An Integrated Approach for Gear Health Prognostics

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ABSTRACT

In this paper, an integrated approach for gear health prognostics using particle filters is presented. The presented method effectively addresses the issues in applying particle filters to gear health prognostics by integrating several new components into a particle filter: (1) data mining based techniques to effectively define the degradation state transition and measurement functions using a one-dimensional health index obtained by whitening transform; (2) an unbiased *l*-step ahead RUL estimator updated with measurement errors. The feasibility of the presented prognostics method is validated using data from a spiral bevel gear case study.

1. INTRODUCTION

Over the past years, both physics based and data driven approaches for drivetrain component health prognostics have been developed. Examples of physics based approaches included: Bechhofer *et al*. (2008), He *et al*. (2007), Qiu *et al*. (2002), Li *et al*. (2000), and Ray and Tangirala (1996). Some recent developments in data driven approaches has also been reported in the literature: Dong and He (2007), Hung *et al*. (2007), and Bechhoefer *et al*. (2006). Physics based approaches need specific system knowledge and theory relevant to the monitored components in order to perform prognostics. The data driven approaches do not necessarily need to understand the physics of the monitored applications. Thus the complexity of designing the data driven prognostic methodology is greatly reduced, especially for new and complex components. Both physics based and data driven approaches have their own advantages and disadvantages for component health prognostics. Therefore, developing integrated prognostics approaches that combine the advantages of both the physics based and data driven approaches becomes attractive to condition based maintenance practitioners. A recent development is the use of particle filter to combine both physics based and data driven approaches for prognostics.

Applications of particle filters to prognostics have been reported in the literature, for example, remaining useful life (RUL) predication of a mechanical component subject to fatigue crack growth (Zio and Peloni, 2011), on-line failure prognosis of UH-60 planetary carrier plate subject to axial crack growth (Orchard and Vachtsevanos, 2011), degradation prediction of a thermal processing unit in semiconductor manufacturing (Butler and Ringwood, 2010), and prediction of lithium-ion battery capacity depletion (Saba *et al*., 2009). The reported application results have shown that particle filters represent a potentially powerful prognostics tool due to its capability in handling non-linear dynamic systems and non-Gaussian noises using efficient sequential importance sampling to approximate the future state probability distributions. Particle filters were developed as an effective on-line state estimation tool (see Doucet *et al*., 2000; Arulampalam *et al*., 2002). In order to apply particle filter to RUL prediction of a mechanical component such as gears, a few practical

implementation problems have to be solved: (1) define a state transition function $\mathbf{f}_k$ that represents the degradation evolution in time of the component; (2) select the most sensitive health monitoring measures or condition indicators (CIs) and define a measurement function $\mathbf{h}_k$ that represents the relationship between the degradation state of the component and the CIs; (3) define an effective *l*-step ahead RUL estimator.

In solving the first problem, research on using particle filters for mechanical component RUL prognostics has used Paris' law, crack growth rate per loading cycles, to define the state transition function $\mathbf{f}_k$ (Zio and Peloni, 2011; Orchard and Vachtsevanos, 2011). As an empirical model, Paris' law can be effective for defining a state transition function that represents a degradation state subject to fatigue crack growth. For other type of failure modes such as pitting and corrosion, effective alternatives for defining the state transition function should be explored.

Regarding the second problem, on the surface, it doesn't seem to be a problem to use multiple CIs to define a measurement function for particle filter as it allows information from multiple measurement sources to be fused in a logical manner (Zio and Peloni, 2011). In particle filter analyses, measurements are collected and used to update the prior state distribution via Bayes rule so as to obtain the required posterior state distribution. Subsequently, various kinds of uncertainties arise from different sources that are correlated. In most real applications, no single CI is sensitive to every failure mode of a component. This suggests that defining the measurement function $\mathbf{h}_k$ will have some form of de-correlated sensor fusion.

For the third problem, in order to apply particle filter to estimate the RUL, an *l*-step ahead estimator has to be defined. Both biased and unbiased *l*-step ahead estimators have been reported by Zio and Peloni (2011) and Orchard and Vachtsevanos (2011). However, as pointed out by Zio and Peloni (2011), one issue related to these estimators is that state estimation and prediction must be accompanied by a measure of the associated error.

