The transport of mass, energy, and entropy in cryogenic support struts for engineering design

Thermal and Fluids Analysis Workshop

August 13-17, 2012, Pasadena, California

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1 Abstract

Engineers working to understand and reduce cryogenic boil-off must solve a variety of transport problems. An important class of nonlinear problems involves the thermal and mechanical design of cryogenic struts. These classic problems are scattered about the literature and typically require too many resources to obtain. So, to save time for practicing engineers, the author presents this essay. Herein, a variety of new, old, and revisited analytical and finite difference solutions of the thermal problem are covered in this essay, along with commentary on approach and assumptions. This includes a few thermal radiation and conduction combined mode solutions with a discussion on insulation, optimum emissivity, and geometrical phenomenon. Solutions to cooling and heat interception problems are also presented, including a discussion of the entropy generation. And the literature on the combined mechanical and thermal design of cryogenic support struts is reviewed with an introduction to the associated numerical methods.

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Presented By
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Table of Contents

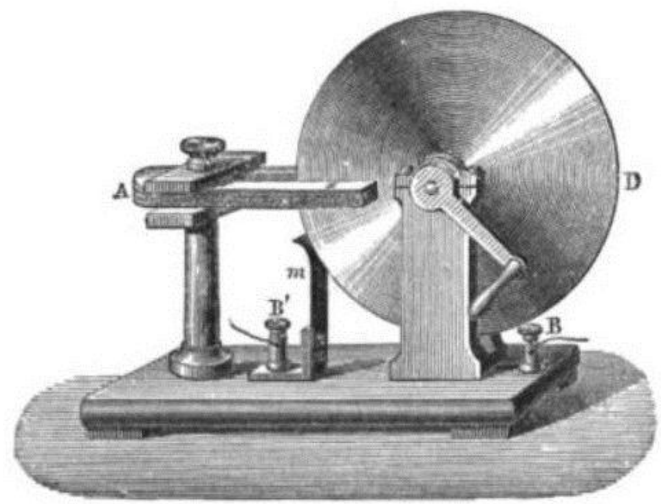
- ✓History
- ✓State of the art
- ✓Future

Royal Institution of Great Britain

Standing on the shoulders of giants

$$\oint_{\partial\Sigma} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} = -\iint_{\Sigma} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{d}\mathbf{S}$$

1831, simple DC generator



Massachusetts's Institute of Technology

More power: superconducting and Cryogenics

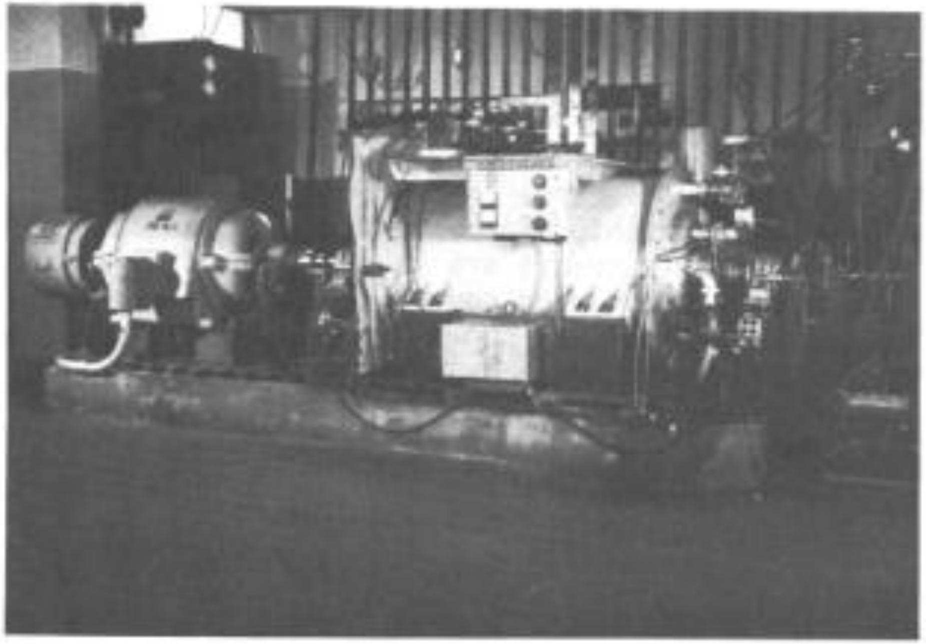


Fig. 9 View of 3-MVA Alternator and 150-HP Drive Motor

- In the 1970s, MIT Electric Power Research Institute did significant research into superconducting synchronous generators, including cryogenics engineering research

