Background noise reduction using adaptive noise cancellation
determined by the cross-correlation

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Background noise due to flow in wind tunnels contaminates desired data by
decreasing the Signal-to-Noise Ratio. The use of Adaptive Noise Cancellation to remove
background noise at measurement microphones is compromised when the reference sensor
measures both background and desired noise. The technique proposed modifies the
classical processing configuration based on the cross-correlation between the reference and
primary microphone. Background noise attenuation is achieved using a cross-correlation
sample width that encompasses only the background noise and a matched delay for the
adaptive processing. A present limitation of the method is that a minimum time delay
between the background noise and desired signal must exist in order for the correlated
parts of the desired signal to be separated from the background noise in the cross-
correlation. A simulation yields primary signal recovery which can be predicted from the
coherence of the background noise between the channels. Results are compared with two
existing methods.

1 INTRODUCTION

Early efforts in the study of time domain Adaptive Noise Cancellation (ANC) were made in
the 1960s\textsuperscript{1,2} and the technique was formalized in 1975\textsuperscript{3}. The method used a minimum of two
channels: a reference channel intended to receive only undesired background noise and a primary
channel containing the signal plus background noise. The reference time-series is fed into a filter
which is adapted to produce a best estimate of the noise present in the primary time-series and
then subtracted from the primary signal leaving a “cleaned” result. Since its publication the

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technique has been widely employed primarily for applications of noise reduction for improved speech recognition.

Background noise in wind tunnels contaminates desired signal data by introducing another noise source into the measurement thereby decreasing the Signal-to-Noise Ratio (SNR). The use of Adaptive Noise Cancellation (ANC) to remove background noise at measurement microphones has a major limitation: a reference sensor intended to only acquire flow noise is usually too spatially separated from the primary microphones to maintain the high coherence necessary for noise attenuation. Further, a reference that maintains this coherence by being placed closer to the primary microphones will contain desired signal as well as flow noise, resulting in cancellation of the desired signal under the classical ANC configuration.

Modifications to the classical ANC implementation addressing this aforementioned constraint have been published. Liang and Malik\textsuperscript{4} used three sensors in a linear arrangement to reject speech arriving from a non-preferred direction. Feder et al.\textsuperscript{5} used a maximum likelihood problem to better estimate the parameters needed to cancel the noise and then solved for them using the iterative Estimate-Maximize technique. Van Compernolle and Van Gerven\textsuperscript{6} decorrelated the outputs of the classic ANC configuration to obtain further noise cancellation. Weinstein et al.\textsuperscript{7} achieved noise cancellation using two sensors that contained both noise and the desired signal by imposing the constraint that the two signal estimates are statistically uncorrelated. Nájar et al.\textsuperscript{8} pre-whitened the input signals before decorrelating the outputs of the classic ANC configuration.

The technique proposed here modifies the classical processing configuration based on the cross-correlation between the reference and primary microphone. A minimum time delay between these two signals must exist in order for the correlated parts of the desired signal to be separated from the background noise in the cross-correlation. Using a correlation sample width that only encompasses the background noise contribution between the two channels as an ANC finite impulse response filter length and an appropriate delay for the primary microphone, noise attenuation is achieved even though the reference signal contains both primary and background noise. Performance is linked to the coherence of the background noise that exists between the channels. The results of this processing are compared to spectral subtraction\textsuperscript{9} and a simplified version of a technique\textsuperscript{10} that produces the autospectrum calculated from the cross-correlation.

2 THEORY

2.1 Adaptive Noise Cancellation determined by the Cross-Correlation

Adaptive Noise Cancellation\textsuperscript{3} uses the Least Mean Squares algorithm to minimize the system output ($e_n$) by removing correlated noise between two channels ($input_{ref}$, $input_{primary}$) from the channel designated as the primary. The formulation of the method can be summarized as

$$y_n = \bar{W} \cdot input_{ref}$$  \hspace{2cm} (1)

$$e_n = input_{primary,n} - y_n$$  \hspace{2cm} (2)

$$\bar{W} = \bar{W} + 2\mu e_n input_{ref}$$  \hspace{2cm} (3)
where $\vec{W}$ is the filter weight vector that is adaptively updated, $y_n$ is the finite impulse response filter output which is created by convolving the weight vector with the reference input, and $\mu$ is the user-defined step size. The process minimizes $e_n$ by removing correlated signal(s) present between the primary and reference input. The SNR of $e_n$ (at a given frequency) is the inverse of the SNR of the reference input. A problem arises when the reference input has a SNR > 1, as the desired signal will then be attenuated.

