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Abstract

During mass properties testing on the X-48B Blended Wing Body aircraft (The Boeing Company, Chicago, Illinois) at the National Aeronautics and Space Administration Dryden Flight Research Center, Edwards, California, large inertia measurement errors were observed in results from compound pendulum swings when compared to analytical models. By comparing periods of oscillations as measured from an average over the test period versus the period of each oscillation, it was noticed that the frequency of oscillation was shifting significantly throughout the test. This phenomenon was only noticed during compound pendulum swings, and not during bifilar pendulum swings. The frequency shift was only visible upon extensive data analysis of the frequency for each oscillation, and did not appear in averaged frequency data over the test period. Multiple test articles, test techniques, and hardware setups were used in attempts to eliminate or identify the cause of the frequency shift. Plotting the frequency of oscillation revealed a region of minimal shift that corresponded to a larger amplitude range. This region of minimal shift provided the most accurate results compared to a known test article; however, the amplitudes that produce accurate inertia measurements are amplitudes larger than those generally accepted in mass properties testing. This paper examines two case studies of the frequency shift, using mass properties testing performed on a dummy test article, and on the X-48B Blended Wing Body aircraft.

Nomenclature

CG	center of gravity
IMU	inertial measurement unit
I_{xx}	roll inertia, slug-ft^2
I_{yy}	pitch inertia, slug-ft^2
I_{zz}	yaw inertia, slug-ft^2
l	distance from pivot point to vertical center of gravity of test article, ft
L	length
MOI	moment of inertia
T	period of oscillation, s
R	reactions at suspension, lbs_f
W	weight of test article, lbs_f
x	longitudinal distance from a reference point, in.
z	vertical distance from a reference point, in.
θ	angle of tilt measured from level, deg

Subscripts

fwd	forward
TA	test article
Rig	test rig
cg	center of gravity

Introduction

The original objective of the mass properties test on the X-48B Blended Wing Body aircraft (The Boeing Company, Chicago, Illinois) was to determine the roll, pitch, and yaw moments of inertia by using a combination of the bifilar pendulum and compound pendulum methods. The compound pendulum method is an experimental technique used to determine the moments of inertia (MOIs) of a test article. The compound pendulum is able to measure multiple MOI terms while the test article is in a level position, unlike other conventional mass properties techniques such as the bifilar pendulum method. The compound pendulum, however, suffers from lower accuracy compared to other techniques, and is more sensitive to errors in measurement (refs. 1 and 2). A dummy test article was used to estimate uncertainty bounds by undergoing highly accurate spin testing, which was then compared to compound pendulum testing. During the first phase (Phase 1) of compound pendulum testing, large errors were noticed in the roll and pitch inertias. Initially, an average measurement of the period over the test block was used to calculate inertias. An inertial measurement unit (IMU) was also on board the X-48B aircraft, and used as secondary data to provide time histories of oscillation. Using the test block averaged period, the large errors (>60 percent from manufacturer-supplied data) were observed. When the IMU time history was analyzed, however, a relatively significant frequency shift was seen throughout each test block. This frequency shift resulted in a change of calculated inertia values of 70 percent. A second phase (Phase 2) of tests was run using an IMU as a primary measurement device on both the dummy test article and the X-48B aircraft. By measuring the frequency of oscillation over the time history and using subsets of the time history where the frequency was stable, accurate measurements of roll and pitch inertias could be made. The cause of the frequency shift is unknown; possible causes are discussed. This report describes the process by which the MOIs were measured, details the frequency shift effect, and describes methods to account for the frequency shift in mass properties measurement.

Test Article Description

Two test articles were used: a dummy test article that approximated some of the mass properties of the X-48B aircraft; and the X-48B aircraft itself. The dummy test article was utilized to determine the accuracy of the test in the specific test fixture and the test technique, and to estimate uncertainty. The dummy test article, shown in fig. 1, is a cross made of two aluminum rectangular beams, with masses on the ends of the lateral beams along with a tail mass; it is also referred to as the “Iron Cross.”

The dummy test article was tested independently using a spin method of calculating inertias, which agreed very well with the analytical model of the dummy test article. The spin method has uncertainties in center of gravity (CG) measurement of .01 in. and in MOI measurement of ±1 percent. Due to the inability of the independent test to accommodate the test article in an I_{yy} (pitch inertia) testing configuration, I_{yy} was not measured. Since the mass properties measured by the spin method correspond well to the analytical model, all mass properties that are not measured by the spin method are assumed to be correct.

The results of this test are presented in table 1.

Table 1. Spin method versus numerical model.

Variable	Percentage or absolute difference
Weight	.04 %
X CG coordinate	.051 in.
Y CG coordinate	.0007 in.
Z CG coordinate	.12 in.
I_{xx} (roll inertia)	2.99 %
I_{yy} (pitch inertia)	N/A
I_{zz} (yaw inertia)	1.47 %

The X-48B aircraft is an 8.5-percent scale model of a blended wing body design. A top and front view of the X-48B aircraft are shown in fig. 2. All mass properties of the X-48B aircraft are in the same order of magnitude as the dummy test article, thus, the uncertainties in measurement using the same hardware and analysis techniques should be similar for both the dummy test article and the X-48B aircraft.

