Phase Coupling between Spectral Components of Collapsing Langmuir Solitons in Solar Type III Radio Bursts

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We present the high time resolution observations of one of the Langmuir wave packets obtained in the source region of a solar type III radio burst. This wave packet satisfies the threshold condition of the supersonic modulational instability, as well as the criterion of a collapsing Langmuir soliton, i.e., the spatial scale derived from its peak intensity is less than that derived from its short time scale. The spectrum of this wave packet contains an intense spectral peak at local electron plasma frequency, $f_{pe}$, and relatively weaker peaks at $2f_{pe}$ and $3f_{pe}$. We apply the wavelet based bispectral analysis technique on this wave packet and compute the bicoherence between its spectral components. It is found that the bicoherence exhibits two peaks at $\sim f_{pe}$ and $\sim 2f_{pe}$, which strongly suggest that the spectral peak at $2f_{pe}$ probably corresponds to the second harmonic radio emission, generated as a result of the merging of antiparallel propagating Langmuir waves trapped in the collapsing Langmuir soliton, and, the spectral peak at $3f_{pe}$ probably corresponds to the third harmonic radio emission, generated as a result of merging of a trapped Langmuir wave and a second harmonic electromagnetic wave.

1. Introduction

Solar flares, which represent the most dramatic energy releases from the Sun, eject electrons into the corona and interplanetary medium. These electrons form bump-on-tail distributions and drive Langmuir waves unstable. These Langmuir waves subsequently couple to type III radio emissions at the fundamental and second harmonic of the electron plasma frequency, $f_{pe}$. This scenario, generally known as the plasma hypothesis [Ginzburg and Zheleznyakov, 1968] is supported by the observed fast negative frequency drift of the type III radio burst, which is interpreted as due to the electron beam propagating radially outward into lower and lower densities of the corona and interplanetary medium. Furthermore, the in situ observations of energetic electrons [Lin et al., 1973, 1981; Ergun et al., 1998] and Langmuir waves [Gurnett and Anderson, 1976, 1977; Lin et al., 1986; Kellogg et al., 1992; Gurnett et al., 1993; Thejappa et al., 1993; Hospodarsky and Gurnett, 1995; Thejappa and MacDowall, 1998; Thejappa et al., 1999; Henri et al., 2009] in the source regions of type III radio bursts have provided additional support for the plasma hypothesis. Some times, faint third harmonic emissions are also observed in type III bursts [Benz, 1973; Takakura and Yousef, 1974]. One of the outstanding questions is the identification of the mechanism/mechanisms responsible for conversion of Langmuir waves into electromagnetic radiation.

Some authors [Papadopoulos et al., 1974; Smith et al., 1979; Goldstein et al., 1979; Goldman et al., 1980; Nicholson et al., 1978; Papadopoulos and Freund, 1978] predict that in type III radio bursts, the Langmuir waves are strongly turbulent, and therefore, the related oscillating two stream instability (OTSIII) and Langmuir collapse [Zakharov, 1972] play significant roles in the stabilization of electron beams as well as in conversion of Langmuir waves into electromagnetic waves. The Fast Envelope Sampler of Ulysses URAP experiment [Stone et al., 1992] have provided some evidence for strong turbulence processes in type III burst sources [Kellogg et al., 1992; Thejappa et al., 1993, 1996; Thejappa and MacDowall, 1998; Thejappa et al., 1999; Thejappa and MacDowall, 2004]. The in situ observations from the improved Time Domain Sampler (TDS) of the STEREO/WAVES experiment [Bougeret et al., 2008] have provided some evidence for strong turbulence processes in type III burst sources [Thejappa et al., 2012a, b, c].

In this paper, we present new observations of a Langmuir wave packet captured by the STEREO TDS in the source region of a local type III burst. This wave packet satisfies the threshold condition of the supersonic modulational instability, as well as the criterion of a collapsing soliton; the spatial scale derived from its peak intensity is less than the spatial scale derived from the observed time scale. The spectrum of this event, which is the focus of this study contains peaks at $2f_{pe}$ and $3f_{pe}$ in addition to an intense peak at $f_{pe}$. To investigate whether any phase coherence exists between these spectral components, we compute the bicoherence using the wavelet based bispectral analysis technique. This bicoherence exhibits two peaks, which strongly suggest that (1) the $2f_{pe}$ spectral peak probably corresponds to the electromagnetic waves generated as a result of the coalescence of anti-parallel propagating Langmuir waves trapped in the collapsing Langmuir soliton, and (2) the $3f_{pe}$ peak corresponds to third harmonic radio emission generated as a result of coalescence of trapped Langmuir wave with second harmonic electromagnetic wave. In section 2, we present the observations, in section 3, we present the bispectral analysis, and in section 4, we present the conclusions.

