Phase Coupling between Spectral Components of Collapsing Langmuir Solitons in Solar Type III Radio Bursts

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We present the high time resolution observations of one of the Langmuir wave packets obtained in the source region of a solar type III radio burst. This wave packet satisfies the threshold condition of the supersonic modulational instability, as well as the criterion of a collapsing Langmuir soliton, i.e., the spatial scale derived from its peak intensity is less than that derived from its short time scale. The spectrum of this wave packet contains an intense spectral peak at $f_{pe}$, and relatively weaker peaks at $2f_{pe}$ and $3f_{pe}$. We apply the wavelet based bispectral analysis technique on this wave packet and compute the bicoherence between its spectral components. It is found that the bicoherence exhibits two peaks at $(\sim f_{pe}, \sim f_{pe})$ and $(\sim f_{pe}, 2f_{pe})$, which strongly suggest that the spectral peak at $2f_{pe}$ probably corresponds to the second harmonic radio emission, generated as a result of the merging of anti-parallel propagating Langmuir waves trapped in the collapsing Langmuir soliton, and, the spectral peak at $3f_{pe}$ probably corresponds to the third harmonic radio emission, generated as a result of merging of a trapped Langmuir wave and a second harmonic electromagnetic wave.

1. Introduction

Solar flares, which represent the most dramatic energy releases from the Sun eject electrons into the corona and interplanetary medium. These electrons form bump-on-tail distributions and drive Langmuir waves unstable. These Langmuir waves subsequently couple to type III radio emissions at the fundamental and second harmonic of the electron plasma frequency, $f_{pe}$. This scenario, generally known as the plasma hypothesis [Ginzburg and Zheleznyakov, 1958] is supported by the observed fast negative frequency drift of the type III radio burst, which is interpreted as due to the electron beam propagating radially outward into lower and lower densities of the corona and interplanetary medium. Furthermore, the in situ observations of energetic electrons [Lin et al., 1973, 1981; Ergun et al., 1998] and Langmuir waves [Gurnett and Anderson, 1976, 1977; Lin et al., 1986; Kellogg et al., 1992; Gurnett et al., 1993; Thejappa et al., 1993; Hospodarsky and Gurnett, 1995; Thejappa and MacDowall, 1998; Thejappa et al., 1999; Henri et al., 2009] in the source regions of type III radio bursts have provided additional support for the plasma hypothesis. Some times, faint third harmonic emissions are also observed in type III bursts [Benz, 1973; Takakura and Yousef, 1974]. One of the outstanding questions is the identification of the mechanism/mechanisms responsible for conversion of Langmuir waves into electromagnetic radiation.

Some authors [Papadopoulos et al., 1974; Smith et al., 1979; Goldstein et al., 1979; Goldman et al., 1980; Nicholson et al., 1978; Papadopoulos and Freund, 1978] predict that in type III radio bursts, the Langmuir waves are strongly turbulent, and therefore, the related oscillating two stream instability (OTSI) and Langmuir collapse [Zakharov, 1972] play significant roles in the stabilization of electron beams as well as in conversion of Langmuir waves into electromagnetic waves. The Fast Envelope Sampler of Ulysses URP experiment [Stone et al., 1992] have provided some evidence for strong turbulence processes in type III burst sources [Kellogg et al., 1992; Thejappa et al., 1993, 1996; Thejappa and MacDowall, 1998; Thejappa et al., 1999; Thejappa and MacDowall, 2004]. The in situ observations from the improved Time Domain Sampler (TDS) of the STEREO/WAVES experiment [Bougeret et al., 2008] improved over that of all similar high time resolution receivers flown in earlier spacecraft in precision, linearity, sample length and rate [Kellogg et al., 2009] and newly available higher order spectral analysis techniques have conclusively shown that the OTSI and related strong turbulence processes are commonly occurring phenomena in type III burst sources [Thejappa et al., 2012a, b, c].