Center of Gravity

The longitudinal and vertical CG of both test articles were measured during bifilar pendulum testing. The longitudinal CG measurement was performed by suspending the test article at two locations, with both suspensions of equal length. A load cell measured the reaction force at both suspensions. Knowing the suspension locations relative to the test article reference frame (depicted in fig. 1), the equations of equilibrium can be used to calculate the longitudinal CG position (ref. 2). The CG calculations are shown in eq. (1).

$$\sum M_{ref} = 0$$

$$R_{fwd}x_{fwd} + R_{aft}x_{aft} - W_{TA}x_{cg} = 0 \qquad (1)$$

$$x_{cg} = \frac{R_{fwd}x_{fwd} + R_{aft}x_{aft}}{W_{TA}}$$

The vertical CG measurement was performed by suspending the test article from two different suspension lengths (L1 and L2), which induced a tilt to the test article based on the difference in the length of each of the suspensions (see fig. 3). A load cell again measured the reaction force at both suspensions. By knowing the angle of tilt, as well as the vertical and horizontal locations of the reaction forces, the equations of equilibrium could be used to calculate the vertical CG position (ref. 3). The value of X_{cg} is assumed to be known from longitudinal CG testing. Equation 2 presents the equations used for calculating the vertical CG using the tilt methodology; fig. 4 presents the terms from eq. (2) in graphical form.

$$\sum M_{ref} = 0$$

$$= R_{fwd}(x_{fwd}R_{fwd}\cos\theta - z_{fwd}\,R_{fwd}\sin\theta) - W(X_{cg}\cos\theta - Z_{cg}\sin\theta) + R_{aft}(X_{aft}R_{aft}\cos\theta - Z_{aft}R_{aft}\sin\theta \qquad (2)$$

In order to calculate inertia using the compound pendulum, the distance from the pivot point (about which the test article is rotating) to the vertical CG must be measured. Since the vertical CG is usually not on the surface of a test article, a reference point is used so that a physical measurement can be obtained. The formula for inertia using the compound pendulum method being highly sensitive to vertical CG position, any errors in measurement will be magnified in the calculation of inertia values (refs. 1 and 2). This concept is explained in further detail in the section, “Determination of Pitching and Rolling Moments of Inertia.” Both the longitudinal and vertical CGs are corrected to remove the contribution of the test rig.

Determination of Pitching and Rolling Moments of Inertia

The determination of the roll inertia, I_{xx}, and the pitch inertia, I_{yy}, involve small perturbations in a pendulum setup. In a simple pendulum, the equations of motion are as shown in eq. (3):

$$lmg\sin\theta = I\ddot{\theta} \tag{3}$$

By realizing that, for small angles, $\sin(\theta) = \theta$, and that $\ddot{\theta}$ is a function of the natural frequency of the pendulum, the equation of motion for a pendulum can be rearranged as shown in eq. (4):

$$\begin{gathered}\ddot{\theta} = \frac{lmg\theta}{I} \\ \ddot{\theta} = \omega^2\theta \\ \omega = \sqrt{\frac{lmg}{I}} \\ T = \frac{2\pi}{\omega} \\ T = 2\pi\sqrt{\frac{I}{lmg}} \\ I = \frac{T^2 lmg}{4\pi^2}\end{gathered} \tag{4}$$

By inducing oscillation about a particular axis, the period of oscillation, or T in eq. (4), can be experimentally determined. Since l and the product mg are known quantities (l is measured directly and mg is measured with a load cell), eq. (4) can be solved for the inertia about the axis of oscillation.

It is important to realize that the above equation represents the total inertia of both the rig and the test article, about the pivot point of oscillation. To determine the inertia about the CG of the test article, the parallel axis theorem must be used, as shown in eq. (5):

$$I_{TA} = I - I_{Rig} - m_{TA}l^2 \tag{5}$$

where I_{Rig} represents the inertia about the suspension point of the rig. The larger the value of l (or the longer the pivot arm), the higher the proportion of the ml^2 is to the total inertia. A large number is therefore subtracted from a slightly larger number, producing a much smaller number, and any error in the measurement of vertical CG can cause large errors in accuracy. For example, if I = 120 slug-ft^2, I_{Rig} = 5 slug-ft^2, m = 9.32 slugs, and l = 3 ft, then I_{TA} = 31.12 slug-ft^2. A 1/8-inch error in measurement of l (l = 2.99 ft) produces I_{TA} = 31.73 slug-ft^2, which is a roughly 1.8-percent error compared to the "nominal" value.

Compound Pendulum Test

The compound pendulum test was chosen to calculate pitch and roll moments of inertia despite the potential inaccuracies because the test article could be kept in a level position, as opposed to using the bifilar pendulum method, in which the test article would need to be rotated to each orthogonal axis. The test articles were suspended by a dual triangular suspension system, as shown in fig. 5.

Both the dummy test article and the X-48B aircraft attach to the yellow trapezoidal frame (attachment rig) shown in fig. 5. The purpose of using dual triangular suspensions was to constrain the system to single-degree-of-freedom motion, thereby reducing cross-coupling. During the Phase 1 testing, the period of oscillation was acquired by using a stopwatch and averaging over the test block. An IMU is internally mounted on the X-48B aircraft and was recorded as a backup system to verify the stopwatch data. Measurements of the vertical CG position were made by measuring from the pivot point to a vertical CG reference point (as detailed in the "Center of Gravity" section above). The test article was then swung in pitch or in roll, and the period of oscillation measured. Any flexure in the support structure or connection points was cause for invalidation of test results; the structure was stiffened to prevent flexure by shimming up joints and adding extra bracing. In addition, any motion in other than the primary axis was cause to stop the test and repeat with different swing technique. The number of oscillations varied between 15-30 per test block. Tests were repeated at least three times for each test article.

Phase 1 Results

Bifilar testing was performed prior to compound pendulum testing. The results from the bifilar testing showed that the yaw inertia could be measured to within ±2.1 percent of the spin-test value. The lateral CG was designed to be centered on the test article (a value of 0.00) by symmetry, and the bifilar testing confirmed this to be the case. An insignificant frequency shift was observed in the bifilar data, therefore the bifilar data from all testing is considered accurate and is not shown in tables 2 and 3. This report will now examine only the pitch and roll data. The initial results from the dummy test article were as shown in table 2.

Table 2. Comparison between analytical model and experimentally-determined mass properties: Dummy test article.