2. Observations

In Fig. 1, we present the dynamic spectrum of one of the local type III solar radio bursts (fast drifting emissions) and associated Langmuir waves (non-drifting emissions), observed on February 11, 2012 by the STEREO B spacecraft. In Fig. 2, we present the frequency-time spectrogram in a narrow frequency range, which shows the bursty structure of the Langmuir waves. The substantial frequency spreading of Langmuir waves seen here is probably due to nonlinear frequency broadening. In this figure, the Langmuir

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wave burst corresponding to the TDS event of current interest is shown by an arrow. In Fig. 3a, we present one of the most intense wave packets detected during this type III event. This wave packet is characterized by the peak electric field strength $E_L$ of 64.6 mV m$^{-1}$ and $\frac{1}{2}$-power duration $D$ of $\sim 11$ ms. Since the $E_x$ and $E_z$ signals are weaker and show the same general features as the $E_y$ signal, we analyze only the $E_y$ signal. We justify one-dimensional treatment by assuming that these Langmuir wave fields are probably aligned along the ambient magnetic field. In Figure 3b, we show the spectrum of this waveform from 12 to 45 kHz. An intense peak at $f_{pe} \approx 14$ kHz, and two secondary peaks, one at $\sim 28$ kHz (second harmonic $\sim 2f_{pe}$) and another one at $\sim 42$ kHz (third harmonic $\sim 3f_{pe}$), are clearly seen in this spectrum. The observed harmonics probably are the natural signals since: (1) the simulations of instrumental nonlinearities [Walker et al., 2002] as well as the ground testing have shown that $3f_{pe}$ signal is stronger than $2f_{pe}$ signal [Malaspina et al., 2010], whereas in this case the higher the harmonic, weaker the signal, and (2) according to Kellogg et al. [2010], the STEREO/WAVES experiment is designed in such a way that the responses in the Langmuir wave frequency range are as accurately linear as possible, and no definite relation exists between the harmonic and fundamental amplitudes.

Assuming that the frequency of main spectral peak in Fig. 3b corresponds to the $f_{pe}$, we estimate the electron density $n_e$ during this event as $\sim 2.4 \times 10^6$ m$^{-3}$ using the relation $f_{pe} \approx 9n_e^{1/2}$. The solar wind speed $v_{sw}$ is measured as $\sim 500$ km s$^{-1}$ by the STEREO PLASTIC experiment [Galejs et al., 2008], and we assume that the electron temperature $T_e$ during this event is $\sim 10^5$ K. We estimate the speed of the electron beam $v_b$ as $\sim 0.28c$ by fitting the frequency drift curve to the dynamic spectrum of the type III event ($c$ is the speed of light). Using these values, we estimate: (1) Debye length $\lambda_{De} = 69T_e^{1/2}n_e^{-1/2} \sim 14$ m for $T_e \sim 10^5$K and $n_e \sim 2.4 \times 10^6$ m$^{-3}$, (2) wave number of Langmuir waves, $k_L = \frac{2\pi f_{pe}}{v_b} \sim 10^{-3}$ m$^{-1}$ for $f_{pe} \sim 14$ kHz and $v_b \sim 0.28c$, (3) $k_L \lambda_{De} \sim 1.4 \times 10^{-2}$, (4) normalized peak energy density, $\frac{W_p}{n_e E_L^2} \sim 5.6 \times 10^{-3}$ for $E_L = 64.6$ mV m$^{-1}$, $n_e = 2.4 \times 10^6$ m$^{-3}$ and $T_e = 10^5$ K, and (5) the $\frac{1}{2}$ level spatial scale $S \sim Dv_{sw} \sim 393\lambda_{De}$ for the measured values of $D = 11$ ms and $v_{sw} = 500$ km s$^{-1}$.

![Figure 1](image1.png)

**Figure 1.** Dynamic spectrum of the local type III radio burst (fast drifting emission from $\sim 3$ MHz down to $\sim 20$ kHz) and associated Langmuir waves (non-drifting emissions in the frequency interval 10-14 kHz.)

