# COMPARISON OF AIRCRAFT MODELS AND INTEGRATION SCHEMES FOR INTERVAL MANAGEMENT IN THE TRACON

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## Abstract

Reusable models of common elements for communication, computation, decision and control in air traffic management are necessary in order to enable simulation, analysis and assurance of emergent properties, such as safety and stability, for a given operational concept. Uncertainties due to faults, such as dropped messages, along with non-linearities and sensor noise are an integral part of these models, and impact emergent system behavior. Flight control algorithms designed using a linearized version of the flight mechanics will exhibit error due to model uncertainty, and may not be stable outside a neighborhood of the given point of linearization. Moreover, the communication mechanism by which the sensed state of an aircraft is fed back to a flight control system (such as an ADS-B message) impacts the overall system behavior; both due to sensor noise as well as dropped messages (vacant samples). Additionally simulation of the flight controller system can exhibit further numerical instability, due to selection of the integration scheme and approximations made in the flight dynamics.

We examine the theoretical and numerical stability of a speed controller under the Euler and Runge-Kutta schemes of integration, for the 'Maintain' phase for a "Mid-Term (2035-2045) Interval Management (IM) Operational Concept" for descent and landing operations. We model uncertainties in communication due to missed ADS-B messages by vacant samples in the integration schemes, and compare the emergent behavior of the system, in terms of stability, via the boundedness of the final system state. Any bound on the errors incurred by these uncertainties will play an essential part in a composable assurance argument required for real-time, flight-deck guidance and control systems,. Thus, we believe that the creation of reusable models, which possess property guarantees, such as safety and stability, is an innovative and essential requirement to assessing the emergent properties of novel airspace concepts of operation.

## Introduction

Currently, traffic controllers retain overall separation responsibility in National Airspace System (NAS), and will continue to do so in the FAA-defined IM operational concept. In the longer term, Next Generation Air Traffic Operational Concepts may potentially encounter a shifting locus of control from today's centralized model, adding novel communication and control concerns. While these concepts will likely not be realized in the near future, we wish to build publically available, reusable models of common computation, decision and control elements that will encompass the behavior exhibited by these elements in near-term (2015-2025) ConOps, along with enabling the functionality to be exhibited by these components in the as-of-yet unrealized future systems.

We wish to consider a 'Mid-Term' (2035-2045) concept of operations for the 'Maintain' portion of IM, when aircraft will have both ADS-B-in and ADS-B-out to support trajectory management [1,2]) and situation awareness. FAA, MITRE and NASA Aeronautics research has been primarily focused on the 'achieve' phase of the current IM concept, as that is believed to be the phase where the greatest benefit is to be accrued, as is the only phase where trajectory based operations is regarded as being necessary. For the 'Mid-Term', ADS-B enabled operational concept, the fault assumption that ADS-B/GPS messages can be omissive (or even incorrect), thus requiring the IM velocity control algorithm to use stale or estimated data, becomes an important consideration even in the 'Maintain' phase. In the simplest approach, we wish to maintain fixed time spacing between the IM and target aircraft (referred to as 'station keeping'). This form of velocity controller can be modified to assist in any 'required time of arrival' (RTA) approach currently employed to sequence aircraft arriving at the terminal area.

It is impossible to eliminate uncertainties in physically-realized control systems. Uncertainty can be classified into two categories for these systems:

disturbance signals and dynamic perturbations. The former includes input and output disturbance (such as a gust on an aircraft), sensor noise (such as corrupt or missing messages) and actuator noise. The latter represents the discrepancy between the mathematical model and the actual dynamics of the system in operation. A mathematical model of any real system is always just an approximation of the true, physical reality of the system dynamics. Typical sources of the discrepancy include unmodeled (usually high-frequency) dynamics, neglected nonlinearities in the modeling, effects of deliberate reduced-order models, and system-parameter variations due to environmental changes and torn-and-worn factors. These modeling errors adversely affect the stability and performance of a flight control system.

Ultimately, our reusable models must be executable and analyzable, in order to enable simulation and facilitate the creation of assurance arguments for overall systems properties. These models should carry guarantees of their behavior, such as safety or stability proofs, which do not have to be re-derived in an overall system context, as long as all assumptions and fault hypotheses are met. For example, any operational concept enabled by ADS-B is enhanced by considering the notion of dropped or incorrect messages, as the correctness of any algorithms implemented relying upon the measurements contained in ADS-B messages (and the constant 1 Hz broadcast rate) can greatly impact emergent properties..

In this paper, we derive a theoretically and numerically stable IM velocity controller that is robust to the uncertainties presented due to model nonlinearities, integration scheme inaccuracies, and communication (sensor feedback) errors. In the next section, we provide a description of the Interval Management concept, and focus on stability concerns in the 'Maintain' phase. The third section is used to examine and bound uncertainties due to non-linearities arising in differing modeling formalisms. In the fourth section, we compare the numerical stability of various integration schemes used in the simulation of the aircraft trajectories. We then derive a robust velocity controller that is theoretically and numerically stable by design. The subsequent section discusses a low fidelity simulation environment used to simulate the aircraft, as well as outline the flight profiles flown. We then compare and contrast the fidelity of the simulations and the efficacy of the theoretically and numerically stable controller under both nominal operating conditions, as well as dropped communications. Finally, conclusions are drawn, and avenues for further inquiry are proposed.