In this paper, we present new observations of Langmuir wave packet captured by the STEREO TDS in the source region of a local type III burst. This wave packet satisfies the threshold condition of the supersonic modulational instability, as well as the criterion of a collapsing soliton; the spatial scale derived from its peak intensity is less than the spatial scale derived from the observed time scale. The spectrum of this event, which is the focus of this study contains peaks at $2f_{pe}$ and $3f_{pe}$ in addition to an intense peak at $f_{pe}$. To investigate whether any phase coherence exists between these spectral components, we compute the bicoherence using the wavelet based bispectral analysis technique. This bicoherence exhibits two peaks, which strongly suggest that (1) the $2f_{pe}$ spectral peak probably corresponds to the electromagnetic waves generated as a result of the coalescence of anti-parallel propagating Langmuir waves trapped in the collapsing Langmuir soliton, and (2) the $3f_{pe}$ peak corresponds to third harmonic radio emission generated as a result of coalescence of trapped Langmuir wave with second harmonic electromagnetic wave. In section 2, we present the observations, in section 3, we present the bispectral analysis, and in section 4, we present the conclusions.

2. Observations

In Fig. 1, we present the dynamic spectrum of one of the local type III solar radio bursts (fast drifting emissions) and associated Langmuir waves (non-drifting emissions), observed on February 11, 2012 by the STEREO B spacecraft. In Fig. 2, we present the frequency-time spectrogram in a narrow frequency range, which shows the bursty structure of the Langmuir waves. The substantial frequency spreading of Langmuir waves seen here is probably due to nonlinear frequency broadening. In this figure, the Langmuir

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wave burst corresponding to the TDS event of current interest is shown by an arrow. In Fig. 3a, we present one of the most intense wave packets detected during this type III event. This wave packet is characterized by the peak electric field strength $E_L$ of 64.6 mV m$^{-1}$ and ½-power duration $D$ of $\sim 11$ ms. Since the $E_x$ and $E_y$ signals are weaker and show the same general features as the $E_y$ signal, we analyze only the $E_y$ signal. We justify one-dimensional treatment by assuming that these Langmuir wave fields are probably aligned along the ambient magnetic field. In Figure 1, we present one of the TDS events, which shows the peaks at local electron plasma frequency, $\omega_p$ as well as at $\omega_p/2$ and $\omega_p/3$. We estimate the electron density $n_e$ during this event as $\sim 2.4 \times 10^6$ m$^{-3}$ using the relation $f_{pe} = \omega_p/2\pi = 500$ kHz. We estimate the speed of the electron beam $v_b$ as $\sim 0.28c$ by fitting the frequency drift curve to the dynamic spectrum of the type III event ($c$ is the speed of light). Using these values, we estimate: (1) Debye length $\lambda_{De} = 69.5\,n_e^{1/2} \sim 14$ m for $T_e = 10^5$ K and $n_e = 2.4 \times 10^6$ m$^{-3}$, (2) wave number of Langmuir waves, $k_L = \omega_{pe}/v_b \sim 10^{-3}$ m$^{-1}$ for $f_{pe} \sim 14$ kHz and $v_b \sim 0.28c$, (3) $k_L\lambda_{De} \sim 1.4 \times 10^{-2}$, (4) normalized peak energy density, $\frac{W_p}{n_e^2 e^2} \sim 5.6 \times 10^{-3}$ for $E_L = 64.6$ mV m$^{-1}$, $n_e = 2.4 \times 10^6$ m$^{-3}$ and $T_e = 10^5$ K, and (5) the ½ level spatial scale $S \sim \frac{1}{2}v_b \tau \sim 393\lambda_{De}$ for the measured values of $D = 11$ ms and $\tau = 500$ ms$^{-1}$.

Assuming that the frequency of main spectral peak in Fig. 3b corresponds to the $f_{pe}$, we estimate the electron density $n_e$ during this event as $\sim 2.4 \times 10^6$ m$^{-3}$ using the relation $f_{pe} = \omega_p/2\pi = 500$ kHz. The solar wind speed $v_{sw}$ is measured as $\sim 500$ kms$^{-1}$ by the STEREO PLASTIC experiment [Galvin et al., 2008], and we assume that the electron temperature $T_e$ during this event is $\sim 10^5$ K. We estimate the speed of the electron beam $v_b$ as $\sim 0.28c$ by fitting the frequency drift curve to the dynamic spectrum of the type III event ($c$ is the speed of light). Using these values, we estimate: (1) Debye length $\lambda_{De} = 69.5\,n_e^{1/2} \sim 14$ m for $T_e \sim 10^5$ K and $n_e \sim 2.4 \times 10^6$ m$^{-3}$, (2) wave number of Langmuir waves, $k_L = \omega_{pe}/v_b \sim 10^{-3}$ m$^{-1}$ for $f_{pe} \sim 14$ kHz and $v_b \sim 0.28c$, (3) $k_L\lambda_{De} \sim 1.4 \times 10^{-2}$, (4) normalized peak energy density, $\frac{W_p}{n_e^2 e^2} \sim 5.6 \times 10^{-3}$ for $E_L = 64.6$ mV m$^{-1}$, $n_e = 2.4 \times 10^6$ m$^{-3}$ and $T_e = 10^5$ K, and (5) the ½ level spatial scale $S \sim \frac{1}{2}v_b \tau \sim 393\lambda_{De}$ for the measured values of $D = 11$ ms and $\tau = 500$ ms$^{-1}$.