![Figure 2](image2.png)

**Figure 2.** The frequency-time spectrogram of the Langmuir waves observed during the type III burst of February 11, 2012. The Langmuir wave emissions are very bursty and show substantial frequency spreading. The arrow shows the Langmuir wave burst corresponding to the current TDS event.

![Figure 3](image3.png)

**Figure 3.** (a): One of the most intense Langmuir wave packets captured by the Time Domain Sampler (TDS) during the type III event of Fig. 1. The $\frac{1}{2}$ power duration of 11 ms which is equivalent to the spatial scale of $393\lambda_{De}$ is also shown. (b): The FFT spectrum of the TDS event, which shows the peaks at local electron plasma frequency, $f_{pe}$ as well as at $2f_{pe}$ and $3f_{pe}$.
The threshold for the supersonic modulational instability [Zakharov, 1972]

\[ \frac{W_L}{n_e T_e} > \frac{m_a}{m_i} > (k_l \lambda_{De})^2 \]

is easily satisfied in this case, since \( \frac{W_L}{n_e T_e} \sim 5.6 \times 10^{-3} \), \( \frac{m_a}{m_i} \sim 5.5 \times 10^{-4} \) and \((k_l \lambda_{De})^2 \sim 2 \times 10^{-4}\). The wave packet also satisfies the criterion of the collapsing envelope soliton [Thornhill and ter Haar, 1978; Garnett et al., 1981]

\[ \frac{W_L}{n_e T_e} \geq (\Delta k \lambda_{De})^2, \]

where \( \Delta k = \frac{2 \pi}{L} \) is the wavenumber characteristic of the envelope, since the observed \( \frac{W_L}{n_e T_e} \sim 5.6 \times 10^{-3} \) is greater than \((\Delta k \lambda_{De})^2 \sim 2.6 \times 10^{-4} \) estimated for the spatial scale of \( S \sim 393 \lambda_{De} \).

3. Bispectral Analysis

For nonlinear wave-wave interactions, the frequencies \((f_1, f_2, f_3)\) and wave vectors \((k_1, k_2, k_3)\) must satisfy the resonance conditions: \( f_1 + f_2 = f_3 \) and \( E_1 + k_2 = k_3 \). Since the wave number \( k_i \) is related to the phase \( \phi_i \) of the wave, the phases of three waves are also related as \( \phi_1 + \phi_2 = \phi_3 \). Bispectrum, a higher order spectrum, permits the distinction of the spontaneously excited normal modes from the coupled modes by measuring the degree of phase coherence between the modes. The FFT based bi-spectrum is defined as

\[ B(f_1, f_2) = E[X(f_1)X(f_2)X^*(f_1 + f_2)], \]

where \( E \) denotes the expected value, \( X(f) \) is the Fourier transform of \( x(t) \) and * denotes complex conjugation. This equation clearly indicates that the bispectrum is zero unless three waves at frequencies \( f_1, f_2 \) and \( f_1 + f_2 \) are present in the time series with phase coherency amongst the waves. For spontaneously generated non-interacting modes, the phases would be random and therefore the statistical averaging will be zero. The bicoherence, which is the normalized bispectrum, provides the quantitative measure of the phase coherence. It is defined as:

\[ b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{E[X(f_1)X(f_2)X^*(f_1 + f_2)]^2}. \]

A unit value for the bicoherence indicates perfect coupling, a zero value indicates no coupling, and any value between zero and one indicates partial coupling. Periodogram estimation (interval segmentation) is usually used to estimate the bispectrum. The approximate variance of the bicoherence estimator is defined as [Kim and Powers, 1979]:

\[ \text{var}(b^2) \sim \frac{M}{2N} [1 - b^2(f_1, f_2)], \]

where \( N \) is the number of total data points and \( M \) is the number of data records into which the time series is divided. To assess the low bicoherence levels, one needs long and stationary time series with \( N' \gg 1 \). The bispectral analysis has been applied on the data from laboratory [Kim and Powers, 1979] as well as space plasma experiments [Lagoutte et al., 1989; Dudok de Wit and Krasnoselskikh, 1995; Bale et al., 1996; Balikhin et al., 2001; Walker et al., 2003; Henri et al., 2009].