## Near-Term Interval Management Concept s

An important construct in enabling new air traffic operations is ensuring that the system provides the agents with the flexibility necessary to robustly respond to disruptions and disturbances, while increasing overall system capacity[3]. As clearly defined in [4], the IM concept involves an air traffic controller issuing an IM clearance to a candidate aircraft. The onboard software of the IM cleared aircraft then provides a precise speed for the aircraft to maintain. The pilot is responsible for implementing and maintaining the precise speed profile provided by the onboard software. The controller, however, maintains responsibility for assuring separation between aircraft.

The main objective of the operational concept is for IM aircraft to achieve and maintain an assigned spacing goal relative to a target aircraft. The provision of precise guidance within the flight deck enables the flight crew to actively manage the spacing relative to the target aircraft, without further overt controller action, thereby reducing controller workload. The fundamental enabling concept behind IM is the provision of velocity commands derived by a flight deck velocity control algorithm to maintain a localized relative spacing in the face of operational uncertainties and environmental effects, such as varying aircraft flight modes, performance characteristics and winds.

The interval management concept of operations can be implemented during any phase of flight. In this paper, we examine the IM concept applied to the terminal area, more specifically, in the context of the optimal profile descent (OPD) maneuver [5]. An optimal profile descent acts to minimize the fuel consumption of the aircraft during the descent phase of flight, in the terminal area. The IM concept attempts to achieve the assigned spacing interval (defined as a time or distance) by a specified 'Achieve By Point'. Examples of such points are the

meter fix to the terminal airspace, the runway threshold, or any other waypoint.

### *Abbreviated Survey of Prior Research In Interval Management*

Interval Management in its various forms has been investigated by several organizations, such as MITRE [6-8], EuroControl [9-12], NASA [4, 13-14] and the FAA [15,16]. Evaluations performed by MITRE address both human-in-the-loop simulations as well as theoretical modeling approaches. A thorough definition of an interval management concept of operations is outlined by Levitt and Weitz [6,7].

Eurocontrol has worked on the 'CoSpace' concept, which studies terminal area procedures. Eurocontrol's sequencing and spacing concept addresses controller workload, as opposed to aircraft efficiency. Furthermore, algorithm(s) used in the CoSpace Concept are time-history based, and not based on 4-D trajectories, thus limiting the study to traffic arriving from a single direction. The CoSpace Concept includes both a ground based as well as airborne component. The ground based components provide sequencing and spacing functions, whereupon the cleared aircraft achieve the desired spacing through adherence to the speed commands generated by the flight deck speed controller. Mohleji and Wang perform an analysis on position and velocity errors introduced by ADS-B in the context of Airborne Spacing-Flight deck Interval Management (ASPA-FIM) [8]. The authors formulate a Gauss-Markov model for the GPS position errors in order to perform Monte-Carlo simulation to validate that the number of speed commands generated is acceptable to the flight crew. The simulation model is complex, as it encompasses models for flight dynamics, winds and position/velocity errors, as well as rudimentary communication failures. The formulation possesses a mechanism for modeling dropped ADS-B messages, along with other common modes of failure in communications, which are prevalent with ADS-B technology in the current airspace. While addressing the time correlation between position measurement errors, the models used do not account for message collisions, which may occur in a densely populated airspace, such as a terminal area.

Penhallegon et al. [17] perform a human-in-the-loop study of the IM concept during departure. Sixteen airline pilots participated in a variety of operational scenarios (seven nominal, two off-nominal) in the Hartsfield Jackson Atlanta International Airport (KATL) region, during departure from 10,000 ft to cruise altitudes, using either precise goals or open-with-capture (OWC) directives. The precise goals required that a specific value be used to achieve or maintain an altitude, while the OWC goals required that IM speeds be provided to achieve the assigned spacing goal at the achieve by point only if the IM aircraft was predicted to achieve a spacing interval less than the assigned spacing goal. Several key issues that were found during the study concerned the use of unrestricted climbs out of the terminal area. The ability to have accurate measurements at regular intervals (i.e., ADS-B) directly influences the number of velocity changes that must be made in order to adhere to the climb profile. Thus, any verification of an IM speed controller must encompass models of dynamic (aircraft and otherwise) as well as environmental uncertainty, and should be robust enough to capture relevant coupling effects in both operational procedures as well as flight dynamics. The issue of dropped or incorrect time, position, or velocity measurements must be considered carefully in assessing the correctness (or robustness) of any designed velocity controller.

A similar study was performed at NASA Langley [4], this time in reference to arrival operations, which considered the Interval Management concept with respect to spacing on parallel dependent runways. The scenario concerned the Dallas Fort-Worth Terminal Area (KDFW), where pilots attempted closely spaced parallel approaches under the Interval Management operational concept. Again, several standard assumptions were made during the experiment, including the lack of error in the ADS-B and CPDLC messages. Subsequent interval management research efforts maintain this assumption regarding communication models [31].

### *Stability of the Velocity Controller in 'Maintain' Phase of IM*

Velocity control algorithms for the IM concept of operations have been studied in depth. Abbot [28] has proposed a PD controller which does not require

any special ADS-B message format, compensates for dissimilar final approach speeds between aircraft pairs, and provides guidance for a stable final approach. This algorithm has been extensively tested in Monte Carlo simulation and has been evaluated in piloted simulation. Weitz and Hurtado [29] specify both a PD controller and a sliding mode controller (switched PD controller). Tracking an arbitrary path can lead to string instabilities, or the growth of spacing errors along the formation, which necessitated the application of a string-stable control law. A sliding-mode controller was then implemented, which uses information for the reference trajectory and the ideal trajectory to achieve the desired formation. This controller is not a time-based spacing controller, and is dependent on a reference trajectory as well as the relative distance to the target aircraft.