Figure 1. Dynamic spectrum of the local type III radio burst (fast drifting emission from $\sim 3$ MHz down to $\sim 20$ kHz) and associated Langmuir waves (non-drifting emissions in the frequency interval 10-14 kHz.)

Figure 2. The frequency-time spectrogram of the Langmuir waves observed during the type III burst of February 11, 2012. The Langmuir wave emissions are very bursty and show substantial frequency spreading. The arrow shows the Langmuir wave burst corresponding to the current TDS event.

Figure 3. (a): One of the most intense Langmuir wave packets captured by the Time Domain Sampler (TDS) during the type III event of Fig. 1. The ½ power duration of 11 ms which is equivalent to the spatial scale of 393$\lambda_{De}$ is also shown. (b): The FFT spectrum of the TDS event, which shows the peaks at local electron plasma frequency, $f_{pe}$ as well as at $2f_{pe}$ and $3f_{pe}$.
The threshold for the supersonic modulational instability [Zakharov, 1972]
\[
\frac{W_L}{n_e T_e} \geq \frac{m_a}{m_i} > (k_L \lambda_{De})^2
\]
(1)
is easily satisfied in this case, since \( \frac{W_L}{n_e T_e} \sim 5.6 \times 10^{-3} \), \( \frac{m_a}{m_i} \sim 5.5 \times 10^{-4} \) and \((k_L \lambda_{De})^2 \sim 2 \times 10^{-4}\). The wave packet also satisfies the criterion of the collapsing envelope soliton [Thornhill and ter Haar, 1978; Garnett et al., 1981]
\[
\frac{W_L}{n_e T_e} \geq (\Delta k \lambda_{De})^2,
\]
(2)
where \(\Delta k = \frac{2\pi}{S}\) is the wavenumber characteristic of the envelope, since the observed \(\frac{W_L}{n_e T_e} \sim 5.6 \times 10^{-3}\) is greater than \((\Delta k \lambda_{De})^2 \sim 2.6 \times 10^{-4}\) estimated for the spatial scale of \(S \sim 393 \lambda_{De}\).

3. Bispectral Analysis

For nonlinear wave-wave interactions, the frequencies \((f_1, f_2, f_3)\) and wave vectors \((k_1, k_2, k_3)\) must satisfy the resonance conditions: \(f_1 + f_2 = f_3\) and \(k_1 + k_2 = k_3\). Since the wave number \(k_i\) is related to the phase \(\phi_i\) of the wave, the phases of three waves are also related as \(\phi_1 + \phi_2 = \phi_3\). Bispectrum, a higher order spectrum, permits the distinction of the spontaneously excited normal modes from the coupled modes by measuring the degree of phase coherence between the modes. The FFT based bi-spectrum is defined as
\[
B(f_1, f_2) = E[X(f_1)X(f_2)X(f_1 + f_2)^*],
\]
(3)

where \(E\) denotes the expected value, \(X(f)\) is the Fourier transform of \(x(t)\) and * denotes complex conjugation. This equation clearly indicates that the bispectrum is zero unless three waves at frequencies \(f_1, f_2\) and \(f_1 + f_2\) are present in the time series with phase coherency amongst the waves. For spontaneously generated non-interacting modes, the phases would be random and therefore the statistical averaging will be zero. The bicoherence, which is the normalized bispectrum, provides the quantitative measure of the phase coherence. It is defined as:
\[
b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{(E[|X(f_1)X(f_2)X^*(f_1 + f_2)|])^2}.
\]
(4)