In this paper, an integrated prognostic approach using particle filters for gear health prognostics is presented. In particular, in presenting the method, the three integration issues will be addressed: (1) define the state transition function using data mining approach; (2) use an one-dimensional health index (HI) obtained by a whitening transform to define the measurement function; (3) an *l*-step ahead RUL estimator incorporated with a measure of the associated error. The feasibility of the presented method is validated using fatigue testing data from a spiral bevel gear case study performed in the NASA Glenn Spiral Bevel Gear Test Facility.

2. THE METHOD

The general framework of the integrated gear health prognostics approach using particle filter is shown in Figure 1.

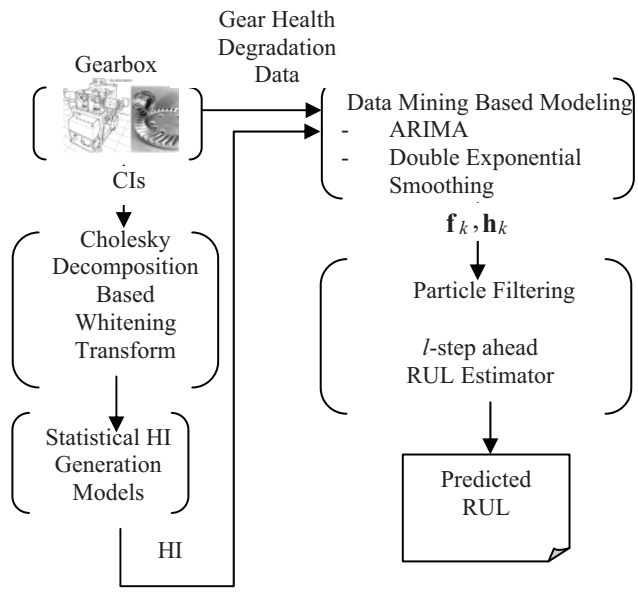

Figure 1. Integrated gear prognostics framework using particle filter

As shown in Figure 1, to predict the RUL of the gears, condition monitoring data that reflect the gear health degradation need to be collected and correlated with health indicators (HI) to build the gear health state transition function $\mathbf{f}_k$ in a particle filter. Inductance type oil debris sensors have been used to monitor the health of gearbox mechanical components (Dempsey, 2002). Inductance type, oil debris sensors count particles and approximate debris size and mass based on disturbances of a magnetic field caused by passage of a metallic particle. As the oil debris sensor data such as oil debris mass (ODM) generally provides a gear degradation indication it can be used to build the gear degradation state transition function $\mathbf{f}_k$ by mining the ODM data. Typical statistical analysis based data mining techniques such as autoregressive integrated moving average (ARIMA), double exponential smoothing, and etc. can be used.

Vibration sensors have been the most commonly used sensors in mechanical systems health monitoring

applications. Condition indictors (CIs) are obtained from vibration data and used for mechanical fault detection and diagnosis. For example, the current health and usage monitoring systems (HUMS) installed in helicopters utilize a large number of vibration based condition indictors. As pointed out in (Bechhoefer *et al*., 2011), there is no single CI that is sensitive to every failure mode of a gear. Some form of sensor fusion is required for the condition based maintenance system of gears. Combining multiple CIs into one health index (HI) is an attractive sensor fusion approach for gear health prognostics.

In the framework presented in Figure 1, a one-dimensional HI obtained by the applying a Cholesky decomposition based whitening transform and statistical generation models *i*s used to define the measurement function $\mathbf{h}_k$ by correlating the gear health degradation data with the one-dimensional HI using a double exponential smoothing approach. Based on the defined functions $\mathbf{f}_k$ and $\mathbf{h}_k$, an *l*-step ahead RUL estimator with measurement error is used in particle filter to provide accurate prediction of RUL. The generation of the one-dimensional HI and the *l*-step ahead RUL estimator used in particle filter are explained in the following sections. A detailed analysis of these methods for this application can also be found in reference Ma 2011.