Ivanescu [10] uses a nonlinear dynamical model for the aircraft, but provides a "spacing director" which is a quantized (Discrete) PD controller. Similarly, Lambregts [27] uses Energy Methods (Hamiltonian) as a dynamic formulation for the equations of motion, but does not consider computational stability (the controller derived cannot be stably integrated under explicit integration methods for large step size or large gains or large disturbances).

We wish to investigate whether, in the presence of dropped messages, an aircraft that can fly a trajectory based approach to a required degree of precision, and thus possibly continue the IM function in the 'maintain' phase of the concept. Maintaining a fixed interval is called station-keeping, and is fundamentally different than the 'achieve' part of IM.

Crucial to this speed control concept is the feedback control mechanism by which the onboard speed controller compares the measured position and velocity of the IM aircraft to the desired 4-D trajectory values. Any velocity control algorithm will be extremely sensitive to failures in measurement technology, be it dropped or incorrect ADS-B messages, or even Inertial Measurement Unit (IMU) inaccuracies that directly impact the Inertial Navigation System (INS). Current efforts [28,29] utilize varying forms of proportional derivative control, which are unstable under high gains. The sliding mode controller proposed in [29] is not easily amenable to standard Lyapunov stability analysis; assuring its stability may prove to be intractable.

We use the technique of *feedback* linearization, in order to design a theoretically stable robust velocity controller model that employs the pitch angle as the control input. This allows us to derive an outer-loop PID controller which is guaranteed to be stable as long as there are no right half plane poles, and no non-minimum phase zeros. We can then bound the error incurred during numerical simulation, due to the dynamic model and integration scheme employed. Furthermore, the resilience of the controller to a limited number of faulty messages (i.e., dropped messages) can be determined, by bounding the state error of the aircraft's trajectory at the final waypoint. We recognize that current flight operations are adequately ensured by today's flight control systems. However, we anticipate that 'Mid-Term' operational concepts enabled by the use of ADS-B In and Out, will place additional requirements in terms of emergent behaviors of these controllers, in regard to overall system stability and safety. We believe that provably demonstrating the stability of such controllers in a compositional fashion will be one of the necessary steps to ensure safe emergent behavior of multiple aircraft participating concurrently in 'Mid-Term' IM operations.

The baseline profile used in the following simulation work in this paper details the aircraft's descending profile in the terminal area is as follows, and is based on [14]. The aircraft transitions from its cruising altitude at the Top of Descent (TOD) to an idle-thrust constant Mach descent profile. At the requisite altitude it captures the constant thrust calibrated airspeed (CAS) descent profile as specified in [14], until the aircraft is approximately at 12,000-10,000 ft MSL, and where it then undergoes a powered deceleration maneuver (some combination of drag devices and throttle movement) in order to shallow out the flight path angle while meeting the required time of arrival at the desired waypoint for final approach [14].

We will now derive a model for a theoretically and numerically stable velocity controller that is robust to the uncertainties presented due to model nonlinearities, integration scheme inaccuracies, and sensor/actuator (communications feedback) errors. With the possibility of stale or inaccurate

measurements being used by the velocity controller, the trajectory modeling techniques employed must be resilient to uncertainties and disturbances present in the environment. The correctness of the velocity control algorithm will be sensitive to the modeling dynamics used to denote the aircraft (plant), the integration scheme used to propagate time and measurements forward, and the controller dynamics (i.e., number of differentiator and/or integrators used).

## Stability Issues and Non-linearity in Aircraft Trajectory Models

Trajectory modeling is a rich field [6, 8, 29, 30]. Using the informal notion of "best equipped, best served", aircraft which can achieve a higher conformance to the "required performance" of trajectory prediction, negotiation, and guidance in a 4-D trajectory-based operations environment, may be able to execute procedures such as IM. We briefly enumerate several simple models, kinematic and dynamic, of the aircraft trajectory, ***provide all the constants necessary for the replication of the simulations***, and discuss the impact of uncertainty due to non-linearity on the stability of the system. Additionally, the utility of the Hamiltonian representation over the Newtonian, and its effect on the numerical stability and precision of potential integration schemes is explained, especially in relation to higher fidelity models.

For the dynamic models, we are able to mitigate the error incurred due to non-linearity. We then simulate the descent phase of flight to examine theoretical and numerical stability of a controller designed through linearized feedback techniques.

### *Bounding Non-Linearities: Kinematic Model*

The current version of the modular environment used in this research for simulation purposes, employs a kinematic model to project the aircraft trajectory between the flight plan's waypoints. A simple averaging function is used to project the aircraft trajectory forward between waypoints as follows:

$$\vec{a}(t) = \frac{\vec{v}(t_{i+1}) - \vec{v}(t_i)}{t_{i+1} - t_i}, t \in [t_i, t_{i+1})$$

where the velocities and positions are specified at the successive $i^{th}$ and $(i+1)^{th}$ waypoints. The distance and velocity for values between waypoints $[t_i, t_{i+1})$ are then found using the corresponding kinematic equations:

$$\vec{v}(t) = \vec{v}(t_i) + \vec{a}(t) * (t - t_i)$$

$$\vec{d}(t) = \vec{d}(t_i) + \frac{1}{2}\vec{a}(t)(t - t_i)^2$$

Note that this is a piecewise linear function for the velocity. Any bound on the error due to non-linearity in the flight dynamics is directly dependent on the point of linearization (i.e., mode of flight and system state around which it is linearized). The effect of external forces and moments is not accounted for in this representation.

