating two stream instability (OTSI) [Papadopoulos et al., 1974; Smith et al., 1979; Goldstein et al., 1979], and related soliton formation and spatial collapse [Zakharov, 1972; Nicholson et al., 1978] are proposed as the most effective beam stabilization mechanisms. The OTSI excites a low frequency ion density perturbation of frequency and wave number \( (\Omega, q) \), which can beat with two of the beam-excited Langmuir waves of frequency and wave number \( (f_{pe}, k_L) \) and produce down-shifted \( (f_{pe} - \Omega, k_L + q) \) (anti-Stokes) and up-shifted \( (f_{pe} + \Omega, k_L + q) \) (Stokes) modes, respectively. The spatial collapse, on the other hand occurs due to intensification of the localized Langmuir wave packet in the self generated shrinking density cavity. Some possible evidence for the strong turbulence processes in the Jupiter’s foreshock [Gurnett et al., 1981], in the solar wind [Kellogg et al., 1992], and in the source regions of type III bursts [Thejappa et al., 1993; Thejappa and MacDowall, 1998; Thejappa et al., 1999; Thejappa and MacDowall, 2004] was reported.

In a recent study, Thejappa et al. [2012] have reported the STEREO/SWAVES [Bougeret et al., 2008] high time resolution observations of an isolated localized type III associated Langmuir wave packet with short duration of \( \sim 3.2 \) ms. These authors have shown that (1) the spectrum of this wave packet contains the characteristics of OTSI, namely, an intense peak at \( f_{pe} \), and two side bands at slightly lower and higher than \( f_{pe} \), and a low frequency enhancement below a few hundred Hz, (2) the frequencies and wave numbers of these spectral components satisfy the resonance conditions of the OTSI, and (3) the peak intensity of the wave packet is well above the thresholds for the OTSI as well as for the formation of envelope solitons collapsed to a few hundred Debye lengths. Based on these observations, it has been argued that the OTSI and spatial collapse control the beam plasma interactions in the solar type III radio bursts.

In this study, for the first time, we will apply trispectral analysis on this Langmuir wave packet and compute the tricoherence. We will show that the tricoherence spectrum exhibits a peak, which is an indicative of the four-wave interaction of type OTSI. We argue that the high degree of phase coherence between the spectral components of the wave packet provides an additional evidence for the OTSI and related strong turbulence processes in the solar type III radio bursts as correctly concluded by Thejappa et al. [2012]. In section 2, we review the observations, in section 3, we present the trispectral analysis and in section 4, we present the conclusions.
2. Review of Observations

In Fig. 1, the fast drifting emission from very high frequencies to the local electron plasma frequency, \( f_{pe} \sim 30 \) kHz is identified as the local type III burst, and the non-drifting emissions in the interval 27-32 kHz are identified as the Langmuir waves. Time Domain Sampler (TDS) of the SWAVES experiment [Kellogg et al., 2009], which samples the A/C electric field from 3 orthogonal antennas has resolved these Langmuir waves into intense waveforms, each of which contains 16384 samples with an acquisition rate of 250,000 samples per second (a time step of 4 \( \mu s \) for a total duration of 65 ms). The most intense wave packet, captured by the E\(_3\) antenna is shown in Fig. 2a. The peak electric field strength \( E_L \) and \( \nu^2 \)-power duration \( \tau \) of this event are 56.5 mVm\(^{-1}\) and \( \sim 3.2 \) ms, respectively. The narrow spectrum of this event, as seen in Fig. 2b shows an intense peak (L) at \( f_{pe} \sim 30 \) kHz, corresponding to \( \nu_e \sim 1.1 \times 10^4 \) m\(^{-3}\), a Stokes peak (S) at \( \sim 29.54 \) kHz, which is slightly less than \( f_{pe} \) and an anti-Stokes peak (AS) at \( \sim 30.41 \) kHz, which is slightly higher than \( f_{pe} \). The spectrum, as shown in Fig. 2c clearly shows a low frequency enhancement corresponding to ion-sound waves below 450 Hz. The STEREO/PLASTIC experiment [Galeev et al., 2008] has measured the solar wind speed \( v_{sw} \) as \( \sim 450 \) km\( \)s\(^{-1}\). We assume that the electron temperature \( T_e \) is \( \sim 10^4 \) K during this event. Assuming that the type III electrons propagate along the Parker’s spiral field lines, we fit a frequency drift curve to the dynamic spectrum \( v_b \) as \( \sim 0.22c \) for the RAE density model [Fainberg and Stone, 1971], where \( c \) is the velocity of light. However, the pitch angle scattering is known to increase the path length of electron beams by a factor of \( \alpha = 1.3 \) to 1.7 [Alvarez et al., 1975; Lin et al., 1973]. This implies that if we incorporate these corrections into the estimates of the beam speeds, they will lie in the range from \( \sim 0.29c \) to \( \sim 0.37c \). Accordingly, the wave number of the Langmuir waves \( k_L = \frac{2\pi}{\lambda_{DL}} = 2.9 \times 10^{-3} \) m\(^{-1}\) estimated for \( v_b = 0.22c \) will decrease to the values from \( \sim 2.2 \times 10^{-3} \) m\(^{-1}\) to \( \sim 1.7 \times 10^{-3} \) m\(^{-1}\). We estimate the rest of the parameters as: (1) Debye length, \( \lambda_{De} = 694e^{-3/2}v_e^{-1/2} \approx 6.6 \) m, and (2) the normalized peak energy density \( w_L = \frac{\nu_e^2}{2e} \approx 10^{-1} \), and (3) the \( \nu^2 \)-power spatial scale of the wave packet \( S \sim 219\lambda_{De} \) (using the relation \( S \sim \tau v_{sw} \)).