If the desired signal and noise to be attenuated have correlations that are sufficiently separated in the time domain, a modification to the classical configuration given above can be made: by choosing a weight vector that encompasses only the background noise cross-correlation width (# of samples) and delaying the primary input appropriately, the correlated signal between the reference and primary input will only be due to the background noise.

The resulting attenuation from this modified processing can be calculated as follows. The primary input is given as channel number 1 and the reference 2. For a simplified scenario, only three signals would be present on the channels: the primary signal (correlated between channels), the background noise (correlated between channels), and other noise sources that are uncorrelated between channels. The autospectra and cross-spectrum, calculated at discrete frequencies $f$ (Hz), are

$$G_{11}(f) = G_{p1p1}(f) + G_{b1b1}(f) + G_{n1n1}(f) \quad G_{22}(f) = G_{p2p2}(f) + G_{b2b2}(f) + G_{n2n2}(f) \quad (4)$$

$$G_{12}(f) = G_{p1p2}(f) + G_{b1b2}(f) \quad (5)$$

The cross-spectrum does not contain the uncorrelated noise sources but does maintain the correlated primary signal and background noise. The resulting coherence between the channels is

$$C_{12}(f) = \frac{|G_{p1p2}(f)+G_{b1b2}(f)|^2}{(G_{p1p1}(f)+G_{b1b1}(f)+G_{n1n1}(f))(G_{p2p2}(f)+G_{b2b2}(f)+G_{n2n2}(f))} \quad (6)$$

However, if the method only attenuates background noise correlated between the channels the coherence becomes

$$C'_{12}(f) = \frac{|G_{b1b2}(f)|^2}{(G_{p1p1}(f)+G_{b1b1}(f)+G_{n1n1}(f))(G_{p2p2}(f)+G_{b2b2}(f)+G_{n2n2}(f))} \quad (7)$$

The result of this ANC processing determined by the cross-correlation between the channels can then be predicted, as attenuation of the primary input is linked to coherence $^{11,12}$ as

$$G_{11,\text{ANC det.by} \, c-c}(f) \, dB = G_{11}(f) \, dB + 10 \log_{10}(1 - C'_{12}(f)) \quad (8)$$

where $G_{11,\text{ANC det.by} \, c-c}$ is the expected autospectrum of channel 1 with the background noise removed. The attractiveness of this method compared to the following two is that that noise is removed in the time-domain resulting in a “cleaned” time-history.

### 2.2 Spectral Subtraction

The one-sided autospectrum of a signal is calculated as
\[ G_{xx}(f) = \frac{2}{Kw_s T} \sum_{k=1}^{K} X^*_k(f, T) X_k(f, T) \]  

where \( X(f) \) is the Fourier-transformed signal using \( T \) data block lengths, \( K \) is the number of block averages taken, \( w_s \) is a data-window weighting constant, and * denotes the complex conjugate. The spectral subtraction method relies on a separate acquisition of the background noise in the testing environment from that when the primary noise source under study is present. The background noise only autospectrum is subtracted (on a pressure-squared basis) from that of the combined autospectrum (primary signal plus background noise)

\[ G_{xx,\text{Spectral Subtraction}}(f) = G_{xx,\text{primary+background}}(f) - G_{xx,\text{background}}(f) \]  

The method relies on the assumption that the spectral character of the background noise does not vary between acquisitions. Note that this is a frequency domain technique and a time-history with the background noise removed cannot be recovered.

### 2.3 Autospectrum calculated from the Cross-Correlation

The estimated cross-correlation for discrete, real-valued signals is defined as

\[ R_{xy}(\tau) = \begin{cases} 
\sum_{n=0}^{N-\tau-1} x_{n+\tau} y_n & \tau \geq 0 \\
R_{xy}(-\tau) & \tau < 0
\end{cases} \]  

where \( n \) are the discrete samples, \( N \) the total number of samples, and \( \tau \) the lag (samples) at which the cross-correlation is computed. The technique proposed by Brooks et al. assumes the correlation of the background noise between two sensors to be invariant between a background noise only and combined acquisition. Thus, the cross-correlation resulting from the background noise only acquisition is subtracted from that due to the combined acquisition

\[ R_{xy,\text{Background Noise Removed}}(\tau) = R_{xy,\text{primary+background}}(\tau) - R_{xy,\text{background}}(\tau) \]  

The resulting cross-correlation should be free of correlations due to the background noise. If there is only one primary source being measured, only one peak due to this source should remain. Simplifying the technique from, a section of the cross-correlation centered on the primary signal peak is used to create an equivalent autospectrum

\[ G_{xx,\text{from c–c}}(f) = \frac{2}{Kw_s T} \sum_{f=-F_s/2}^{F_s/2} R_{xy,\text{Background Noise Removed}}(\tau) e^{-i \pi f \tau} \]  

As with spectral subtraction, this is a frequency domain technique and no cleaned time-history is recovered.