### *Bounding Non-Linearities: Dynamic Model*

Here, we summarize the body fixed coordinate equations of motion, found in [18], and define all the necessary constants employed in the model, in order to reproduce the simulation results (whose values are given in Tables 1 and 2). We have the following equation of motion:

$$T - D - mg\sin(\gamma) + \frac{dW}{dt} = m\dot{V}_t$$

where $V_t$ is the true airspeed, T is the thrust, D is the drag, m is the aircraft mass, g is gravitational acceleration, and W is the wind parallel to the path [18]. We approximate the drag as:

$$D = \frac{1}{2}\rho {V_t}^2 A_s C_D$$

$$D = \frac{1}{2}\rho {V_t}^2 A_s \left(C_{D,0} + \frac{C_L^2}{\pi e AR}\right)$$

where ρ is the air density, $A_s$ is the gross wing area, and $C_D$ is the non-dimensionalized drag co-efficient. $C_D$ is then further expanded, where e is the wing span efficiency (approximately 0.85-0.95), AR is the aspect ratio (ratio of the square of the wingspan to its planform area) and $C_{D,0}$ is the drag at zero lift conditions. Recall that at zero power conditions, $C_{D,0} \approx \frac{1}{3}C_{D,i}$ [19].

This allows us to express the governing nonlinear differential equation of motion as:

$$\dot{V}_t = \frac{1/2\,\rho V_t^2 K_D}{m} - g\sin(\gamma(x))$$

under zero wind conditions.

***Employing feedback linearization using the input variable $\gamma$ allows us to strictly bound the error due to non-linearities in the system dynamics.*** If there is no model uncertainty in the non-linear dynamics, and the relative degree of the output is the same as the degree of the state equation (with respect to the input), then the approximation is exact. However, the measured state providing feedback to the system, given by the ADS-B message detailing its position, means that the Newtonian dynamics result in a second order non-linear differential equation, which poses problems both for controller design and integration scheme stability.

Use of the Hamiltonian formulation is preferable, instead of arriving at a second order equation of motion in an n-dimensional coordinate space, the use of Hamilton's Equation yields a set of first order differential equations on a 2n-dimensional coordinate space. This is exceedingly useful, as most classical integration methods, such as the Runge-Kutta and Euler methods, require the system to be expressed as a set of first order differential equations [21].

By choosing the principle coordinate $q$ to be along the path direction, the momentum along this path is expressed as $p = mV_t$. Assuming the aircraft mass remains constant, we obtain the pair of first order differential equations:

$$\dot{p} = -mg\sin\gamma - \frac{\kappa p^2}{2m}$$

$$\frac{dq}{dt} = \frac{p}{m}$$

where $\kappa = 4\rho A_s C_L^2/3m\pi eAR$. ***Thus we have two first order differential equations, which can be used under feedback linearization techniques to design a theoretically stable controller for the non-linear system.*** This alternate derivation to Newtonian dynamics, which produces first order differential equations, can be used directly in the two main integration schemes such that stable (fully reversible) *implicit integration* can be performed.

## Integration Schemes

For the simulation of the 4-D trajectories, we consider the two most common schemes: the Euler Method of integration and the Runga-Kutta integration scheme. Further details of these approaches can be found in [20].

The stability of an integration method refers to a methods' performance against the solution to the test system $\dot{x} = Ax,\ (A - \lambda I) = 0$, where λ is a complex number (an eigenvalue of the square matrix A, I is the identity matrix) whose real part is negative. The solution to this system for values complex values of λ whose real part is nearly zero, is dependent on integration step size and truncation/rounding error. A method is zero stable if all of the eigenvalues are in the left half plane. We wish to ensure that the speed controller designed is both theoretically and numerically stable with respect to the assumptions required in the derivation of the corresponding equations of motion used.

The step size used in Euler integration is limited by stability. In general, explicit time marching integration methods are not suitable systems where computation with large steps may be necessary when the solution changes slowly (i.e., when the accuracy does not require small steps). Instead we utilize the backward (implicit) Euler method in our implementation [20], in conjunction with the Hamiltonian formulation.

Euler's implicit method yields fairly accurate results for reasonably small step sizes Δt. The local truncation error is bounded by:

$$\varepsilon_L(t_n) = \frac{1}{2}\ddot{x}(t_n)\Delta t + O(\Delta t^3)$$

and global truncation error bounded by:

$$\varepsilon_L(t) = \frac{\Delta t M}{2L}\left(e^{L(t-t_0)} - 1\right)$$

where M is the upper bound on $\ddot{x}$ for the given interval ($t$-$t_0$) and F is Lipschitz, continuous in its second argument, with Lipschitz constant L. ***Thus, we achieve a global error bound proportional only to the maximum rate of deceleration in the fixed time horizon ($t$-$t_0$), which is the length of the simulation.*** Thus, for a fixed maximum deceleration, the global truncation error is bounded from above by a constant, fixed by the maximum deceleration rate.

For most problems, the fourth-order Runge-Kutta method gives the highest accuracy-effort ratio [20]. We simultaneously iterate the pair of differential equations derived from the Hamiltonian, by using the fourth-order Runge-Kutta algorithm

outlined in [20]. Thus, the local truncation error (the error induced for each successive stage of the iterated algorithm) will behave like $\varepsilon_L = C\Delta t^5$, where $C$ is a number independent of $\Delta t$, but dependent on $t_0$ and the fourth derivative of the exact solution at $t_0$ (the constant factor in the error term corresponding to truncating the Taylor series for $q(t_0 + h)$ about $t_0$ at which has order $\Delta t^4$). Unfortunately, explicit Runge-Kutta methods, especially when used in the solution of partial differential equations, are not globally stable (non-reversible). Implicit fourth order Runge-Kutta methods are computationally expensive, and are not often suitable for real time applications. Thus, we wish to benchmark the accuracy of the numerically stable implicit Euler method, with that of the locally stable (but more accurate) Runge-Kutta method.