Summary of data	Percentage or absolute difference
Longitudinal CG	-0.03 in.
Vertical CG	-0.009 in.
Test article weight	0.29 %
I_{xx} (roll inertia)	5.73 %
I_{yy} (pitch inertia)	2.39 %

Errors below 5 percent were expected during testing. The roll inertia stands as a slight outlier, but not enough to cause concern. No IMU data was available on the dummy test article. During X-48B flight-testing, the longitudinal CG, vertical CG, and weight measurements were made using conventional methods after each configuration change of the X-48B aircraft. The longitudinal CG, vertical CG, and weight measurements obtained during mass properties testing corresponded to a high degree of accuracy (<.5 percent) to the manufacturer-supplied mass properties, and are considered correct. The X-48B results are shown in table 3.

Table 3. Comparison between experimentally-determined mass properties and manufacturer-supplied mass properties: X-48B aircraft.

Variable	Difference, percent
I_{xx} (roll inertia)	56.18
I_{yy} (pitch inertia)	65.01

The percent error shown in roll and pitch inertias seems to point toward an error in the experiment. It is true that these tests were performed because of unknown accuracy in the manufacturer's mass properties testing (pre-flight test) of the X-48B aircraft; however, errors of 56 percent and 65 percent warranted further investigation. Since the Phase 1 results were obtained by using the average frequency of oscillation, examining the IMU data may show some more insight into potential problems. Analyzing the IMU data using a Fourier transform across the time history produces the frequency-versus-amplitude plot shown in fig. 6.

The overall shift in frequency is approximately .05 Hz; however, the difference between using the initial period and the final period produces a 70-percent change in the determined inertias. The pitch data exhibit the same problems as those for roll. Because there were no IMU data for the dummy test article, it was unknown whether the inertia estimates of the dummy test article were sensitive to the same types of problems.

Historically, mass properties tests have pointed to smaller amplitudes having higher accuracy than swing tests with larger amplitudes (ref. 2). In this testing, the frequency is most stable in the region of larger amplitudes, between approximately 3 and 9 deg. Data with amplitudes below 1 deg, which is historically considered highly accurate, contains the greatest frequency shift and largest inertial errors.

A second series of compound pendulum tests were conducted, to determine the frequency shift characteristics of the dummy test article, which would determine whether the frequency shift is isolated to the X-48B aircraft; and, by using the frequency shift characteristics of the dummy test article, to analyze the effect on the accuracy of MOI.

Second Compound Pendulum Tests (Phase 2)

The second set of compound pendulum tests were planned to be very similar to the first set of tests; both the dummy test article and X-48B aircraft would be tested in the same rig, and the period of oscillation would be measured in both pitch and roll. During the initial compound pendulum tests, cross-coupling was noticed in pitch and the test fixture was slightly modified to better constrain the test articles. A wireless IMU (different from the onboard X-48B IMU) was attached to the attachment rig, providing IMU measurements on both the dummy test article and the X-48B aircraft. All distance and weight measurements were repeated to ensure the proper values were being used. The X-48B aircraft is shown in roll and pitch compound pendulum test setup in fig. 7 and fig. 8, respectively.

The wireless IMU has a gyroscopic bias stability of .2 deg/s, and the onboard IMU on the X-48B has a gyroscopic bias stability of .00083 deg/s. In the Phase 2 tests, both the wireless IMU and the X-48B IMU were operated concurrently for the X-48B aircraft. A sample time history of the roll rate during a compound swing test from the wireless IMU is provided in fig. 9.

Phase 2 Results

The frequency-versus-amplitude plot produced by the second series of compound pendulum tests is detailed in fig. 10 and shows that the dummy test article exhibits a similar trend in frequency shift with a smaller magnitude than that of the X-48B aircraft. The stable frequency region between 3 and 6.5 deg was used to calculate new inertia values. The frequency shift exceeded a 5-percent difference from the stable frequency value at 3 deg and below. The percent difference shown in table 4 refers to the inertia calculated using the stabilized frequency versus the inertia calculated analytically. Both the roll and pitch inertias are highly accurate; in fact, the roll inertia is half that of the error from the first set of compound pendulum tests.

Table 4. Comparison between analytical and experimental data: dummy test article Phase 2 using stabilized frequency.

Summary of data	Difference, percent
I_{xx} (roll inertia)	-2.20
I_{yy} (pitch inertia)	-2.75

Retesting the X-48B aircraft, the Phase 2 plot of frequency versus amplitude for pitch is shown in fig. 10 and for roll in fig. 11. All of the frequency-versus-amplitude plots are overlaid for both the X-48B aircraft and the dummy test article in fig. 12. The same trend is occurring in pitch and roll on both test articles, with the only difference being the magnitude of the frequency shift.

The X-48B onboard IMU data for Phase 2 are comparable to the Phase 1 results; however, by limiting the frequency to between 3 and 8.5 deg (as in the dummy test article), the results shown in table 5 are calculated.

Table 5. Comparison between manufacturer-supplied data and experimental data: X-48B Phase 2 using amplitude-limited frequency.

	Difference, percent
Roll	-4.04
Pitch	-2.95

Discussion

By comparing the IMU data to the average frequency data, the errors shown by using the stabilized frequency are within expected values. Seemingly, by using an amplitude-limited subset of the test data, inertia measurements within a few percent of “truth” are attainable; however, the source of the frequency shift is unknown, and potential causes are discussed here: experimental setup, cross-coupling, aerodynamic effects, small angle approximations, and IMU limitations.

One theory is that the friction in the bearings that were used to suspend the test article was causing the frequency shift. The bearings are the same bearings used in the bifilar tests; X-48B IMU data were available and analyzed for the bifilar method with an insignificant frequency shift. Several higher-order friction models (coulomb, stribeck, and viscous) were analyzed without correlating any results properly, possibly due to a lack of data.