### 3 SIMULATION

In order to evaluate the proposed technique and compare it to existing methods a simplified scenario with conditions allowing for non-restrictive processing was created. A diagram to accompany it is given in Figure 1. This intends to depict an anechoic, open wind tunnel with two
microphones located out of the flow. The set up represents a typical wind tunnel aeroacoustic test
of an aircraft component in an airflow.

In accordance with section 2.1, three components were included on channels 1 and 2: background noise, the primary signal, and uncorrelated noise. All were created with a pseudo-
random number generator in order to provide broadband energy. The background noise was
filtered to simulate a slowly decaying broadband signal. The primary signal was filtered to have
a peak that is above the background noise level but which decays before and after that peak
frequency range. The uncorrelated noise on each channel is an unfiltered, low-level, broadband
signal. The “time-histories” of the primary signal and background noise were manipulated to
mimic a longer delay between channel 2 and 1 for the background noise relative to the primary
signal due to their “directions of arrival”. In order to create an uncorrelated yet statistically
equivalent “background only” acquisition, another signal was generated with the same power as
the background signal, filtered and delayed equally. Autospectra of the signals are given in Fig. 2
and the cross-correlation between the channels in Fig. 3. The sampling rate was 20 kHz, the
block size was 512 samples, the bandwidth was ~39.1 Hz, and 193 averages were used when
computing the Fast Fourier Transforms (FFT). The goal of the processing is to recover the
primary signal (red line, Fig. 2) on channel 1 by removing the background noise (blue line, Fig.
2).

It is seen in Fig. 3 that the correlation peaks for the background noise and primary signal
are separated substantially in samples to facilitate the requirement that the ANC filter length will
only encompass correlated background noise.

4 RESULTS

The simulated signals were processed using the techniques outlined in Section 2 with the
goal of removing the background noise from channel 1 using channel 2 as a reference signal
(Eqns. (1-3)). The ANC filter length was chosen to be 512 samples to match the resolution of the
FFT processing. A delay of 1024 samples was used for channel 1 (input_{primary,n}, Eqn. (2))
resulting in the maximum background noise cross-correlation being centered in the weight vector
(Fig. 4a). A small step size (\( \mu = 6e-6 \), Eqn. (3)) was chosen in order to avoid filter output
divergence. In order to reach an optimum value, the system (Eqn. (1-3)) was left to iterate at this
step size. A converged filter weight vector (\( \vec{W} \), Eqn. (3)) was gradually reached through iteration
(Fig. 4b).

Figure 5 displays the cross-correlations of the ANC results with channel 2 (compare to Fig.
3). The classical processing configuration of ANC has decorrelated its output with channel 2 at
lags where the background noise and primary signal appeared. Using ANC determined by the
cross-correlation produced a result which is uncorrelated with the background noise yet still
maintains its original correlation with the primary signal.

Figure 6 shows the results using the classical ANC configuration versus those obtained by
using a filter width and delay which allow only the background noise to be correlated between
the input signals (ANC det. by cross-correlation). It also shows the predicted results from Eqn.
(8). It is seen that in the frequency range where the primary signal’s power is greater than that of
the background noise, classical ANC attenuates the primary signal. This illustrates the hazard of
using the classical ANC configuration when the primary signal is present on the reference
channel in equal or greater magnitude than the background noise. The predicted results from
Eqn. (8) match the ANC determined by the cross-correlation results well in the frequency range
where the primary signal is strong. Some variation is seen at frequencies with negative SNR but
the spectral characteristics are still maintained. Note that full recovery of the target (primary signal + uncorrelated noise) on channel 1 was not possible, as predicted by Eqn. (8). This is due to the presence of uncorrelated primary signal between the inputs of the adaptive noise canceller. The coherence (Eqn. (7)) of the background noise between the inputs is reduced resulting in less attenuation than is needed to fully recover the target signal (Eqn. (8)).