## Velocity Controller

We wish to consider the global asymptotic stability of a controller along with its corresponding numerical stability, with respect to disturbances (i.e., dropped messages) in tandem. It is non-trivial to prove the stability of a nonlinear system under PD control; ***we wish to employ feedback linearization in order to assure both numeric and theoretical stability by design***.

As a quick aside, we mention that there are several types of velocity associated with aircraft in flight. The true airspeed (TAS) of the aircraft is the speed of the aircraft relative to the atmosphere. In the absence of wind, the true airspeed is equal to the groundspeed. The calibrated airspeed (CAS) is the indicated airspeed of the aircraft corrected for instrumentation error, at standard temperature and pressure. The CAS and TAS are related through an implicit function that must be iteratively solved. The Mach number of the aircraft, usually defined while the aircraft is at high speeds and altitude, is given by the ratio of the true speed of the aircraft to the speed of sound at that altitude. We will deal predominantly with the TAS and the CAS in the remaining sections of this paper.

Recall that the dynamics for the aircraft in descent conditions is represented by a non-linear differential equation. We wish to design a controller that uses the flight path angle as the control variable, in order to enable feedback linearization, thereby facilitating the construction of a stable-by-design controller for the non-linear system. We note that it is equally valid to employ the throttle (or thrust) as the control variable, but do not consider this avenue.

The pilot controls the flight path angle by adjusting the pitch angle ($\alpha$), through deflection of the elevator ($\delta_e$). We acknowledge that this will change the vertical path, possibly adding difficulty in making the final altitude constraint.

### *Feedback Linearization*

The central idea in feedback linearization is to algebraically transform nonlinear systems dynamics into (fully or partly) linear ones, so that linear control techniques can be applied [32]. This differs entirely from conventional (Jacobian) linearization, because feedback linearization is achieved by exact state transformation and feedback, rather than by linear approximations of the dynamics. The basic idea of simplifying the form of a system by choosing a different state representation is not completely unfamiliar; rather it is similar to the choice of reference frames or coordinate systems in mechanics. The Hamiltonian formulation, with its symmetry in state variables, lends itself particularly to this approach.

If we regard the position of the aircraft (given by an ADS-B message) as the measured output of the system $y = \hat{q}$, feedback linearization requires that we differentiate the output until the physical input $\gamma$ appears in the $r^{th}$ derivative of y. Thus, in our case, r=2. Then $\gamma$ is chosen to yield a transfer function from the "synthetic input", v(t), to the output y which is given by:

$$\frac{Y(s)}{V(s)} = \frac{1}{s^r} = \frac{1}{s^2}$$

whereby if r is the same as the order of the system, as in our case, input/output feedback linearization is equivalent to input/state linearization, and we do not have to worry about internal dynamics.

In state space formulation, we have:

$$\dot{x} = f(x) + g(x)\sin\gamma$$

$$y = h(x)$$

where:

$$x = \begin{bmatrix} p \\ q \end{bmatrix}$$

$$f(x) = \begin{bmatrix} \frac{\rho A_s \dot{q}^2}{2m^2}\left(\frac{4C_L^2}{3\pi eAR}\right) \\ \frac{p}{m} \end{bmatrix}$$

$$g(x) = \begin{bmatrix} -g \\ 0 \end{bmatrix}$$

$$h(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} x$$

.

Here, $\gamma$ and y are scalars. Using the standard notation from operator theory [32], we have:

$$\dot{y} = \frac{\partial h}{\partial x}\dot{x}L_f^h(h) + L_g(h)\sin\gamma = L_f^1(h)$$

but:

$$L_g(h) = 0$$

so:

$$\dot{y} = L_f^1(h)$$

thus we must take:

$$\ddot{y} = L_f^2(h) + L_g\left(L_f^1(h)\right)\sin\gamma$$

Now, set:

$$\sigma(x) = L_f^2(h)$$

$$\varsigma(x) = L_g\left(L_f^1(h)\right)$$

then:

$$v(x) = \sigma(x) + \varsigma(s)\sin\gamma$$

where v(x) is the synthetic input (or synthetic control).

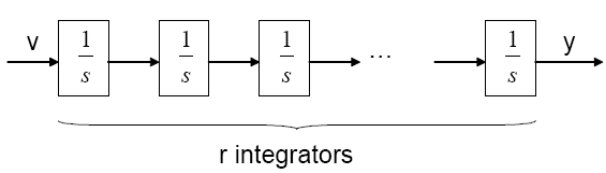


**Figure 1: Synthetic Input For Feedback Linearization**

Thus, in our case we have a double integrator linear system. We can now design a stable controller for this system using any linear controller design method. We chose, for robustness:

$$v(x) = -\sum_{k=0}^{2} c_k L_f^k(h) + K_C\left[\left(y_{wp} - y\right) + \frac{1}{\tau}\int_0^{\tau}\left(y_{wp} - y\right)d\tau\right]$$

Here the first term in the expression is the standard feedback linearization term (with $c_k$ being the coefficients of the second order differential equation expanded around the waypoint $y_{wp}$), and the second term is tuned online for robustness, as is performed in the next subsection. That controller is then implemented and obtained through:

$$\sin\gamma = \frac{1}{\varsigma(x)}\left[-\sigma(x) + v(x)\right]$$

where the only error encountered will result from the Taylor series approximation of sin($\gamma$), which can be made precise to within the desired floating point precision. Thus, the controller is stable by construction.