MIT’s prototype synchronous generator

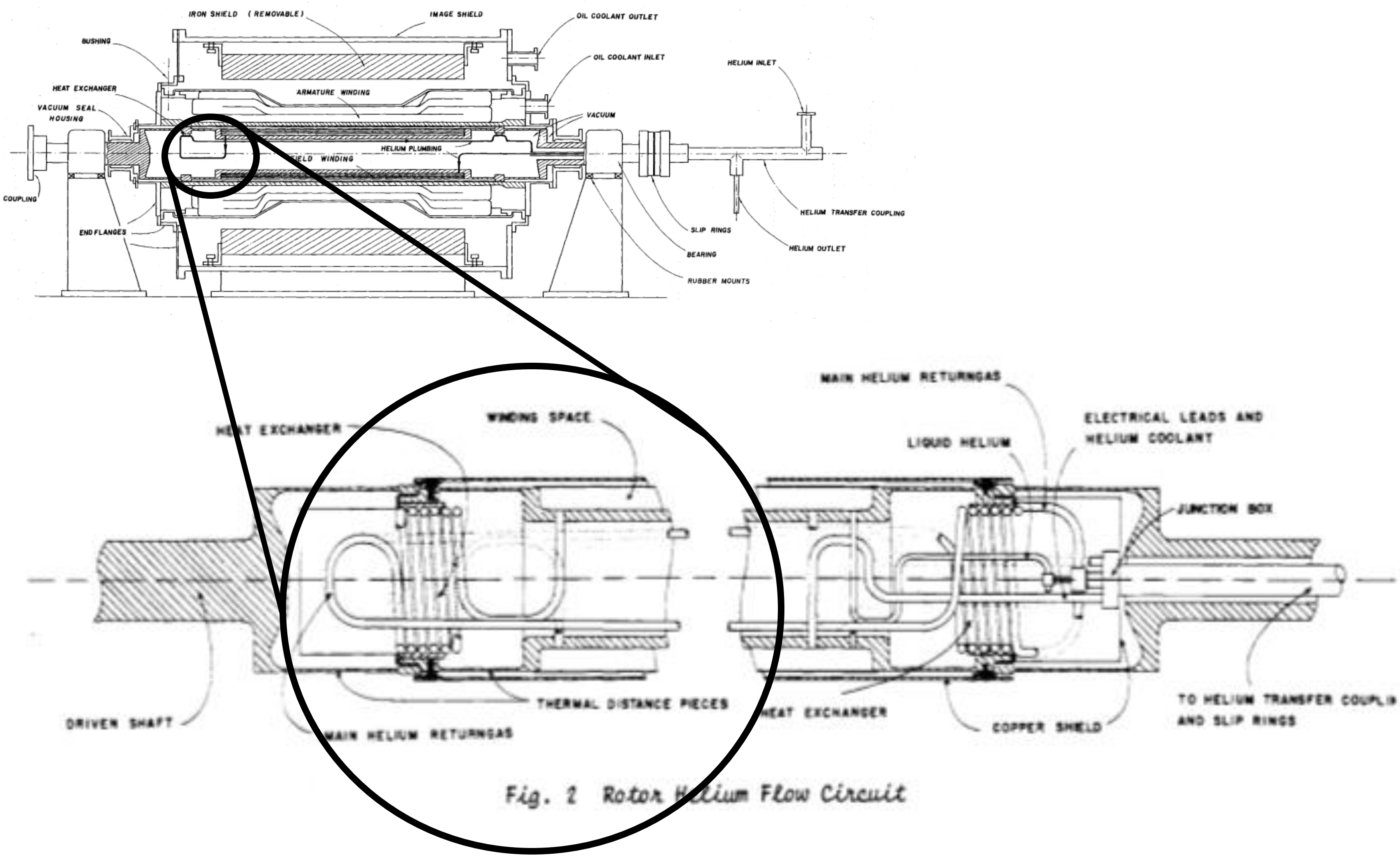