Figure 7 shows the ANC determined by the cross-correlation results in comparison with spectral subtraction. In the frequency range where the SNR > 1, spectral subtraction recovers the primary signal more accurately than ANC. As the SNR becomes less than 1 the estimation provided by spectral subtraction becomes imprecise; the level of the background noise only acquisition is near equivalent to the combined acquisition of primary signal and background noise. The resulting pressure-squared value thus tends toward zero. Spectral subtraction is thus limited to cases where the SNR > 1, and provides only frequency-domain results. Note that the ANC result was obtained in the time-domain, and plotted in the frequency-domain for comparative purposes.

Figure 8 shows the 512 samples of the cross-correlation obtained using Eqns. (11-12). Figure 9 gives the autospectrum resulting from that cross-correlation compared to the ANC result. The autospectrum resulting from the cross-correlation of Fig. 8 is very similar to the spectral subtraction result (Fig. 7), however it exhibits less inaccuracy as the SNR falls below 1 than spectral subtraction. Both result in a more accurate recovery of the primary signal + uncorrelated noise than the ANC result where the SNR > 1. The autospectrum obtained from the cross-correlation is limited to cases where the SNR ~ 1 or greater. Similar to spectral subtraction, it provides only frequency-domain results.

4 DISCUSSION AND CONCLUSIONS

This paper detailed a modified application of the classical Adaptive Noise Cancellation configuration which addressed the situation where the primary signal is present in the reference channel. For background noise which is sufficiently separated from the primary signal in the cross-correlation between two channels, the ANC filter length can be set and input signal delayed such that the only correlated signal between the inputs becomes the background noise. Thus, only background noise is cancelled although both primary and reference inputs contain comparable levels of both primary signal and background noise.

A limitation of the processing is that the background noise coherence between the two inputs is reduced due to the presence of the primary signal on both. This inhibits noise attenuation and thus full primary signal recovery. An equation was given for estimation of the coherence of the background noise between the inputs, which then was used to predict the ANC output. Good matching between results and this prediction were seen.

A test case intended to mimic background noise in an open, anechoic wind tunnel with out-of-flow microphones was created. The relative delays of the background noise and primary signal were set to allow for sufficient separation of the two in the cross-correlation. It was shown that where the SNR > 1 the classical ANC configuration will attenuate the primary signal. Background noise removal through spectral subtraction provided more accurate recovery of the primary signal than the modified ANC approach where the SNR > 1. However, recovery of the primary signal using spectral subtraction became inaccurate as the SNR fell below 1. Results were also given for a technique that produces an autospectrum from a background-subtracted cross-correlation. This result had similar accuracy to spectral subtraction where the SNR > 1, and was more accurate as the SNR < 1, yet still contained inaccurate level variations.
The modified ANC technique provides a method of recovering a signal buried in noise with accuracy governed by the coherence of the background noise between the inputs. Where the SNR > 1 spectral subtraction and an autospectrum generated from the cross-correlation of the primary signal serve as more accurate level estimators. However, when the SNR falls below 1 the modified ANC method produces a more accurate spectral shape than either of the aforementioned methods. It also provides a means of time-domain noise attenuation.

It is important to note that the modified ANC method presented here relies on having a sharp autocorrelation function for either the desired signal or background noise so that they can be separated. Thus, it is suitable for broadband/broadband situations which have a sufficient number of samples separating the signals’ arrival such that their cross-correlation functions will not overlap. In the case of a broadband/tone mixture, the signal to be removed/extracted depends on the cross-correlation of the inputs. If the tone is above the “noise floor” of the cross-correlation it should be extracted avoiding the specific delay and cross-correlation width of the broadband signal. If the tone’s correlation is not above the “noise floor”, the specific delay and cross-correlation width of the background noise should be used to remove the background noise, leaving the tone intact.

5 REFERENCES


Fig. 1 – Diagram of simulated experimental setup.

Fig. 2 – Autospectra of simulated signals.
Fig. 3 – Cross-Correlation between channel 2 and 1.

Fig. 4 – a) Converged weight vector, b) Convergence of max tap value.

Fig. 5 – Cross-Correlations between channel 2 and ANC results.
Fig. 6 – Autospectra detailing ANC results and prediction from Eqn. (8).

Fig. 7 – Autospectra detailing ANC and spectral subtraction results.
Fig. 8 – Cross-Correlation used to determine autospectrum.

Fig. 9 – Autospectrum calculated from Fig. 7 compared with ANC results.