### *Outer Loop Control: Proportional Derivative Integral (PID) Control*

Proportional-Integral-Derivative control is the most common form of control applied in industry today [19,21]. A PID controller takes the form:

$$u(t) = K_p e(t) + \int_0^1 K_i e(t)dt + K_d\frac{de(t)}{dt}$$

where e(t) is the error, and the gains must be tuned concurrently. The proportional component combats present error. By raising the proportional gain $K_p$ we attain a faster response to changes in plant dynamics and a lower e(t); however, increasing the gain too much leads to overshoot and instability. The derivative component $K_d$ combats future error, by counteracting the overshoot. The integral component combats past (cumulative) error, and eliminates the steady state error by modulating the plant input so that the time-averaged error is zero.

For our system, we use the classical Ziegler-Nichols Tuning Method, and take $K_p = 1/2\,K_c$, $K_I = 2K_p/T_c$, and $K_D = T_c K_p/8$, where $K_C$ is the critical gain at which the plant under pure proportional control begins to oscillate, and $T_C$ is the oscillation period at that gain. For our given dynamics, we take $K_P = 2.5, T_c = 4$. This allows us to maintain the desired flight path angle throughout the descent to ensure a constant Mach and constant CAS under idle-thrust. For the outlined PID controller and the Runga-Kutta integration scheme with step-size of 1 second and 60 seconds, the system is stable, as the integration scheme is zero stable.

### *Linear Quadratic Gaussian (LQG) Control*

An LQG controller is the combination of an optimal estimator (Kalman Filter) and an optimal regulator (linear quadratic), which, in special cases, can be designed independently using the separation principle [22]. The LQG controller is a unique controller and constitutes a linear dynamic feedback law that is computable in real time. It is also the fundamental solution to the optimal perturbation control problem of a corresponding non-linear system.

While our PID velocity controller is sufficiently robust to maintain our desired Interval Tolerance throughout the constant thrust portion of the descent, we have neglected in this preliminary study, the vector field which describes the wind's impact on the flight path (both in the along path and cross-track directions). If the wind is accurately modeled as an additive Gaussian white noise distribution, and we model the non-linear portion of the dynamics as a linear system with bounded uncertainties, and formulate the control variable as being subject to quadratic cost (i.e., limit the maximum energy of the control signal), we would then choose to implement a LQG controller

## Simulation Results

We will now consider two aircraft flying Optimal Profile Descents (see Figure 2). The IM aircraft is assigned a target aircraft to follow from the Top of Descent (TOD) point. The aircraft assigned the same waypoint fixes (which is not a requirement for IM, but made processing easier), and are spaced by an Interval Tolerance (IT) $\tau$, which is the time based equivalent of a fixed distance separation. The aircraft transition from a constant cruise, at a velocity approximately of 340 knots (0.87M) at FL390 to a constant Mach descent. At approximately FL250, the aircraft then captures a constant CAS descent, to finish at approximately 12,000 ft MSL with a speed of 250 knots. The aircraft then performs a (possibly powered) maneuver to decelerate in order to attain the desired position, altitude and CAS at the metered fix. We assume that the target aircraft passes through the TOD point at the desired Mach with at least a 3 nmi equivalent time-based separation from the IM aircraft, at which point the IM velocity controller then takes over and provides velocity/angle of attack input guidance to the IM aircraft to maintain this minimum separation up until the deceleration phase of the profile (at approximately 12,000 ft MSL). This differs from convention, as currently IM software is slated to provide speed inputs, and that the aircraft's current FMS system will then establish the pitch.

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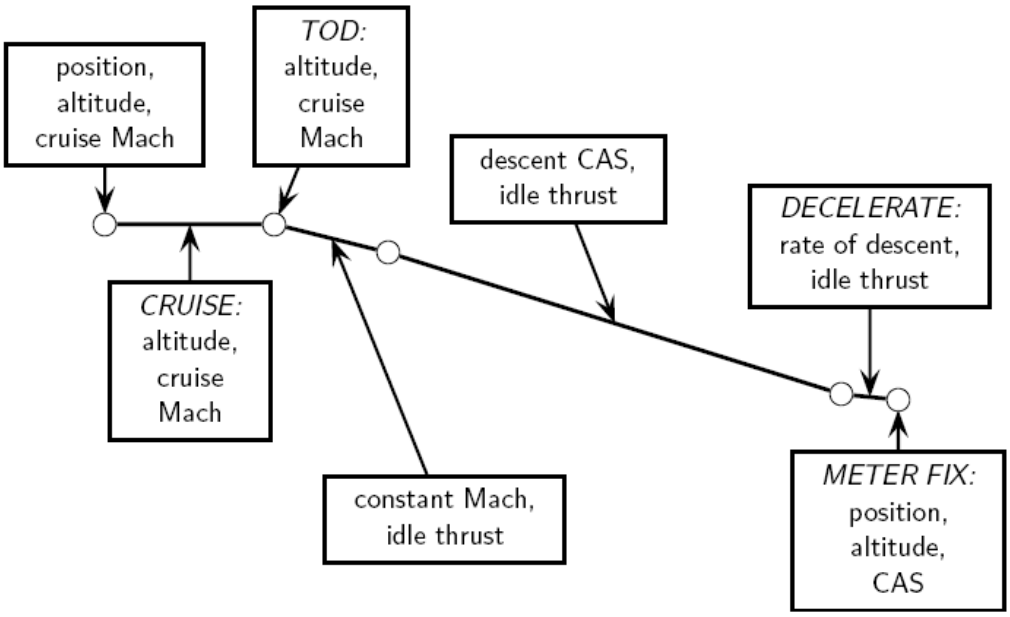