2.1 The One-Dimensional Health Index

The concept of using Cholesky decomposition to develop an one-dimensional gear health index and its threshold setting based on a probability of false alarm was first reported in Bechhoefer *et al*. (2011). Cholesky decomposition is a linear transformation that can de-correlate a covariance matrix. To develop the one-dimensional health index, a set of correlated CIs are first de-correlated by applying the Cholesky decomposition. The Cholesky decomposition of Hermitian, a positive definite matrix results in **A** = **LL***, where **L** is a lower triangular, and **L*** is its conjugate transpose. By definition, the inverse covariance is positive definite Hermitian. Let **F** be a set of correlated CIs. It then follows that:

$$\mathbf{LL}^* = \mathbf{\Sigma}^{-1} \tag{1}$$

and

$$\mathbf{Y} = \mathbf{L} \times \mathbf{F}^T \tag{2}$$

where **Y** is a vector of *n* independent CIs with unit variance and $correlation(\mathbf{Y}) = 0$. The Cholesky decomposition, in effect, creates the square root of the inverse covariance. This in turn is analogous to dividing feature by its standard deviation (the trivial case of one feature). In turn, Eq. (2) creates the necessary independent and identical distributions required to define the health index for a function of distributions.

Assuming that the distributions of the CIs follow a Gaussian distribution, then three statistical HI generation models can be developed: (1) the Gaussian order statistic; (2) the sum of *n* Gaussian; and (3) the total energy of *n* Gaussian. These three models are explained as follows (Bechhoefer *et al*. 2011):

(1) When the HI is defined as the Gaussian order statistic, it can be generated as following:

$$\begin{aligned} &\mathbf{Y} = \mathbf{L} \times \left(\mathbf{F}^T - \mathbf{m}\right) \\ &HI = \left(\max\{\mathbf{Y}\} + .34\right) \times 0.5 \big/ \left(3.41 + 0.34\right) \end{aligned} \tag{3}$$

where **m** is the mean of **F**. Subtracting the mean and multiplying by **L** transforms the features into *n*, *Z* distributions (zero mean, IID Gaussian distributions).

(2) When the HI is defined as the sum of *n* Gaussian, it can be generated as following:

$$\begin{aligned} &\mathbf{Y} = \mathbf{L} \times \mathbf{F}^T \\ &HI = 0.5 \big/ \left(8.352 - 0.15\right)\left(-0.15 + \textstyle\sum_{i=1}^{n} \mathbf{Y}_i\right) \end{aligned} \tag{4}$$

(3) When the HI is defined as the total energy of *n* Gaussian, it can be generated as following:

$$\begin{aligned} &\mathbf{Y} = \mathbf{L} \times \mathbf{F}^T \\ &HI = 0.5 \big/ 3.368 \sqrt{\textstyle\sum_{i=1}^{n} \mathbf{Y}_i^2} \end{aligned} \tag{5}$$

2.2 RUL Prediction using Particle Filters

2.2.1 Particle filter for state estimation

Applying particle filters to state estimation will be discussed first. Particle filters are used to estimate the state of a dynamic system using state and observation parameters. The state transition function represents the degradation in time of the gears. The observation or measurement represents the relationship between the degradation state of the component and the CIs.

Consider a system described by the discrete time state space model:

$$\mathbf{x}_k = \mathbf{f}_k\left(\mathbf{x}_{k-1}, \boldsymbol{\omega}_{k-1}\right) \tag{6}$$

$$\mathbf{z}_k = \mathbf{h}_k\left(\mathbf{x}_k, \boldsymbol{\upsilon}_k\right) \tag{7}$$

where:

$\mathbf{f}_k : \Re^{n_x} \times \Re^{n_\omega} \rightarrow \Re^{n_x}$ is the state transition function

$\boldsymbol{\omega}_k$ is an independently and identically distributed (*iid*) state noise vector of known distribution

$\mathbf{h}_k \; \Re^{n_x} \times \Re^{n_\upsilon} \rightarrow \Re^{n_z}$ is the measurement function

$\boldsymbol{\upsilon}_k$ is an *iid* measurement noise vector

The problem of state estimation is to estimate the dynamic state $\mathbf{x}_k$ in terms of probability density function (pdf) $p(\mathbf{x}_k|\mathbf{z}_{0:k})$, given the measurement up to time k. The initial distribution of the state $p(\mathbf{x}_0)$ is assumed known.