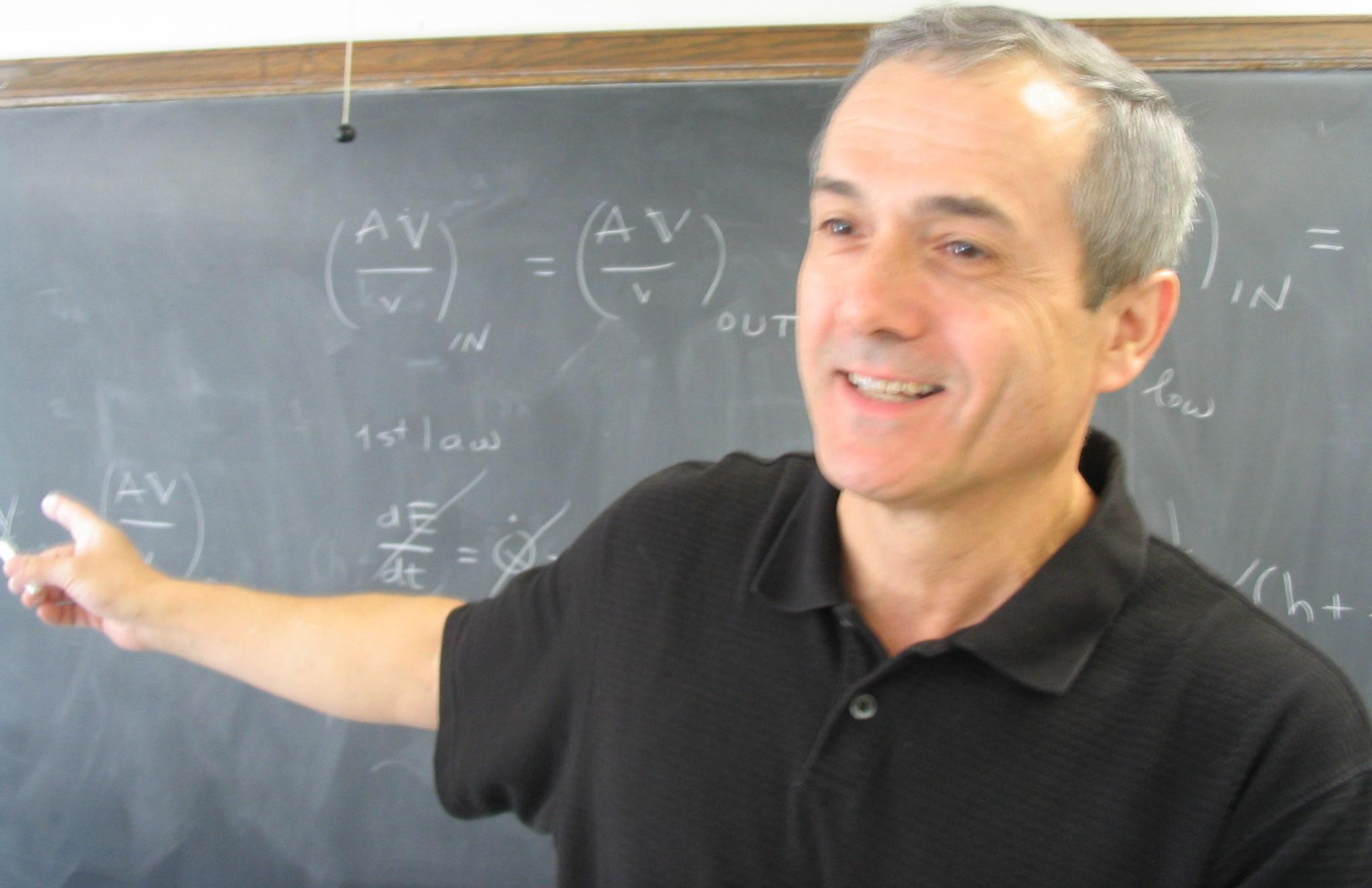