Another theory is that cross-coupling may have been present, which could introduce errors in period measurement. The IMU allowed real-time monitoring of motion in all three axes, however, and cross-coupling was on the order of 15:1 (primary oscillation versus other axes oscillation).

Another theory is that aerodynamic effects are being seen in the roll and pitch axes that are not significant in the yaw axis. Analysis was done trying to correlate apparent mass and entrapped mass into the results, with no improvement on the results. It is possible that unaccounted-for aerodynamic effects are occurring.

Another observation is that the derived equations use small angle theorem and, therefore, the oscillations should be kept as small as practical. Oscillations on the order of 1 deg are historically considered to be accurate for proper mass properties measurement; however, only data obtained when the oscillations were between 3 and 10 deg proved to be the best source of data in this experiment. Looking at the small angle theorem, 14 deg is the point at which the relative error exceeds 1 percent. Thus, angles of 3-10 deg are still relatively accurate when using the small angle approximation.

A possibility that the IMU data were below the noise floor, and therefore inaccurate, was discussed. After data analysis, both the onboard X-48B IMU data and the wireless IMU data were overlaid, as shown in figure 13. It is clear that the frequency shift begins well above the noise floor. A separate series of tests to verify IMU data accuracy were performed using a non-contact solution to measure oscillations. A photogrammetry system (high-speed photography data acquisition system) was used to obtain oscillation data and the wireless IMU was concurrently measuring the same oscillations. A simple pendulum was constructed using hardware similar to the X-48B simple pendulum (rod end bearings). The simple pendulum was made of a turnbuckle at an approximately 2.5-ft extension, connected to a rod end bearing at the pivot point and a simple mass at the end of the pendulum of approximately 25 lb. The trends showing increasing frequency at lower amplitudes are nearly identical between the photogrammetry system and the wireless IMU, with the differences in magnitude being attributed to the higher resolution and accuracy of the photogrammetry system.

Conclusions

During mass properties testing on the X-48B, large errors were seen when using the compound pendulum method to calculate roll and pitch inertias. By looking at the frequency for each oscillation rather than the average frequency over the test block, a frequency shift was observed. This frequency shift was small enough to not be visible to the naked eye, yet large enough to shift the inertia value by 70 percent. The frequency shift occurred in multiple test articles across multiple test setups. By using stable frequencies rather than shifting frequencies, the error in roll and pitch inertias can be minimized.

The conclusion reached in this paper is that for any mass properties test, a method of obtaining the period through a recorded time history is necessary, in order to monitor for any potential frequency shift. The frequency shift is not occurring as an artifact due to data analysis; the shift is a real phenomenon. Because the cause of the frequency shifts is unknown, it is recommended that all mass properties tests that use pendulum methods (including bifilar and compound pendulum) use an inertial measurement unit, a photogrammetry system, or any method that obtains a time history with enough resolution to obtain the frequency of oscillation, rather than using a stopwatch, self-timing system, or other non-recording method of measurement. To obtain accurate mass properties measurement, only stable frequency values should be used to calculate inertia, and regions in which the frequency is not stable should be ignored. With great care in measurement, and knowledge of how the frequency is shifting through the course of oscillation, the compound pendulum method can reduce error on full-scale test articles to as little as 2 percent for this test setup.

References

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2. Wolowicz, Chester H., and Roxanah B. Yancey, *Experimental Determination of Airplane Mass and Inertial Characteristics*, NASA TR R-433, 1974.

3. Peterson, Wayne, “Mass Properties Measurement in the X-38 Project,” SAWE Paper No. 3325, 2004.

Figures

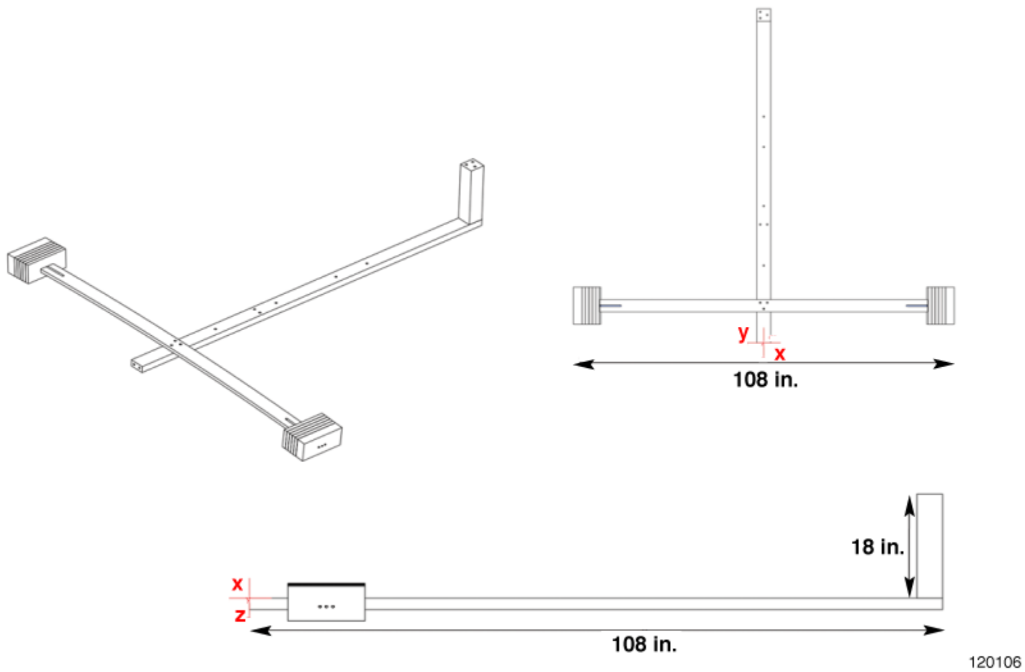

Figure 1. The dummy test article and reference frame.