The Bayesian solution to the state estimation problem normally consists of two steps: prediction and update. In the prediction step, the prior probability distribution of the state $\mathbf{x}_k$ at time k, starting from the probability distribution $p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})$ at time k-1, is obtained as:

$$\begin{aligned} p(\mathbf{x}_k|\mathbf{z}_{0:k-1}) &= \int p(\mathbf{x}_k|\mathbf{x}_{k-1},\mathbf{z}_{0:k-1})p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})d\,\mathbf{x}_{k-1} \\ &= \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_k|\mathbf{z}_{0:k-1})d\,\mathbf{x}_{k-1} \end{aligned} \quad (8)$$

In the update step, at time k, a new measurement $\mathbf{z}_k$ is collected and used to update the prior distribution to obtain the posterior distribution of the current state $\mathbf{x}_k$ as:

$$p(\mathbf{x}_k|\mathbf{z}_{0:k}) = \frac{p(\mathbf{x}_k|z_{0:k-1})p(\mathbf{z}_k|\mathbf{x}_k)}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})} \quad (9)$$

where the normalizing constant is:

$$p(\mathbf{z}_k|\mathbf{z}_{0:k-1}) = \int p(\mathbf{x}_k|\mathbf{z}_{0:k-1})p(\mathbf{z}_k|\mathbf{x}_k)d\,\mathbf{x}_k \quad (10)$$

Obtaining exact state estimation solutions for Eq. (8) and Eq. (9) is not realistic for most cases. Therefore, particle filter is used to obtain the heuristic solutions. The prediction at time k can be accomplished by particle filter by performing the following two tasks: (1) sampling N number of random samples (particles) $x_{k-1}^i, i = 1,...,N$ from the probability distribution of the state noise $\boldsymbol{\omega}_{k-1}$ and (2) generating new set of samples $x_k^i, i = 1,...,N$ using Eq. (6). In the update step, each new sampled particle x_k^i is assigned a weight w_k^i based on the likelihood of the new measurement $\mathbf{z}_k$ at time k as:

$$w_k^i = \frac{p(\mathbf{z}_k|x_k^i)}{\sum_{i=1}^N p(\mathbf{z}_k|x_k^i)} \quad (11)$$

Then the approximation of the posterior distribution $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ can be obtained from the weighted samples $\{x_k^i, w_k^i, i = 1,...,N\}$ (Doucet *et al.*, 2000).

2.2.2 Particle filter for RUL prediction

In order to apply particle filter to estimate the RUL, an l-step ahead estimator has to be developed. An l-step ahead estimator will provide a long term prediction of the state pdf $p(\mathbf{x}_{k+l}|\mathbf{z}_{0:k})$ for $l = 1,...,T-k$, where T is the time horizon of interest. In making an l-step ahead prediction, it is necessary to assume that no information is available for estimating the likelihood of the state following the future l-step path $\mathbf{x}_{k+1:k+l}$, that is, future measurements $\mathbf{z}_{k+l}, l = 1,...,T-k$ cannot be used for making the prediction. Therefore, one can only project the initial condition $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ using state transition pdf $p(\mathbf{x}_j|\mathbf{x}_{j-1}), j = k+1,...,k+l$ along all possible future paths weighted by their probability $\prod_{j=k+1}^{k+l} p(\mathbf{x}_j|\mathbf{x}_{j-1})d\,\mathbf{x}_{j-1}$. By combining Eq. (6) and Eq. (9), an unbiased l-step ahead estimator can be obtained (Zio and Peloni, 2011; Orchard and Vachtsevanos, 2011):

$$p(\mathbf{x}_{k+l}|\mathbf{z}_{0:k}) = \int...\int \prod_{j=k+1}^{k+l} p(\mathbf{x}_j|\mathbf{x}_{j-1})p(\mathbf{x}_k|\mathbf{z}_{0:k})\prod_{j=k}^{k+l-1} d\,\mathbf{x}_j \quad (12)$$

In theory, an unbiased estimator would give the minimum variance estimation. However, solving Eq. (12) can be either difficult or computationally expensive. A particle filtering approximation procedure of the l-step ahead estimator is provided in (Zio and Peloni, 2011).

Assume that the state $\mathbf{x}_k$ represents a mono-dimensional health indicator and RUL is the time remained before its crossing of a pre-specified critical value λ. At each time $k+l$ projected l steps from current time k, estimating $\hat{p}(RUL \le l|\mathbf{z}_{0:k})$ is equivalent to estimating $\hat{p}(\mathbf{x}_{k+l} \ge \lambda|\mathbf{z}_{0:k})$.