The wavelet based bispectral analysis technique is superior to the FFT based technique, since it can handle both short and non-stationary time series. It has been developed to study the strong turbulence processes at the Earth’s quasi-parallel bow shock [Dudok de Wit and Krasnoselskikh, 1995], and it has been successfully used to study the electrostatic decay in type III sources [Henri et al., 2009]. In this study, we apply this wavelet based bispectral analysis technique on the wave packet of Fig. 3a. The continuous wavelet transform (CWT), which is used in this technique is defined as

\[ W(a, \tau) = \frac{1}{\sqrt{|a|}} \int x(t) \psi^*(\frac{t-\tau}{a}) dt, \]

where the scale \( a \) is related to the frequency \( f \) as \( a = \frac{1}{f} \) and \( \tau \) is the time. The Morlet wavelet \( \psi(t) = e^{2\pi ft} e^{-t^2/2} \) is generally used to study nonlinear wave phenomena. The wavelet based bispectrum in the frequency space can be defined as

\[ B(f_1, f_2) = E[W(f_1)W(f_2)W^*(f_1 + f_2)], \]

where \( W(\nu \delta t, f = 1/a) \) is a function of time \( t \) as well as frequency \( f \). The wavelet bicoherence can be defined as

\[ b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{E[|W(f_1)W(f_2)W^*(f_1 + f_2)|]^2}. \]

The discrete wavelet coefficients \( W_l \) of the signal are computed using the Morlet wavelet. The discretization frame of the wavelet coefficients is taken regular with a constant frequency step to ensure the frequency resonance conditions.

In Figure 4a, we present the modulus of the wavelet transform applied to the waveform data presented in Fig. 3a. The spectral features corresponding to the fundamental, second
and third harmonic components are clearly seen in this spectrogram, which show that the higher the harmonic, weaker the emission. It also shows the temporal coincidence of the second and third harmonic emissions with collapsing Langmuir soliton. In Figure 4b, we present the results of wavelet-based bicoherence analysis. This bicoherence spectrogram clearly shows evidence for three wave-wave interactions. The bicoherence peaks at (~14 kHz, ~14 kHz), and (~14 kHz, ~28 kHz) suggest that these spectral components interact to produce the emissions at 28 kHz and 42 kHz, respectively. The most probably three wave interactions are \( L + L' \rightarrow T_{fpe} \), and \( L + T_{2fpe} \rightarrow T_{3fpe} \), respectively (\( L \) and \( L' \) are the opposite propagating Langmuir waves, and \( T_{fpe} \), \( T_{2fpe} \), and \( T_{3fpe} \) are the second and third harmonic electromagnetic waves, respectively).

The Langmuir waves involved in the interaction \( L + L' \rightarrow T_{2fpe} \) are probably the anti-parallel propagating waves trapped in the collapsing soliton. This is consistent with the wavelet spectrogram (Fig. 4a), which shows the temporal coincidence of the second and third harmonic emissions with the peak of the Langmuir wave packet. The wave-wave interaction \( L + L' \rightarrow T_{2fpe} \) agrees with that of Papadopoulos et al. [1974], who suggested that the anti-parallel propagating modes generated during oscillating two stream instability (OTSIs), get trapped in the collapsing soliton and couple to each other generating electromagnetic waves at \( 2f_{pe} \). The interpretation of the spectral peak at 28 kHz in terms of \( T_{2fpe} \) is consistent with that of Bale et al. [1999], who interpreted the harmonic spectral peak of the wave packet observed by the WIND TDS in the upstream solar wind in terms of \( T_{2fpe} \). One should note that the observed phase coherence between the fundamental and harmonic spectral components does not support the mechanism invoked by Papadopoulos and Freund [1978] as well as Goldman et al. [1980], which invoke the spontaneous emission of electromagnetic waves at \( 2f_{pe} \) by the stable and collapsing solitons, respectively; the primary waves at \( f_{pe} \) and harmonics at \( 2f_{pe} \) in such a case are not phase correlated.

