The Pursuit of K:
Reflections on the Current State-of-the-Art in Stress Intensity Factor Solutions for Practical Aerospace Applications

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Abstract Submitted for Presentation at ICAF 2013

The elastic stress intensity factor (SIF, commonly denoted as *K*) is the foundation of practical fracture mechanics (FM) analysis for aircraft structures. This single parameter describes the first-order effects of stress magnitude and distribution as well as the geometry of both structure/component and crack. Hence, the calculation of *K* is often the most significant step in fatigue analysis based on FM. This presentation will provide several reflections on the current state-of-the-art in SIF solution methods used for practical aerospace applications, including a brief historical perspective, descriptions of some recent and ongoing advances, and comments on some remaining challenges.

Newman and Raju made significant early contributions to practical structural analysis by developing closed-form SIF equations for surface and corner cracks in simplified geometries, often based on empirical fits of finite element (FE) solutions. Those solutions (and others like them) were sometimes revised as new analyses were conducted or limitations discovered. The foundational solutions have exhibited striking longevity, despite the relatively "coarse" FE models employed many decades ago. However, in recent years, the accumulation of different generations of solutions for the same nominal geometry has led to some confusion (which solution is correct?), and steady increases in computational capabilities have facilitated the discovery of inaccuracies in some (not all!) of the legacy solutions. Some examples of problems and solutions are presented and discussed, including the challenge of maintaining consistency with legacy design applications.

As computational power has increased, the prospect of calculating large numbers of SIF solutions for specific complex geometries with advanced numerical methods has grown more attractive. Fawaz and Andersson, for example, have been generating literally millions of new SIF solutions for different combinations of multiple cracks under simplified loading schemes using p-version FE methods. These data are invaluable, but questions remain about their practical use, because the tabular databases of key results needed to support practical life analysis can occupy gigabytes of storage for only a few classes of geometries. The prospect of using such advanced numerical methods to calculate in real time only those *K* solutions actually needed to support a specific crack growth analysis is also tempting, but the stark reality is that the computational cost is still so high that the approach is not practical except for specific, critical application problems. Some thoughts are offered about alternative paradigms.

Compounding approaches are some of the earliest building blocks of SIF development for more complex geometries. These approaches are especially attractive because of their very low computational cost and their conceptual robustness; they are, in some ways, an intriguing contrast and complement to the brute-force numerical methods. In recent years, researchers at NRC–Canada have published remarkable results showing how compounding approaches can be used to generate accurate solutions for very difficult problems. Examples are provided of some successes—and some limitations—using this approach.

These closed-form, tabulated numerical, and compounding approaches have typically been used for simple remote loading with simple load paths to the crack. However, many significant cracks occur in complex stress gradient fields. This is a job for weight function (WF) methods, where the arbitrary stress distribution on the crack plane in the corresponding uncracked body (typically determined using FE methods) is used to determine *K*. Several significant recent advances in WF methods and solutions are highlighted here. Fueled by advanced 3D numerical methods, many new solutions have been generated for classic geometries such as surface and corner cracks with wide ranges of geometrical validity. A new WF formulation has also been developed for part-through cracks considering the arbitrary stress gradients in <u>all</u> directions in the crack plane (so-called bivariant solutions). Basic WF methods have recently been combined with analytical expressions for crack plane stresses to develop a large family

of accurate SIF solutions for corner, surface, and through cracks at internal or external notches with very wide ranges of shapes, sizes, acuities, and offsets. Finally, WF solutions are much faster than FE or boundary element solutions, but can still be much slower than simple closed-form solutions, especially for bivariant solutions that can require 2D numerical integration. Novel pre-integration and dynamic tabular methods have been developed that substantially increase the speed of these advanced WF solutions.

The practical utility of advanced SIF methods, including both WF and direct numerical methods, is greatly enhanced if the FM life analysis can be directly and efficiently linked with digital models of the actual structure or component (e.g., FE models for stress analysis). Two recent advances of this type will be described. One approach directly interfaces the FM life analysis with the FE model of the uncracked component (including stress results). Through a powerful graphical user interface, simplified FM life models can be constructed (and visualized) directly on the component model, with the computer collecting the geometry and stress gradient information needed for the life calculation. An even more powerful paradigm uses expert logic to automatically build an optimum simple fracture model at any and every desired location in the component model, perform the life calculation, and even generate fatigue crack growth life contour maps, all with minimal user intervention. This paradigm has also been extended to the automatic calculation of fracture risk, considering uncertainty or variability in key input parameters such as initial crack size or location. Another new integrated approach links the engineering life analysis, the component model, and a 3D numerical fracture analysis built with the same component model to generate a table of SIF values at a specific location that can then be employed efficiently to perform the life calculation.

Some attention must be given to verification and validation (V&V) issues and challenges: how good are these SIF solutions, how good is good enough, and does anyone believe the life answer? It is important to think critically about the different sources of error or uncertainty and to perform V&V in a hierarchal, building-block manner. Some accuracy issues for SIF solutions, for example, may actually involve independent material behavior issues, such as constraint loss effects for crack fronts near component surfaces, and can be a source of confusion. Recommendations are proposed for improved V&V approaches.

This presentation will briefly but critically survey the range of issues and advances mentioned above, with a particular view towards assembling an integrated approach that combines different methods to create practical tools for real-world design and analysis problems. Examples will be selectively drawn from the recent literature, from recent enhancements in the NASGRO and DARWIN computer codes, and from previously unpublished research.