**Figure 2: Descent Profile of Aircraft for OPD**

### *Simulation Parameters*

We simulate the IM operated aircraft with the feedback linearized IM velocity controller for the descent scenario described above. We exit the cruise phase at approximately FL360 with a speed of 340 kts at a distance of approximately 115 nmi from the airport runway. We wish to arrive at the metered fix, approximately 30 nmi from the runway at a speed of 180 kts at 10000 ft MSL. We wish to terminate the unpowered portion of the descent at a distance of 35 nmi from the runway, at a speed of 240 kts and at 12,000 ft MSL. We use the following general parameters for calculating the relevant altitude dependent quantities necessary for our governing equations of motion.

| | |
|---|---|
| Ideal Gas Constant (J/(mol*K) | 8.31447 |
| Lapse Rate (K/m) | 0.0065 |
| Specific Mass of Air (kg/mol) | 0.0289644 |

**Table 1: Atmospheric Parameters [23-24]**

Furthermore, we use the Naval Atmospheric Tables in order to calculate the air temperature, pressure, density, dynamic viscosity based on the atmospheric altitude, as well as the gravitational constant g [23-24].

| | |
|---|---|
| Landing Mass of 777 (kg) | 223168 |
| Wingspan (m) | 60.9 |
| Chord (m) | 7.02463054 |
| Aspect Ratio (m) | 7.4 |
| Wing Area (m^2) | 427.8 |
| Max Thrust (kN) | 436 |

**Table 2: Boeing 77-300 Parameters [25-26]**

We use the performance characteristics of the Boeing 777-300 for both our IM and target aircraft, as well as for the lift coefficient $c_l$, which is a function of $\alpha$ [26].

### *Simulation Results*

We simulate the 4-D trajectory in the low fidelity simulation environment using the feedback linearized velocity controller in order to maintain a separation of 3 nmi between the IM and target aircraft. We assume that the target aircraft is conforming to its flight plan, and adjust the flight path angle of the IM aircraft to maintain the required true airspeed and position in order to maintain the IM tolerance

between aircraft. This is similar to a concept being used by Boeing-Madrid, referred to as 'speed on pitch'. We forward propagate the aircraft equations of motion via both the Euler and Runga-Kutta integration schemes, under two different timesteps: 1 second and 60 seconds. The concept of dropped messages was simulated by the lack of feedback presented through the measurement y. Thus, if a message is dropped, the controller propagates the control law forward based on its own internal dynamics, without the aid of a rectifying measurement. The choice of the secondary time step of 60 seconds allowed us to simulate a dropped message once every 60 seconds, by starting the following 60 second time step at an offset set of values for the initial integration conditions for the controller. The results are summarized below.

For the described PID velocity controller, we attempt to maintain the desired trajectory under constant thrust until 12,000 ft MSL.

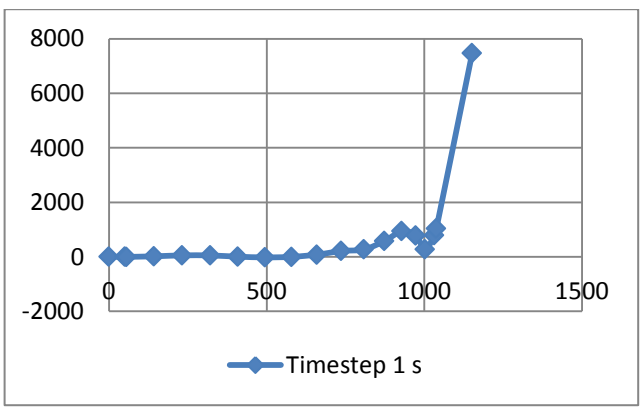


**Figure 3: Difference between Projected Positions (in m) at Waypoints under PID Control during IM OPD maneuver, over time (in seconds)**

The difference in projected position is the total error ($\|q(t_{wp}) - q_{wp}(t_{wp})\|$), between the actual position and desired position at the time the aircraft was to have achieved the waypoint. Note that the final point at 10,000 ft MSL is sufficiently disparate (Figure 3) due to the addition of thrust below 12,000 ft MSL, in order to achieve the required time of arrival with a 3 degree descent profile at 10,000 ft MSL (the standard descent angle). This can be seen at the point at approximately 1000 seconds, which possesses no positional error, in Figure 3. This addition of thrust is not modeled by our controller, which leads to the error seen after approximately 1000 seconds. If we allow the aircraft to descend with flight path angles in excess of 4 degrees at 10,000 ft MSL, we do not have this problem, as thrust need not be applied, and the RTA will be met automatically. However, we are aware that this steep of a descent profile is not generally practiced, and thus we attempted to shallow the profile at the end of the flight, and still meet the RTA.