A unit value for the bicoherence indicates perfect coupling, a zero value indicates no coupling, and any value between zero and one indicates partial coupling. Periodogram estimation (interval segmentation) is usually used to estimate the bispectrum. The approximate variance of the bicoherence estimator is defined as [Kim and Powers, 1979]:
\[
\text{var}(b^2) \approx \frac{M}{2N} [1 - b^2(f_1, f_2)],
\]
(5)

where \(N\) is the number of total data points and \(M\) is the number of data records into which the time series is divided. To assess the low bicoherence levels, one needs long and stationary time series. For periodogram estimation (interval segmentation) is usually used to estimate the bispectrum. In this study, we apply this wavelet based bispectral analysis technique on the wave packet of Fig. 3a. The continuous wavelet transform (CWT), which is used in this technique is defined as
\[
W(a, \tau) = \frac{1}{\sqrt{|a|}} \int x(t) \psi^* \left( \frac{t - \tau}{a} \right) dt,
\]
(6)

where the scale \(a\) is related to the frequency \(f\) as \(a = \frac{1}{f}\) and \(\tau\) is the time. The Morlet wavelet \(\psi(t) = e^{i2\pi \tau t} e^{-t^2/2}\) is generally used to study nonlinear wave phenomena. The wavelet based bispectrum in the frequency space can be defined as
\[
B(f_1, f_2) = E[W(f_1)W(f_2)W^*(f_1 + f_2)],
\]
(7)

where \(W(n\delta t, f = 1/a)\) is a function of time \(t\) as well as frequency \(f\). The wavelet bicoherence can be defined as
\[
b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{(E[|W(f_1)W(f_2)W^*(f_1 + f_2)|])^2}.
\]
(8)

Figure 4. (a): The wavelet based spectrogram of the TDS event of Fig. 3a, which shows the spectral features at \(f_{pe}, 2f_{pe}\) as well as at \(3f_{pe}\). (b): The wavelet based bi-coherence spectrum, which shows peaks at \((\sim 14, \sim 14)\) kHz and \((\sim 14, \sim 28)\) kHz. These bicoherence peaks suggest that their spectral components interact to produce the emissions at 28 kHz and 42 kHz, respectively.
and third harmonic components are clearly seen in this spe­
crogram, which show that the higher the harmonic, weaker
the emission. It also shows the temporal coincidence of the
second and third harmonic emissions with collapsing Lang­
muir soliton. In Figure 4b, we present the results of wavelet
based bicoherence analysis. This bicoherence spectrogram
clearly shows evidence for three wave-wave interactions. The
bicoherence peaks at (~ 14 kHz, ~ 14 kHz), and (~ 14 kHz,
~ 28 kHz) suggest that these spectral components inter­
act to produce the emissions at 28 kHz and 42 kHz, re­
spectively. The most probably wave interactions are
\[ L + L' \rightarrow T_{2pe}, \]
and
\[ L + T_{2pe} \rightarrow T_{3pe}, \]
respectively (L and L' are the opposite propagating Langmuir waves, and \( T_{2pe} \) and \( T_{3pe} \) are the second and third harmonic electromagnetic
waves, respectively).

The Langmuir waves involved in the interaction \( L + L' \rightarrow T_{2pe} \) are probably the anti-parallel propagating waves
trapped in the collapsing soliton. This is consistent with the
wavelet spectrogram (Fig. 4a), which shows the temporal
coincidence of the second and third harmonic emissions with
the peak of the Langmuir wave packet. The wave-wave in­
teraction \( L + L' \rightarrow T_{2pe} \), agrees with that of Papadopoulos et al. [1974], who suggested that the anti-parallel propagating
modes generated during oscillating two stream instability
(OTS1), get trapped in the collapsing soliton and couple to
each other generating electromagnetic waves at \( T_{2pe} \). The
interpretation of the spectral peak at 28 kHz in terms of
\( T_{2pe} \) is consistent with that of Bale et al. [1996], who in­
terpreted the harmonic spectral peak of the wave packet
observed by the WIND TDS in the upstream solar wind in
terms of \( T_{2pe} \). One should note that the observed phase
coherence between the fundamental and second harmonic
spectral components does not support the mechanism in­
voked by Papadopoulos and Freund [1978] as well as Gold­
man et al. [1980], which invoke the spontaneous emission of
electromagnetic waves at \( T_{2pe} \) by the stable and collapsing
solitons, respectively; the primary waves at \( f_{pe} \) and harmon­
ics at \( 2f_{pe} \) in such a case are not phase correlated.