Note that in computing the l-step ahead RUL estimator using particle filter, at each updating step, a weight is computed according to Eq. (11) without considering any measurement of the associated errors. Define $\hat{\mathbf{z}}_k$ the estimated measurement at time k computed from Eq. (7). Then a weighting process in particle filter that takes into account the measurement errors can be defined as:

$$w_k^i = \frac{p((\mathbf{z}_k - \hat{\mathbf{z}}_k)|x_k^i)}{\sum_{i=1}^N p((\mathbf{z}_k - \hat{\mathbf{z}}_k)|x_k^i)} \quad (13)$$

In the integrated prognostic approach using particle filter presented in this paper, the l-step ahead RUL estimator expressed in Eq. (12) is computed using the weights defined by Eq. (13).

3. SPIRAL BEVEL GEAR CASE STUDY

In this paper, data collected from a spiral bevel gear case study conducted in the NASA Glenn Spiral Bevel Gear Test Facility are used to validate feasibility of the presented method.

3.1 Experimental Setup and Data Collection

A detailed description of the test rig and test procedure is given in (Dempsey *et al*., 2002). The rig (as shown in Figure 2) was used to quantify the performance of gear material, gear tooth design and lubrication additives on the fatigue strength of gears. During the testing, vibration CIs and oil debris data were collected in order to detect the pitting damage on the spiral bevel gears.

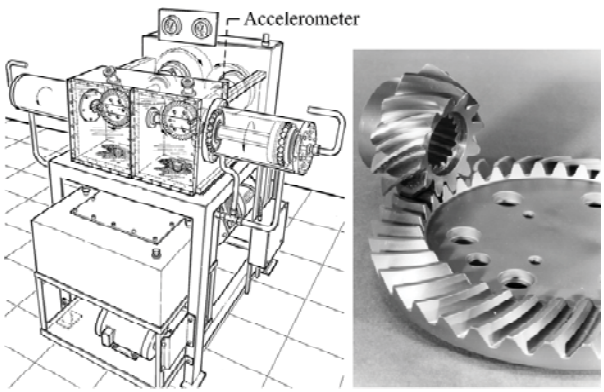

Figure 2. The bevel gear test rig and bevel gears (Dempsey *et al*., 2002)

The tests consisted of running the gears under load through a "back-to-back" closed loop torque regenerative system. Accelerometers were installed on the right and left side of the gearbox per Figure 2. Vibration data was collected once per minute using a sampling rate of 100 kHz for 2 seconds duration. Shaft speed was measured by an optical sensor once per each gear shaft revolution, generating time synchronous averages (TSA) on the gear shaft (36 teeth). The pinion, on which the damage occurred, has 12 teeth. The tests were performed for a specific number of hours or until surface fatigue occurs. In this paper, data collected from experiments 5 and 6 were used. At test completion, destructive pitting could be observed on the teeth of the pinions (see Figure 3).

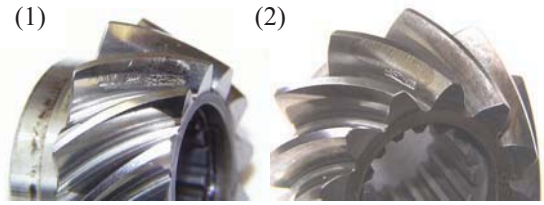

Figure 3. Damaged spiral bevel gears: (1) experiment 5, (2) experiment 6

TSA data was re-processed with gear CI algorithms presented in (Zakrajsek *et al*., 1993) and (Wemhoff *et al*., 2007) to compute the following CIs: (1) TSA: RMS, kurtosis (KT), peak-to-peak (P2P), crest factor (CF); (2) Residual RMS, KT, P2P, CF; (3) Energy operator RMS, KT; (4) Energy Ratio; (5) FM0; (6) Sideband Level factor; (7) Narrowband (NB) RMS, KT, CF; (8) Amplitude modulation (AM) RMS, KT; (9) Derivative AM KT; (10) Frequency modulation (FM) RMS, KT. However, not all the CIs generated from TSA data were good candidates for generating the HI. For the purpose of prognostics, one is interested in selecting the CIs that have shown a good trending correlation. In order to select the best CIs, correlation coefficients of the CIs with the time index were computed. The following 6 CIs with correlation coefficients over 0.5 were selected for the HI calculation: (1) residual RMS, (2) energy operator RMS, (3) FM0, (4) narrowband kurtosis, (5) amplitude modulation kurtosis, and (6) frequency modulation RMS. The HI used for this analysis was the order statistic defined by Eq. (3) as the order statistic gave a more consistent trending of the HI than other statistics.