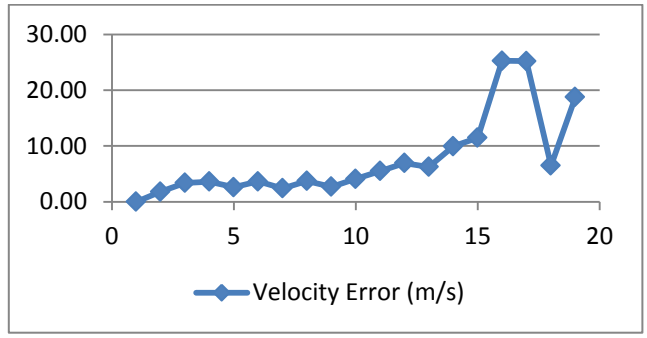


**Figure 4: Difference between Projected Velocity (in m/S) at Waypoints under PID Controller during IM OPD maneuver over time (in min)**

Prior to the addition of thrust, the target velocity of 250 kts at 35 nmi distance from the airport 12,000 ft MSL for the IM aircraft was achieved with a 5 second difference between the estimated time of arrival and actual time of arrival. That is, the aircraft arrived at the desired position with the desired velocity (to within a 2.5 kts tolerance, 5 seconds late). This is seen in Figure 4, which is a graph of the velocity error versus time in flight (given in minutes).

For the case of one-second timesteps, the RK and Euler integration schemes provide almost the exact same 4-D trajectories, with a negligible difference of approximately 10 cm at the metered fix. For the 60 second timestep, the difference in the end position of the two trajectories differs by approximately half a meter (Figure 5). Thus, a sufficiently small step size will result in approximately equivalent results using the RK and Euler methods, even with the non-linear dynamics. Hence, the controller is able to achieve a positional accuracy to within approximately half a meter, and a velocity error of about 2.5 kts, to within 5 seconds of the required time of arrival.

Simulating a dropped message once every 60 seconds during the constant thrust portion of the simulation (i.e., 7 dropped messages) resulted in a larger deviation in position. For this simulation we used the

numerically stable (reversible) Euler method of integration, coupled with the theoretically stable feedback linearized controller, and substituted the feedback measurement with a vacant sample (the estimated position given the controller's internalized dynamics for the aircraft plant). Before the addition of thrust, the controller maintains a descent profile that achieves the desired RTA to within a velocity error of about 2.5 kts at the final fix. The lack of significant increase in velocity error is due to the fact that it is a second order effect. As no two messages are dropped in succession, the inertia inherent in the plant dynamics acts to damp the controller response, thereby mitigating the error. As the messages are resumed, the positional error is driven to the 0.5 m system steady state error. The worst case scenario for the error, under this fault model, occurs when the message is dropped about 4 seconds prior to attaining a waypoint. This results in a positional error of approximately 600 m at the waypoint (Figure 5). This is reflected in the tuning parameters of the controller, as this is approximately the oscillatory period of the critical gain. Thus, the system dynamics do not have sufficient time to damp out any overshoot engendered by the vacant sample. Thus, for a sufficiently small step size (i.e., one second), the PID controller is numerically stable and maintains the IM tolerance to within a 600 m range of error in the presence of a dropped message.

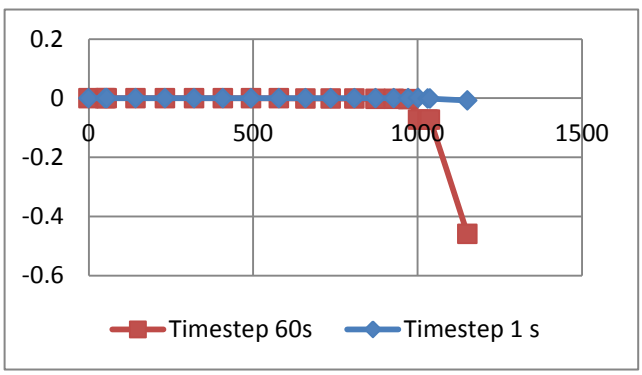


**Figure 5: Difference between Positions (in m) at Waypoints between Controller Performance for Nominal (1 second time step) and Dropped Messages (60 second time step) (in s) over IM OPD maneuver**

## Conclusions

We have designed a reusable model of a theoretically and numerically stable velocity controller for a 'Mid-Term' IM operational concept. The model addresses communication faults and failures, as well as computational error due to integration schemes and non-linearities arising from modeling uncertainties. This controller model possesses stability guarantees, and can be employed in an assessment of the emergent behavior of other current and future ATM operational concepts (i.e., RTA, etc.) which adhere to the enumerated fault hypotheses and environmental assumptions.

The model is executable as well as analyzable. Given the publically accessible parameters outlined in the paper, reproducible simulations of aircraft trajectories under IM can be generated. Throughout the simulation, the controller achieved an upper error bound of 600 m and 2.5 kts of a required fixed point for a 60 second vacant sample period for the descent profile given. The integral component of the controller efficiently reduced cumulative error over the vacant sample period of 60 second, and was tuned to eliminate the steady state error.

As future work, we wish to incorporate disturbances such as variable winds, which can be modeled as additive white Gaussian noise, as well as associated quadratic cost functions to optimize trajectories. Thus, we wish derive a reusable LQG controller model to meet the requirements specifications outlined for the 'Maintain' phase of a 'Mid-Term' IM Operational Concept, and compare its performance to the stable linearized feedback controller. Furthermore, we wish to investigate different communication paradigms which may be employed in future (2045-onwards) ATM Operational Concepts, such as Controller Pilot Data Link Communication (CPDLC), as well as fault models, and analyze their impact on emergent system behavior. We also wish to study if and how we can achieve required levels of precision for the minimum energy required, as well as stability guarantees, under differing communication paradigms.

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