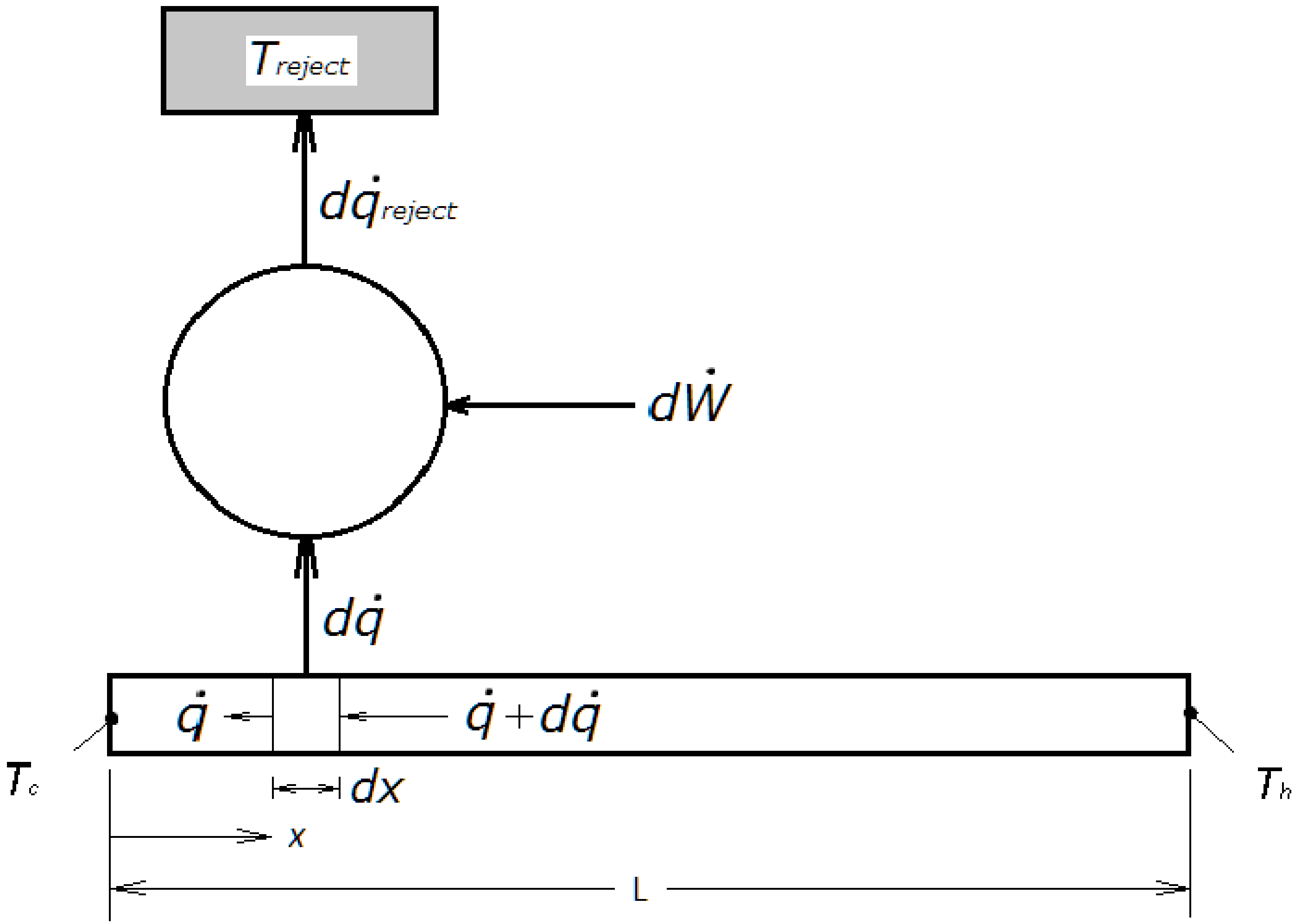
Fig. 2 Rotor Helium Flow Circuit

Dr. Adrian Bejan

J.A. Jones Professor, Duke Department of Mechanical Engineering and Materials Science



Adrian Bejan (1975)