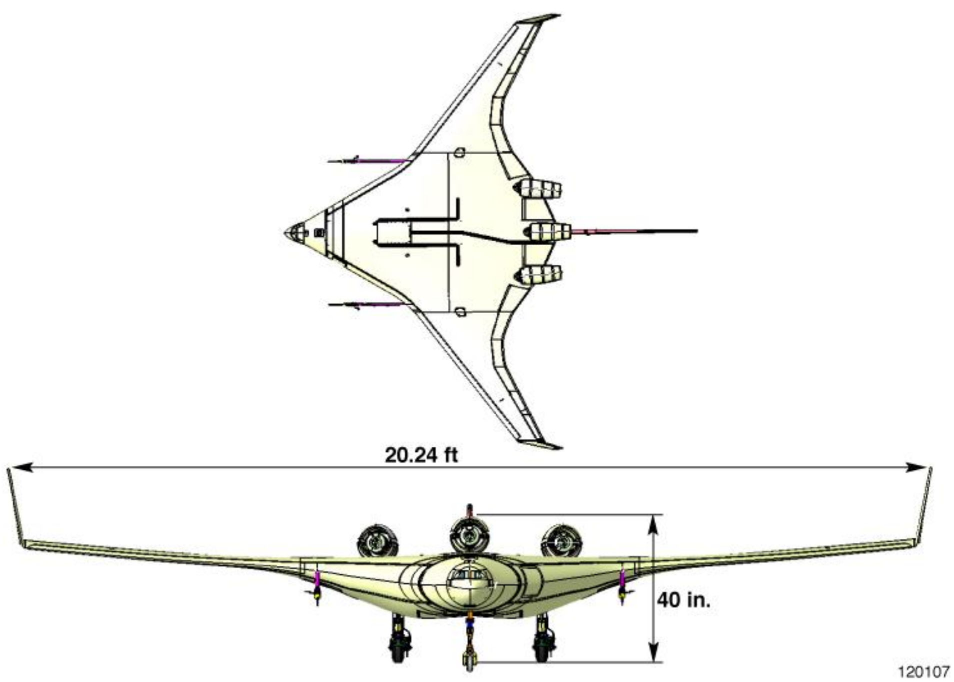

Figure 2. The X-48B aircraft.

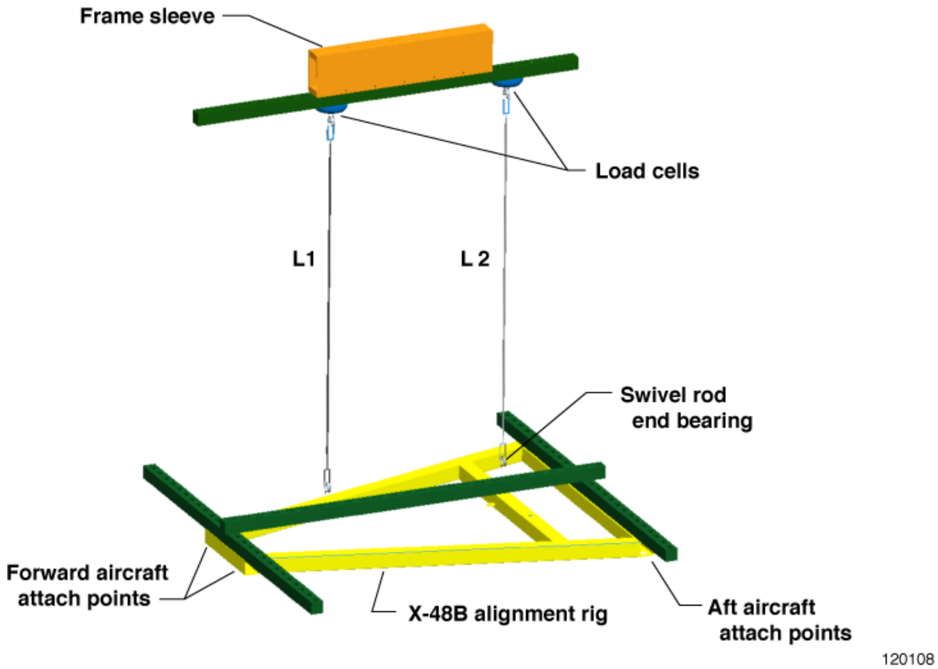

Figure 3. The vertical center of gravity setup.