Malaspina et al. [2010] have interpreted the harmonics of the wave packets captured by the STEREO TDS in the solar wind in terms of localized nonlinear electrostatic modes driven by eigenmode-localized Langmuir waves. Furthermore, these authors have suggested that these nonlinear electrostatic modes probably emit the coherent electromagnetic radiation spontaneously at respective frequencies. However, the observed phase coherence between the fundamental and harmonic components rules out (1) the possibility that the \( 2f_{pe} \) spectral peak corresponds to harmonics Langmuir waves, since the non-linear wave-wave interaction between the fundamental and harmonic Langmuir waves is forbidden (harmonic Langmuir waves are not the eigen modes), and (2) the second harmonic radio emission is not due to any antenna emission but is due to wave-wave interactions between the opposite propagating Langmuir waves.

Kellogg et al. [2010] argued that the harmonics of the wave packets captured by the STEREO TDS in the Earth’s fore shock are probably due to the electrons trapped in the Langmuir wave potential maxima. In this scenario also, the bicoherence which measures the phase coherence is expected to be zero contrary to the observed values, which suggests that the observed harmonic modes in the present case are not due to trapping of electrons in the Langmuir wave maxima. As seen from Figs. 3b and 4a, the third harmonic emission is usually very weak, and, therefore, there are only a handful of observations of third harmonics in solar radio bursts. Zhelleyuzhnyk and Zlotnik [1974] have suggested two kinds of wave-wave interactions, namely, \( L + L' \rightarrow T_{3fpe} \) or \( L + T_{2fpe} \rightarrow T_{3fpe} \), as possible mechanisms for excitation of the third harmonic. However, the observed bicoherence peak at \( (f_{pe}, 2f_{pe}) \) strongly suggests that the third harmonic is probably generated by the later mechanism.

4. Conclusions

We have presented the high time resolution observations of a Langmuir wave packet associated with a local solar type III radio burst. This wave packet is found to satisfy not only the threshold condition of the supersonic modulational instability, but also the criterion for a collapsing Langmuir soliton. Furthermore, the spectrum of this wave packet is found to contain an intense peak at \( f_{pe} \) and weaker peaks at \( 2f_{pe} \) and \( 3f_{pe} \). Since the fundamental is more intense than the second harmonic which is stronger than the third harmonic, contrary to the results obtained by the ground testing and simulations of instrumental effects which have predicted a stronger third harmonic in comparison to the second, it is argued that the observed harmonic peaks probably are not due to instrumental effects. In order to test whether the fundamental and harmonic spectral components are phase coupled with each other, the wavelet based bicoherence analysis technique is applied on this wave packet. It is shown that the bicoherence exhibits two peaks at \( (f_{pe} = 14 \text{ kHz}, ~ 2f_{pe} = 28 \text{ kHz}) \) and \( (f_{pe} = 14 \text{ kHz}, ~ 3f_{pe} = 42 \text{ kHz}) \). The bicoherence peak at \( (f_{pe} = 14 \text{ kHz}, ~ 2f_{pe} = 28 \text{ kHz}) \) strongly suggests that: (1) the three wave-wave interaction \( L + L' \rightarrow T_{3fpe} \rightarrow T_{2fpe} \) occurs in the regime of strong Langmuir turbulence, (2) the anti-parallel propagating Langmuir waves involved in these wave-wave interactions are probably generated by the oscillating two stream instability (OTSIs), which is responsible for the formation of collapsing Langmuir solitons as suggested by Papadopoulos et al. [1974], (3) the observed second harmonic spectral component is not due to the spontaneously emitted coherent emission by the stable and collapsing solitons as suggested by Papadopoulos and Freund [1978] and Goldman et al. [1980], respectively, because in the case of coherent emission, the bicoherence is expected to be very close to zero, and (4) the harmonic modes are not the electrostatic modes spontaneously generated by either the eigenmode-localized Langmuir waves as suggested by Malaspina et al. [2010], or by the trapped electrons in the Langmuir wave potential maxima as suggested by Kellogg et al. [2010]. As far as the bicoherence peak at \( (f_{pe} = 14 \text{ kHz}, ~ 3f_{pe} = 42 \text{ kHz}) \) is concerned, it provides evidence for the merging of the Langmuir waves with the second harmonic electromagnetic wave yielding a third harmonic electromagnetic wave, i.e., \( L + T_{3fpe} \rightarrow T_{3fpe} \), as suggested by Zhelleyuzhnyk and Zlotnik [1974].

Acknowledgments

The research of T. G. is supported by the NASA Grant NNX09AB19G. The SWAVES instruments include contributions from the Observatoire de Paris, University of Minnesota, University of California, Berkeley, and NASA/GSFC.

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