It was argued that these observations of a strong Langmuir wave packet with upper and lower sidebands (Fig. 2b), together with low frequency enhancement (Fig. 2c) are strongly suggestive of OTSI, in which, the beam driven Langmuir wave is the pump wave, the modes corresponding to sidebands and low frequency waves are the nonlinearly excited daughter waves. The frequency matching condition \( 2f_\omega = f_{pe} + f_{AE} \) is easily satisfied, since the frequency shifts of the Stokes and anti-Stokes modes are symmetric with respect to the Langmuir wave pump, being \( \sim 442.5 \) Hz and \( \sim 427 \) Hz, respectively, which in turn are in good agreement with the frequencies of the observed ion sound waves of \( < 450 \) Hz. The wave numbers \( k \) of the side bands are estimated using the expression for the frequency shift [Gurnett et al., 1981] \( \Delta f = \frac{v_{sw}}{2\lambda_{De}}(k\lambda_{De}) \cos \theta + f_{pe}(-1 + (1 + 3(k\lambda_{De})^2)^{1/2}), \) where, the first and second terms correspond to the Doppler shift caused by the motion of the solar wind and intrinsic frequency variation caused by the dispersion relation, respectively, and \( \theta \) is the angle between \( k \) and \( v_{sw} \); \( \theta = 0 \) and \( \theta = \pi \) correspond to the anti-Stokes and Stokes modes propagating away from and toward the Sun, respectively. By plugging the measured \( \Delta f \) of \( \sim 442.5 \) Hz and 427 Hz in this equation, \( k\lambda_{De} \) is estimated as \( \sim 0.03 \) and \( \sim -0.05 \), for the anti-Stokes and Stokes modes, respectively, which suggest that the pump Langmuir waves with \( k_L\lambda_{De} \sim 10^{-2} \) are pumped into those of forward and backward propagating daughter waves with large wave numbers. The upper limit of the wave numbers of the ion sound waves can be estimated using the relation \( q = 2\nu_{sw}/\Omega \), since their phase velocities are usually well below \( v_{sw} \). Thus, for \( \Omega = 450 \) Hz
and \( v_{ew} = 450 \text{ km s}^{-1} \), it is estimated that \( q \approx 5.6 \times 10^{-3} \text{ m}^2 \) and \( q/\lambda_{DR} \approx 0.036 \). These wavenumbers are comparable to those of the sideband emissions, and the matching condition \( k = k_e \pm q \) is easily satisfied, yielding \( |k| \approx |q| \), since \( k_e < q \).

The threshold for the OTSI [Zakharov, 1972] \( \omega_{LTE} > (k_L \lambda_{DE})^2 \) is easily satisfied, since the observed \( \omega_{LTE} \sim 10^{-3} \text{ m}^2 \) is well above \( (k_L \lambda_{DE})^2 \sim 10^{-4} \). For the wave packet to be the collapsed soliton, it should satisfy the condition [Bornhill and Haror, 1978; Barnett et al., 1981] \( \omega_{LTE} \sim (\Delta k^2 \lambda_{DE})^2 \), where \( \Delta k = \frac{k}{2} \) is the wavenumber characteristic of the envelope. In the present case, the observed \( \omega_{LTE} \sim 10^{-3} \text{ m}^2 \) is greater than \( (\Delta k^2 \lambda_{DE})^2 \sim 8 \times 10^{-4} \) obtained for the spatial scale \( S \sim 219 \lambda_{DE} \). This suggests that the observed wave packet is probably the Langmuir envelope soliton, collapsed to the spatial scale of \( 219 \lambda_{DE} \).