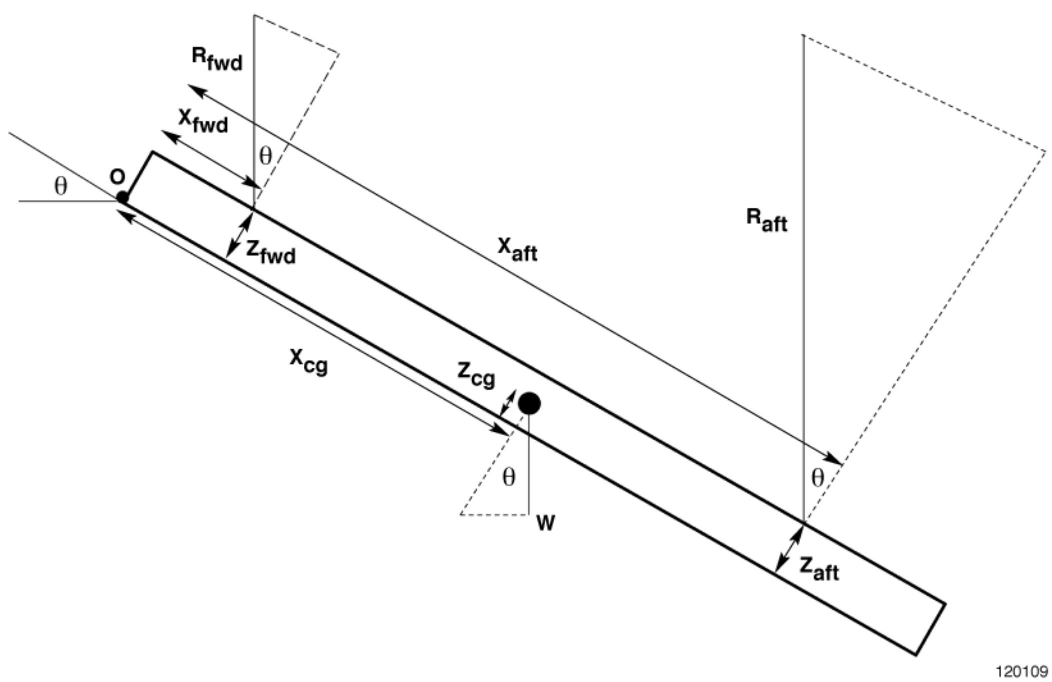

Figure 4. Calculation of the vertical center of gravity, with terms.

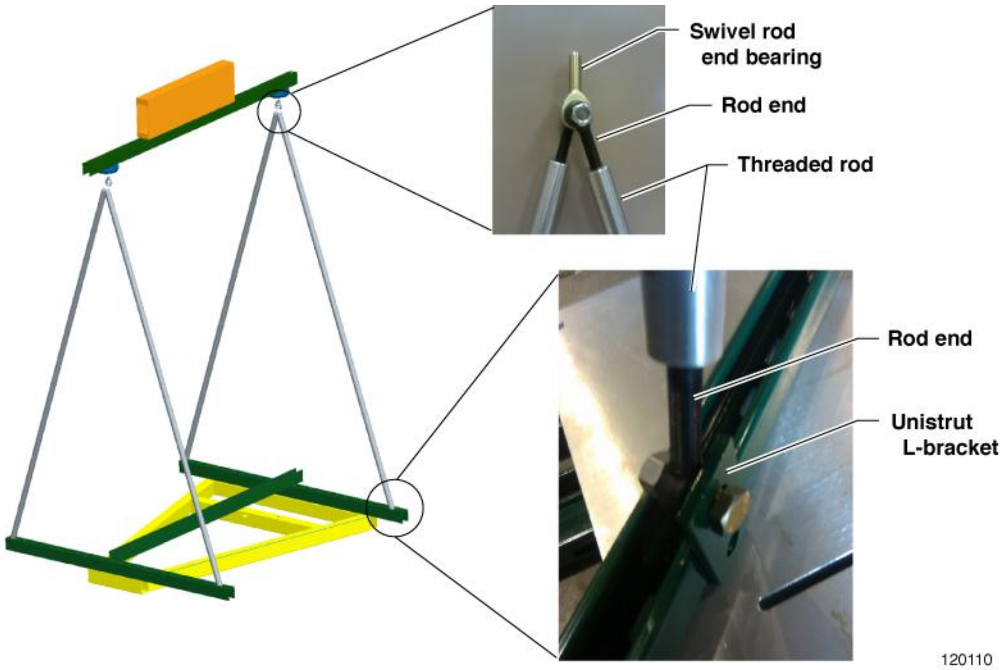

Figure 5. The dual triangular suspension system.

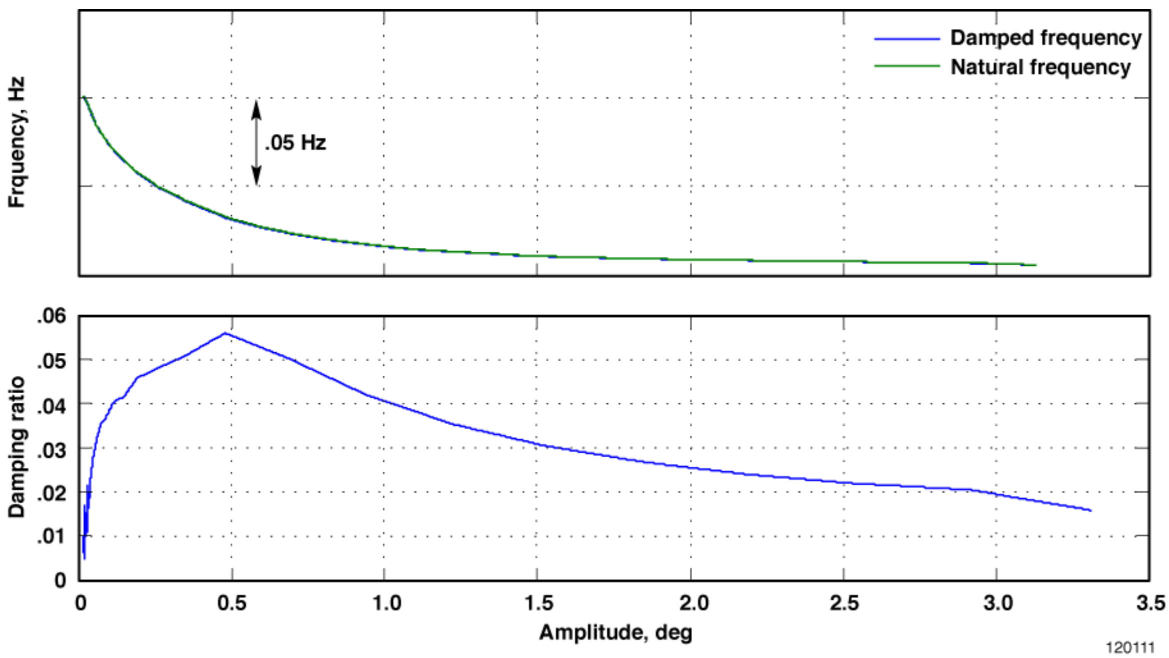

Figure 6. The X-48B inertial measurement unit data: roll axis, frequency, and damping ratio versus amplitude, Phase 1.