Malaspina et al. [2010] have interpreted the harmonics of
the wave packets captured by the STEREO TDS in the
solar wind in terms of localized nonlinear electrostatic
modes driven by eigenmode-localized Langmuir waves. Fur­
thermore, these authors have suggested that these nonlinear
electrostatic modes probably emit the coherent electro­
magnetic radiation spontaneously at respective frequencies.
However, the observed phase coherence between the funda­
mental and harmonic components rules out (1) the possi­
ibility that the \( 2f_{pe} \) spectral peak corresponds to harmonic
Langmuir waves, since the non-linear wave-wave interaction
between the fundamental and harmonic Langmuir waves
is forbidden (harmonic Langmuir waves are not the eigen
modes), and (2) the second harmonic radio emission is not
due to any antenna emission but is due to wave-wave inter­
actions between the opposite propagating Langmuir waves.

Kellogg et al. [2010] argued that the harmonics of the
wave packets captured by the STEREO TDS in the Earth’s
fore shock are probably due to the electrons trapped in the
Langmuir wave potential maxima. In this scenario also, the
bicoherence which measures the phase coherence is expected
to be zero contrary to the observed values, which suggests
that the observed harmonic modes at the present case are not
due to trapping of electrons in the Langmuir wave maxima.

As seen from Figs. 3b and 4a, the third harmonic emis­sion is usually very weak, and, therefore, there are only a
handful of observations of third harmonics in solar radio
bursts. Zheleznyakov and Zlotnik [1974] have suggested two
kinds of wave-wave interactions, namely, \( L + L' \rightarrow T_{3pe} \)
or \( L + T_{2pe} \rightarrow T_{3pe} \), as possible mechanisms for excitation
of higher harmonics. However, the observed bicoherence
peak at \( (f_{pe}, 2f_{pe}) \) strongly suggests that the third harmonic
is probably generated by the later mechanism.

4. Conclusions

We have presented the high time resolution observations of
a Langmuir wave packet associated with a local solar type
III radio burst. This wave packet is found to satisfy not only
the threshold condition of the supersonic modulational insta­
ability, but also the criterion for a collapsing Langmuir
soliton. Furthermore, the spectrum of this wave packet is
found to contain an intense peak at \( f_{pe} \) and weaker peaks at
\( 2f_{pe} \) and \( 3f_{pe} \). Since the fundamental is more intense than
the second harmonic which is stronger than the third har­
monic, contrary to the results obtained by the ground testing
and simulations of instrumental effects which have predicted
a stronger third harmonic in comparison to the second, it is
argued that the observed harmonic peaks probably are not
due to instrumental effects. In order to test whether the
fundamental and harmonic spectral components are phase
coupled with each other, the wavelet based bicoherence ana­
lysis technique is applied on this wave packet. It is shown
that the bicoherence exhibits two peaks at (~ \( f_{pe} \) = 14 kHz,
~ \( f_{pe} \) = 14 kHz) and (~ \( f_{pe} \) = 14 kHz, ~ \( f_{pe} \) = 28 kHz). The
bicoherence peak at (~ \( f_{pe} \) = 14 kHz, ~ \( f_{pe} \) = 28 kHz) strongly
suggests that (1) the three wave-wave interaction ~ 14 kHz
+ ~ 14 kHz ~ ~ 28 kHz, i.e., \( L + L' \rightarrow T_{3pe} \) occurs in the
regime of strong Langmuir turbulence, (2) the anti-parallel
propagating Langmuir waves involved in these wave-wave
interactions are probably generated by the oscillating two
stream instability (OTS1), which is responsible for the for­
mation of collapsing Langmuir solitons as suggested by Pa­
padopoulos et al. [1974], (3) the observed second harmonic
spectral component is not due to the spontaneously emitted
coherent emission by the stable and collapsing solitons as
suggested by Papadopoulos and Freund [1978] and Goldman
et al. [1980], respectively, because in the case of coherent
emission, the bicoherence is expected to be very close to
zero, and (4) the harmonic modes are not the electrostatic
modes spontaneously generated by either the eigenmode­
localized Langmuir waves as suggested by Malaspina et al.
[2010], or by the trapped electrons in the Langmuir wave
potential maxima as suggested by Kellogg et al. [2010]. As
far as the bicoherence peak at (~ \( f_{pe} \) = 14 kHz, ~ \( f_{pe} \) = 28 kHz)
is concerned, it provides evidence for the merging of the
Langmuir waves with the second harmonic electromagnetic
wave yielding a third harmonic electromagnetic wave, i.e.,
\( L + T_{2pe} \rightarrow T_{3pe} \), as suggested by Zheleznyakov and
Zlotnik [1974].

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