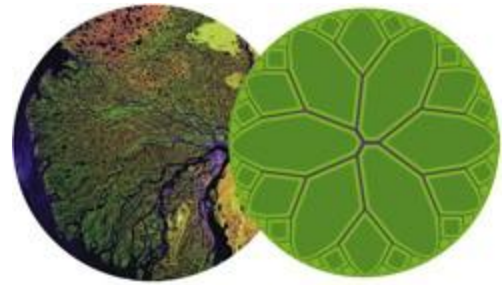
Thermodynamic Minimum

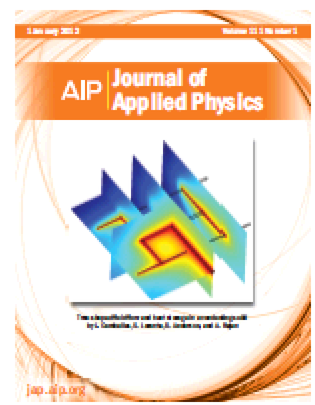
$$\dot{S} = \frac{A}{L}\left(\int_{T_c}^{T_h} \frac{\sqrt{k}}{T}\,\mathrm{d}T\right)^2 = \frac{kA}{L}\left[ln\left(\frac{T_h}{T_c}\right)\right]^2$$

The most efficient cooling scheme generates the least entropy

$$\dot{q}_{optimum} = T\sqrt{k}\left(\frac{A}{L}\right)\left(\int_{T_c}^{T_h} \frac{\sqrt{k}}{T}\,\mathrm{d}T\right)$$

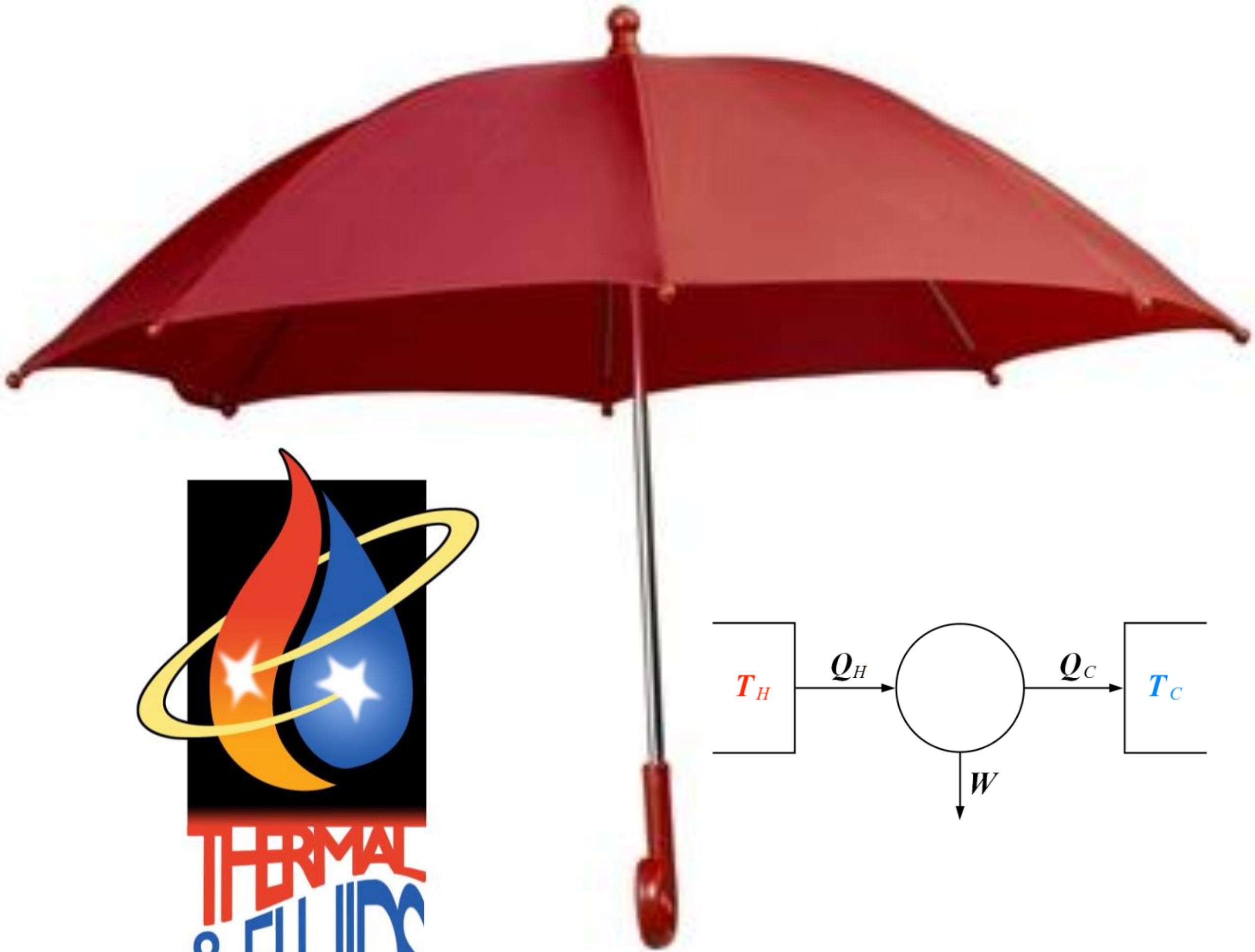
$$\dot{W} = \frac{A}{L}T_h\left(\int_{T_c}^{T_h} \frac{\sqrt{k}}{T}\,\mathrm{d}T\right)^2 = T_h\frac{kA}{L}\left[ln\left(\frac{T_h}{T_c}\right)\right]^2$$