3.2 The Results

The ODM data and the HIs computed using the 6 CIs from experiments 5 and 6 are plotted in Figure 4 and 5, respectively.

From Figure 4 and Figure 5, one can see that for both experiments 5 and 6, the computed HIs have shown a consistent increasing trend over time. The ODM data obtained from both experiments 5 and 6 have also shown the same increasing trend over time.

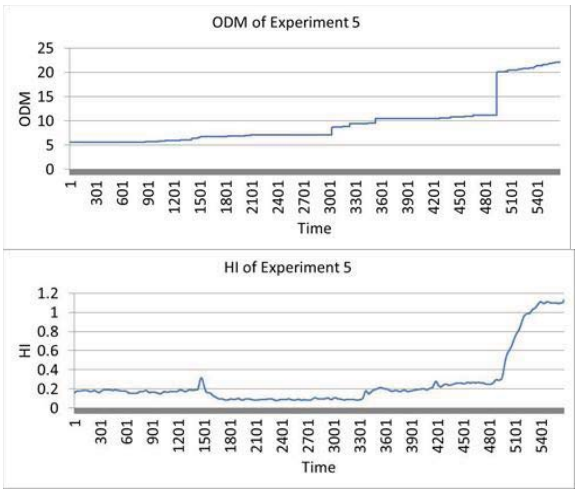

Figure 4. ODM and HI plots of experiment 5

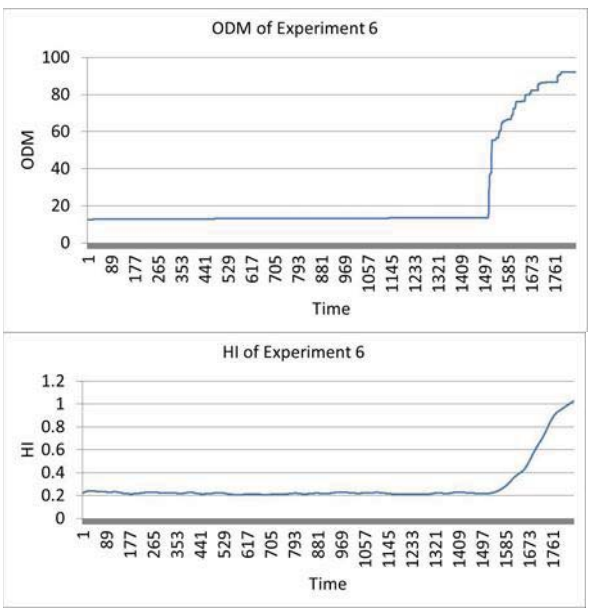

Figure 5. The ODM and HI plots of experiment 6

In this paper, rather than using Paris' law to define the gear degradation state transition function $\mathbf{f}_k$, the data mining approach is used. Autoregressive integrated moving average data mining method was used to identify patterns in the data and forecast the gear degradation state. Various ARIMA models were fitted on the ODM data of experiment 6. The best fitted ARIMA model with a minimum root mean squared errors (RMSE) was determined to be ARIMA(1,1,1). Let: x_k = true ODM value at time k, $\hat{x}_k$ = predicted ODM value at time k. Then the state transition function $\mathbf{f}_k$ can be defined as:

$$x_k = 0.0165 + 1.1415\, x_{k-1} - 0.415\, x_{k-2} - 0.1032\left(x_{k-1} - \hat{x}_{k-1}\right) + \omega_k$$

The plot of actual ODM values against the predicted ODM values is shown in Figure 6. From Figure 6, it is obvious that the ARIMA(1,1,1) model, where (1,1,1) represents the order of the autoregressive, integrated, and moving average, is almost a perfect fit to the ODM data.