### 3. Trispectral Analysis

The phase coherence between the spectral components is one of the important characteristics of the four-wave interaction, such as the OTSI. In the power spectrum estimation, the waveform is treated as a superposition of statistically uncorrelated harmonic components, and the phase relations between the spectral components are suppressed. The information present in the power spectrum is sufficient for the complete statistical description of any Gaussian process of a known mean. However, in order to extract the information regarding the presence of nonlinearities, we should look beyond the power spectrum. Higher order spectra (HOS), which are defined in terms of the higher order moments or cumulants of the signal contain such information. The third-order spectrum is commonly referred to as bispectrum [Kim and Powers, 1970], the fourth-order one as trispectrum. The information about the phase coherence between four spectral components can be extracted from the waveform data using the trispectral analysis. The trispectral method has been developed and applied to synthetic [Kratkchenko-Berejnoi et al., 1995a; b; Lefevre et al., 1995] and to simulated data [Soucek et al., 2003]. The trispectral analysis can detect the phase relationships among four Fourier components, which is the key information regarding four-wave interactions. In OTSI, the sidebands interact with the strong beam excited Langmuir waves simultaneously satisfying the matching rules for the wavenumbers as well as for frequencies. For the OTSI type of four-wave interactions, the cumulant based trispectrum is given by [Kratkchenko-Berejnoi et al., 1995a]

\[
T(1,2,3) = E[X_1 X_2 X_3 X_4] - N(1,2,3,4),
\]

where \( (X_1, X_2, X_3) \) and \( X_4 \) are the complex Fourier components of the signal corresponding to frequencies \( f_1, f_2, f_3 \) and \( f_4 \), \( N(1,2,3,4) = E[X_1 X_2 X_3 X_4] + E[X_1 X_3 X_2 X_4] + E[X_1 X_2 X_4 X_3] + E[X_2 X_3 X_4 X_1] \), \( f_4 = f_1 + f_2 - f_3 \), and \( E[] \) is the expectation operator. The tricoherence, which eliminates the dependence of trispectrum on the amplitude of the signals. The expressions for cumulant based square tricoherence function can be written as [Kratkchenko-Berejnoi et al., 1995a]:

\[
t^2(1,2,3) = |T(1,2,3)|^2 + |N(1,2,3,4)|^2.
\]

A unit value for the tricoherence indicates perfect coupling, a zero value indicates no coupling, and a value between zero and one indicates partial coupling. The trisphare is the phase of the tricoherence. The tricoherence quantifies the fraction of the total product of powers at the frequency quartet, \( (f_1, f_2, f_3, f_1 + f_2 - f_3) \), that is owing to cubic phase-coupled modes. The tricoherence is zero for a Gaussian process.

For Gaussian processes, due to statistical fluctuations, the estimate of the tricoherence from a finite data record will not be zero. Therefore, the method of periodograms is usually used to estimate the trispectrum and tricoherence. This involves the division of the data record of \( N \) samples into \( M \) time intervals and calculation of the ensemble averaged estimates over all intervals. In this study, we have used the segment length \( N = 1000 (0.004s) \), number of segments \( M = 16 \), Hamming windowing, and a cumulant estimator. We have calculated the tricoherence spectrum as a function of three frequencies. Since it is difficult to visualize the results in such a 3-D space, we have made the cross-section of the tricoherence domain at the frequency \( f_3 = 29.5 \text{ kHz} \), corresponding to the frequency of the Stokes mode. In Fig. 3, we present such a cross section. The tricoherence peak shown as circled feature at \( (f_1 = 30 \text{ kHz}, f_2 = 30 \text{ kHz}, f_3 = 29.5 \text{ kHz}) \) is due to the phase relation \( 2\phi_L + \phi_\sigma + \phi_A = \pi \), where \( \phi_L, \phi_\sigma \) and \( \phi_A \) are the phases of the beam-excited Langmuir, Stokes mode and anti-Stokes modes, respectively.

The maximum tricoherence \( t^2 \) is \( \sim 0.51 \). This significant value of tricoherence provides evidence for the cubic nonlinearity, specifically for four-wave interaction, known as the OTSI.

### 4. Conclusions

Thejappa et al. [2012] reported that (1) TDS of the SWAVES experiment on STEREO A has captured very coherent and intense Langmuir wave packets in the source region of a local type III radio burst, (2) The spectrum of this wave packet contains the characteristic signatures of oscillating two stream instability (OTSI), namely, (a) a resonant peak at the local electron plasma frequency, \( f_{pe} \), (b) Stokes peak at a frequency slightly higher than \( f_{pe} \), (c) anti-Stokes peak at a frequency slightly higher than \( f_{pe} \), and (d) low frequency enhancement corresponding to ion sound fluctuations; the frequencies and wave numbers of these spectral components satisfy the resonance conditions of OTSI. (2) The observed \( \omega_{LTE} \sim 10^{-3} \), which is well above the threshold for strong turbulence processes, and the short time scale \( \sim 3.2 \text{ ms} \) indicate that the observed wave packet is a collapsing Langmuir envelope soliton. (3) The Langmuir collapse follows the plane-wave modulational instability studied by Zakharov [1972], and (4) the strong turbulence processes are probably responsible for beam stabilization as well as for the observed type III burst fundamental and harmonic emissions.

In this study, for the first time, using the trispectral analysis, we have shown that the tricoherence between the beam excited Langmuir wave and the side bands is \( \sim 0.51 \), which provides evidence for the four wave interaction of the type OTSI and provides an additional support for the conclusions drawn by Thejappa et al. [2012].

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**Figure 3.** The cross-section at \( f = 29.5 \text{ kHz} \) of the tricoherence spectrum of the TDS event of Fig. 2a.
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