Finite-size systems evolve progressively easier access to the imposed current

Unifies thermal/fluids/thermodynamics



Continuous cooling (solution by Tsao)

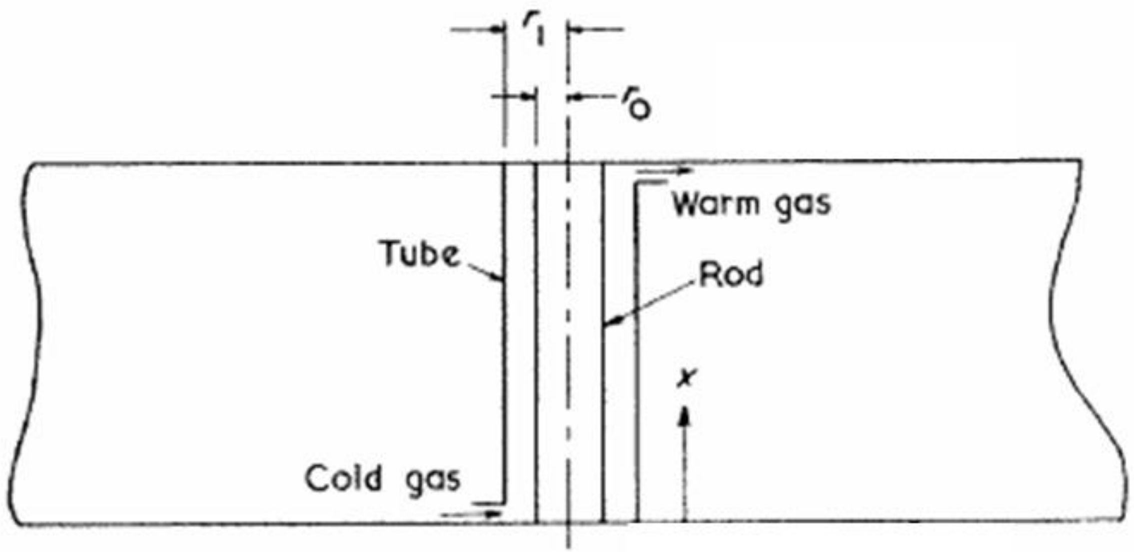
$$\frac{d}{dx}\left(A_c k(T)\frac{dT}{dx}\right) - Ph(T)\left[T(x) - T_m(x)\right] = 0$$

$$\dot{m}c_P \frac{dT_m}{dx} - Ph(T)\left[T(x) - T_m(x)\right] = 0$$

$$T(0) = T_L$$

$$T(L) = T_H$$

$$T_m(0) = T_o$$



Canavan and Miller (constant properties)

$$\eta = x/L \qquad \gamma^2 = 4\alpha^2 + \beta^2$$

$$\tau_f = \frac{(T_f - T_c)}{(T_h - T_c)} \qquad \beta = \frac{hPL}{\dot{m}c_p}$$

$$\tau_s = \frac{(T_s - T_c)}{(T_h - T_c)} \qquad \alpha = \left(\frac{L/(kA)}{1/(hPL)}\right)^{1/2}$$