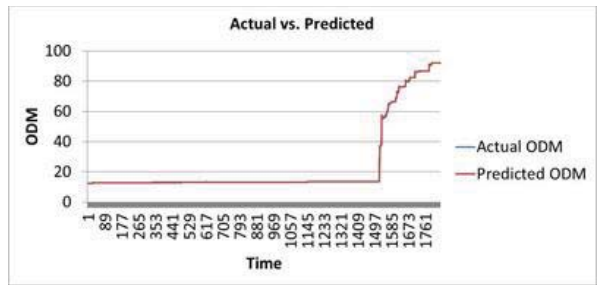

Figure 6. The actual ODM vs. the predicted ODM using ARIMA(1,1,1) model

To define the measurement function $\mathbf{h}_k$, the data mining approach is again used to fit a correlation model between the observation parameter, HI, and the state parameter, the ODM data. Figure 7 shows the plot of HI against ODM for experiment 6.

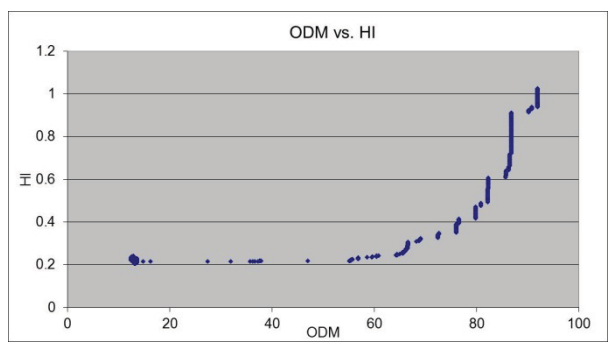

Figure 7. Plot of HI against ODM for experiment 6

The measurement function $\mathbf{h}_k$ was fitted using a double exponential smoothing model. This is a method to fit the data that weighs current observations more than previous observations (Montgomery *et al*., 2008). Figure 8 shows the predicted HI values using the

double exponential smoothing model against the actual HI values.

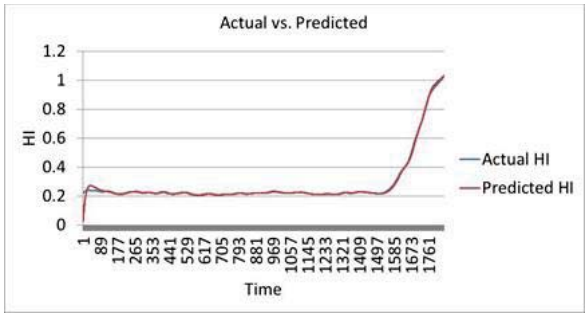

Figure 8. Predicted HI values using double exponential model vs. the actual HI values

Using the state transition function $\mathbf{f}_k$ and the measurement function $\mathbf{h}_k$ defined by the ODM and HI data from experiment 6, the particle filter based l-step ahead RUL estimator was run on the data from experiment 5 using $N = 1000$ particles. To compute the RUL, the critical value λ was set to ODM = 22 mg. This was determined by the amount of debris measured when damage was observed on one pinion tooth per experiment 5. The predicted ODM values are shown as the blue line in Figure 9.