Photograph courtesy of David Wolfe.

Figure 7. The X-48B aircraft in roll setup, Phase 2.

Photograph courtesy of David Wolfe.

Figure 8. The X-48B aircraft in pitch setup, Phase 2.

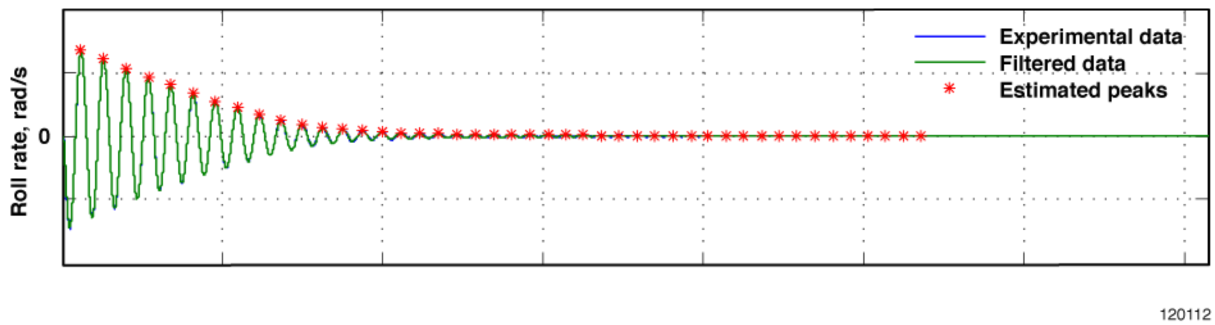

Figure 9. A sample wireless inertial measurement unit time history.

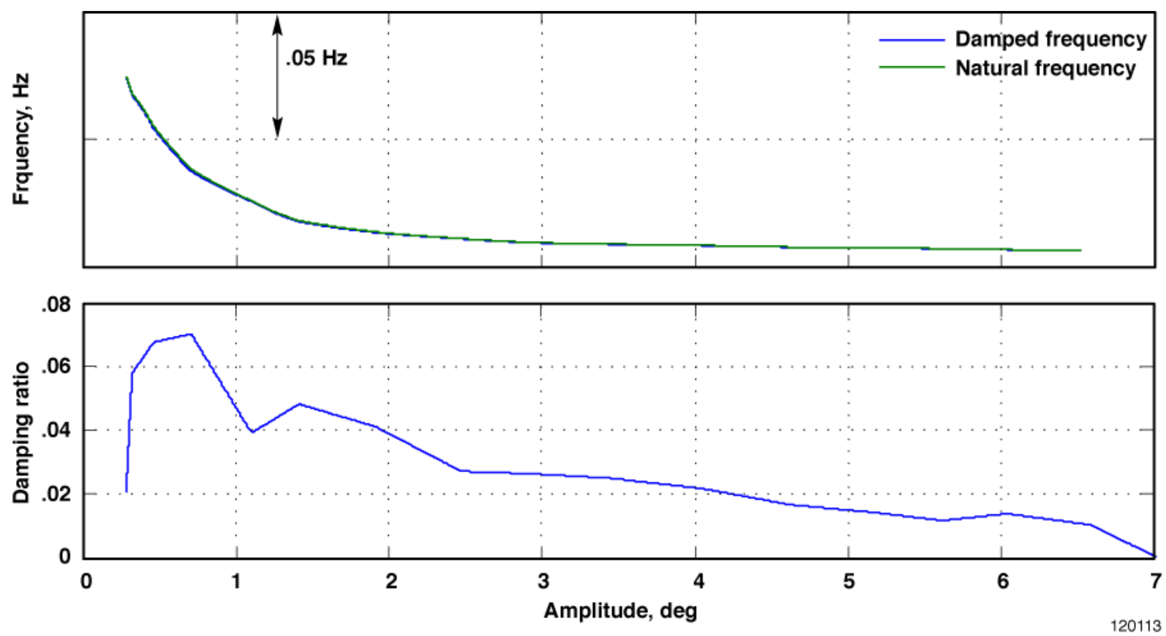

Figure 10. Frequency and damping ratio versus amplitude, Phase 2 testing, dummy test article (roll).