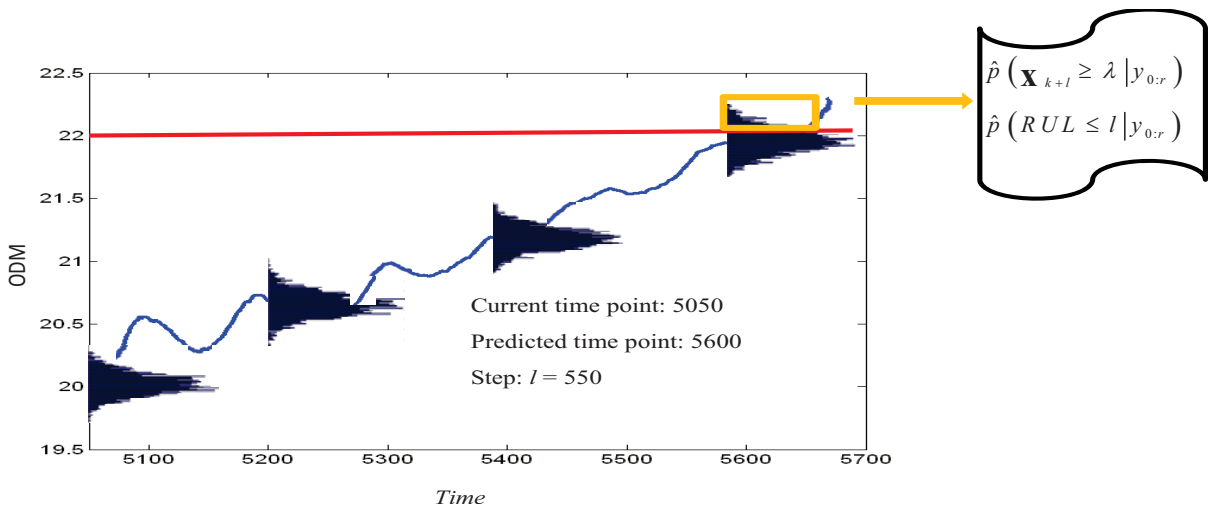

Figure 9. Failure rate and remaining useful life distribution based on the predicted system degradation result

In Figure 9, the red line is the pre-specified threshold. Note that in Figure 9, set $r = k$, then the probability of the remaining useful life less than 550 is computed as the same as the probability of the system is going to fail at future time point 5600 based on current measurement at time point 5050, that is, $\hat{p}(RUL \leq 550|HI_{0:5050}) = \hat{p}(\mathbf{X}_{5600} \geq 22|HI_{0:5050})$. Updating the estimated pdf on the basis of the measurements collected every 100 temporal steps, the estimated mean RUL and corresponding 90% confidence intervals are shown in Figure 10.

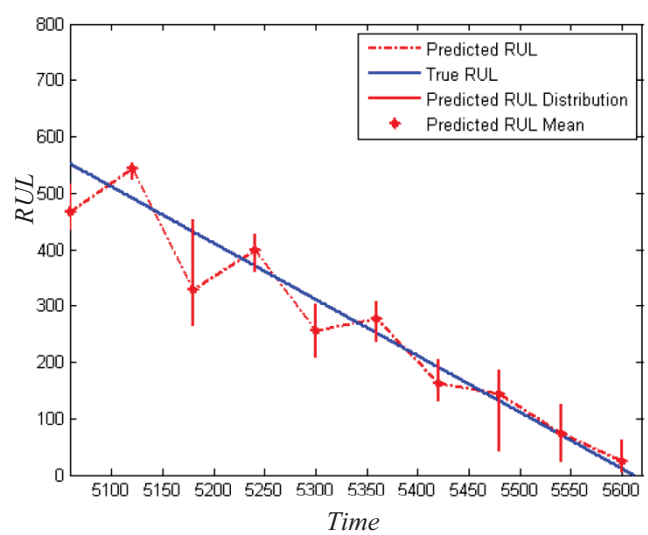

Figure 10. The predicted mean RUL and corresponding 90% confidence intervals using estimator updated with error measurement

From Figure 10, one can see that the RUL prediction made by the integrated approach using particle filter is very close to the true RUL when the RUL is less than 200 temporal steps.

4. CONCLUSIONS

In this paper, an integrated approach for gear health prognostics using particle filters was presented. The two fundamental challenges in applying particle filters to gear health prognostics has been addressed by the presented approach by integrating several new components into a particle filter: (1) data mining based techniques to effectively define the degradation state transition and measurement functions using an one-dimensional health index obtained by a whitening transform; (2) an unbiased *l*-step ahead RUL estimator updated with measurement errors. The feasibility of the presented prognostics method was validated using data from a spiral bevel gear case study. The data were collected from gear surface fatigue tests subject to tooth pitting failure modes. During the testing, both the vibration and ODM data were collected. The vibration data were pre-processed with TSA and then re-processed to compute the CIs. Six CIs were selected based on their trending correlation coefficients to compute the HI. An ARIMA model was fitted to experiment 6 data to define the gear degradation state transition function and a double exponential smoothing model was fitted to the HI and ODM data from experiment 6 to define the measurement function. The gear degradation state transition function and the measurement function defined by the data mining models from experiment 6 were used to predict the RUL of the gear using the HI obtained in experiment 5. The results have shown that the integrated approach using particle filter gave a good RUL prediction performance.

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