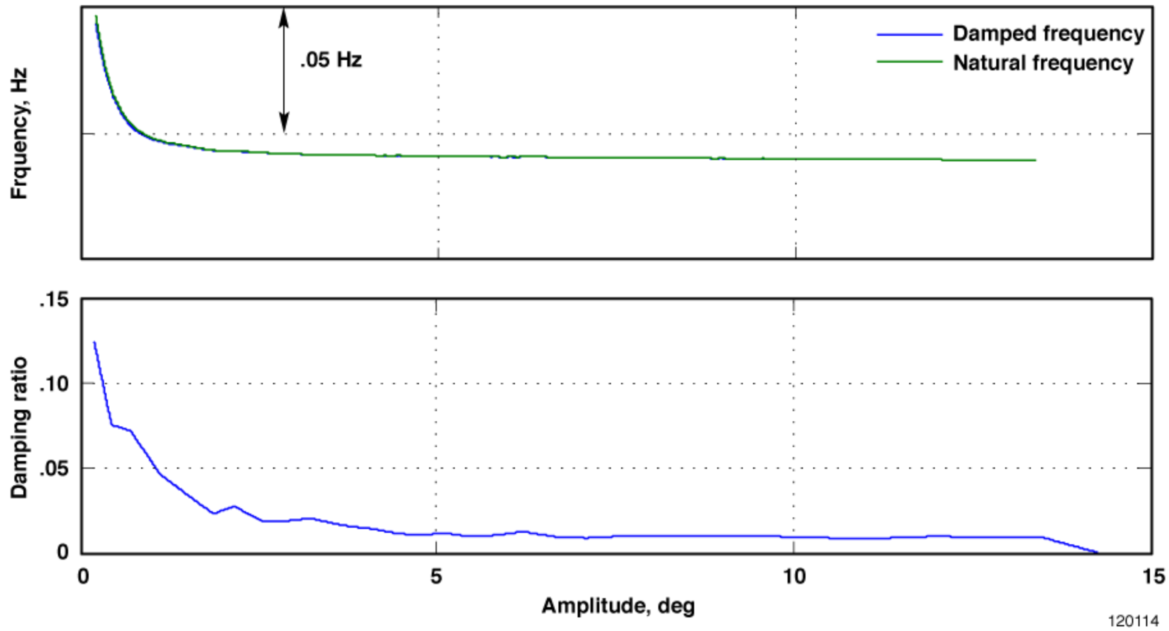

Figure 11. Frequency and damping ratio versus amplitude, Phase 2 testing, X-48B aircraft (pitch).

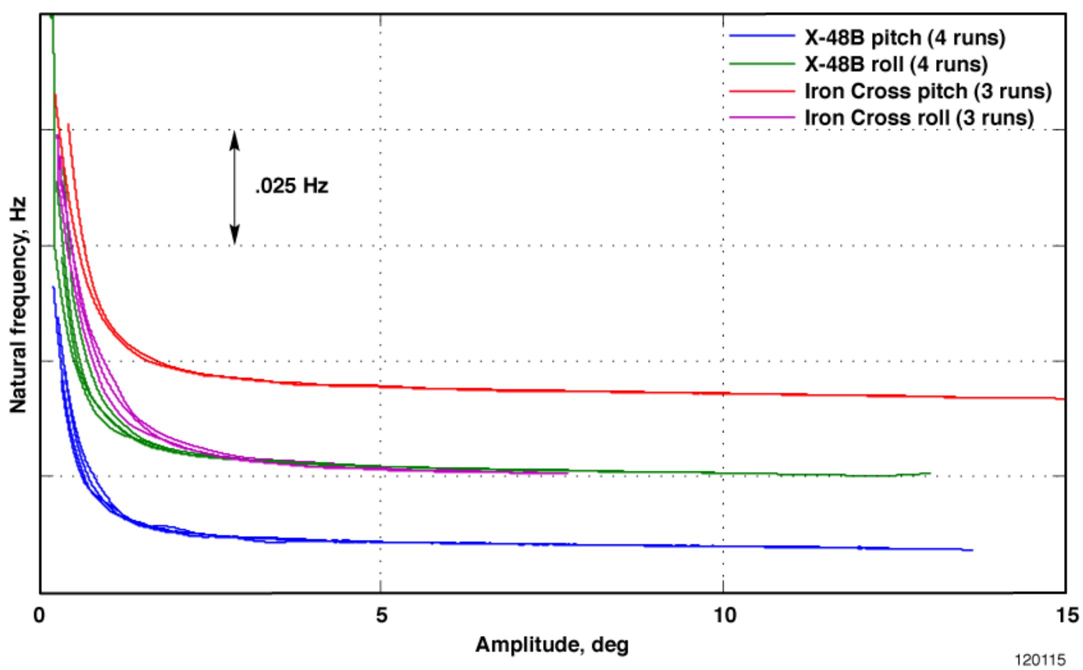

Figure 12. The X-48B aircraft and dummy test article natural frequencies versus amplitude, all runs.

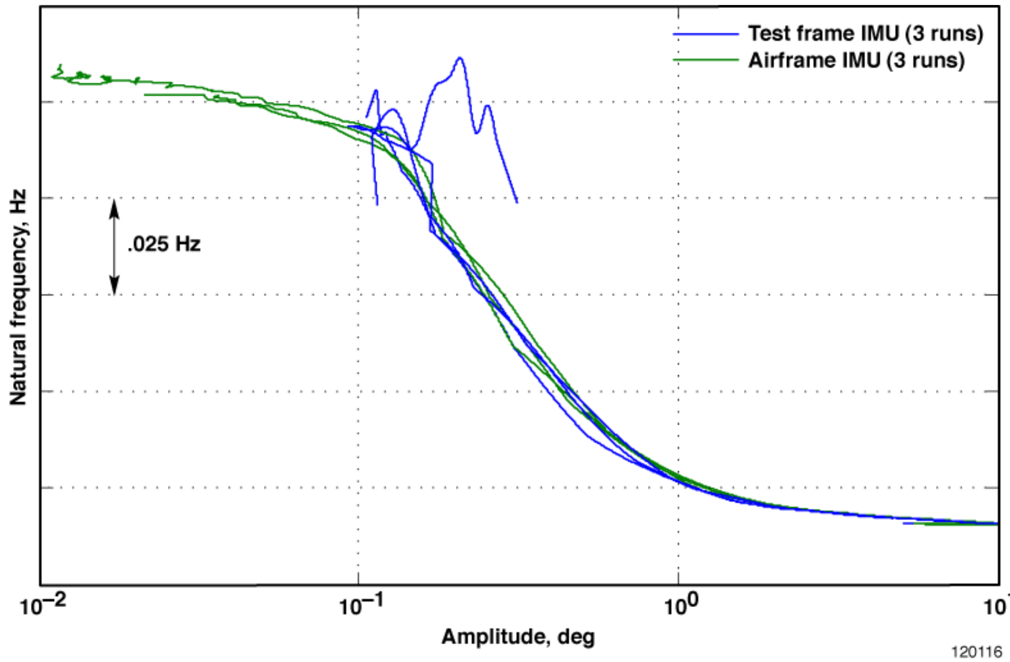

Figure 13. Onboard and wireless inertial measurement unit frequency versus amplitude, log scale.