Boundary conditions

$$T_s(x=0) = T_c \quad \Rightarrow \quad \tau_s(\eta=0) = \frac{T_s(x=0) - T_c}{T_h - T_c}$$

$$T_s(x=L) = T_h \quad \Rightarrow \quad \tau_s(\eta=1) = \frac{T_s(x=L) - T_c}{T_h - T_c}$$

$$T_f(x=0) = T_c \quad \Rightarrow \quad \tau_f(\eta=0) = \frac{T_f(x=0) - T_c}{T_h - T_c}$$

Governing Equations

$$\frac{\mathrm{d}^2\tau_s}{\mathrm{d}\eta^2} = \alpha^2(\tau_s - \tau_f)$$

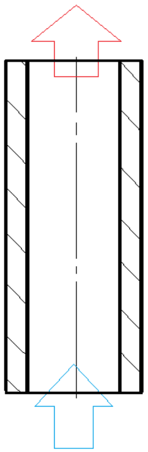
$$\frac{\mathrm{d}\tau_f}{\mathrm{d}\eta} = \beta(\tau_s - \tau_f)$$

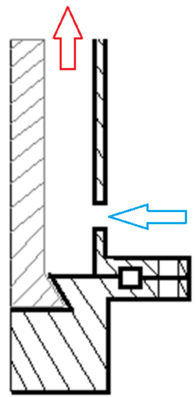
Solutions

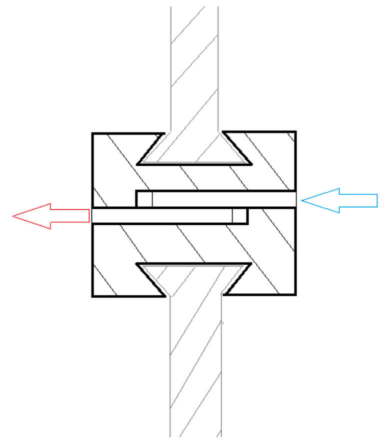
$$\tau_s = \frac{(\gamma+\beta)^2 e^{\frac{\gamma-\beta}{2}\eta} - 4\gamma\beta + (\gamma-\beta)^2 e^{-\frac{\gamma+\beta}{2}\eta}}{(\gamma+\beta)^2 e^{\frac{\gamma-\beta}{2}} - 4\gamma\beta + (\gamma-\beta)^2 e^{-\frac{\gamma+\beta}{2}}}$$

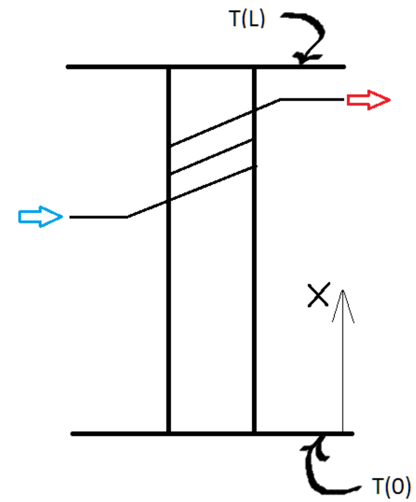
$$\tau_f = 2\beta \frac{(\gamma+\beta) e^{\frac{\gamma-\beta}{2}\eta} - 2\gamma - (\gamma-\beta) e^{-\frac{\gamma+\beta}{2}\eta}}{(\gamma+\beta)^2 e^{\frac{\gamma-\beta}{2}} - 4\gamma\beta + (\gamma-\beta)^2 e^{-\frac{\gamma+\beta}{2}}}$$

$$\frac{\dot{q}''}{k(T_h - T_c)/L} = \left.\frac{\mathrm{d}\tau_s}{\mathrm{d}\eta}\right|_{\eta\to 0} = \frac{\gamma(\gamma^2 - \beta^2)}{(\gamma+\beta)^2 e^{\frac{\gamma-\beta}{2}} - 4\gamma\beta + (\gamma-\beta)^2 e^{-\frac{\gamma+\beta}{2}}}$$









T(L)
X
T(0)

Cooling scheme application: shield mounted

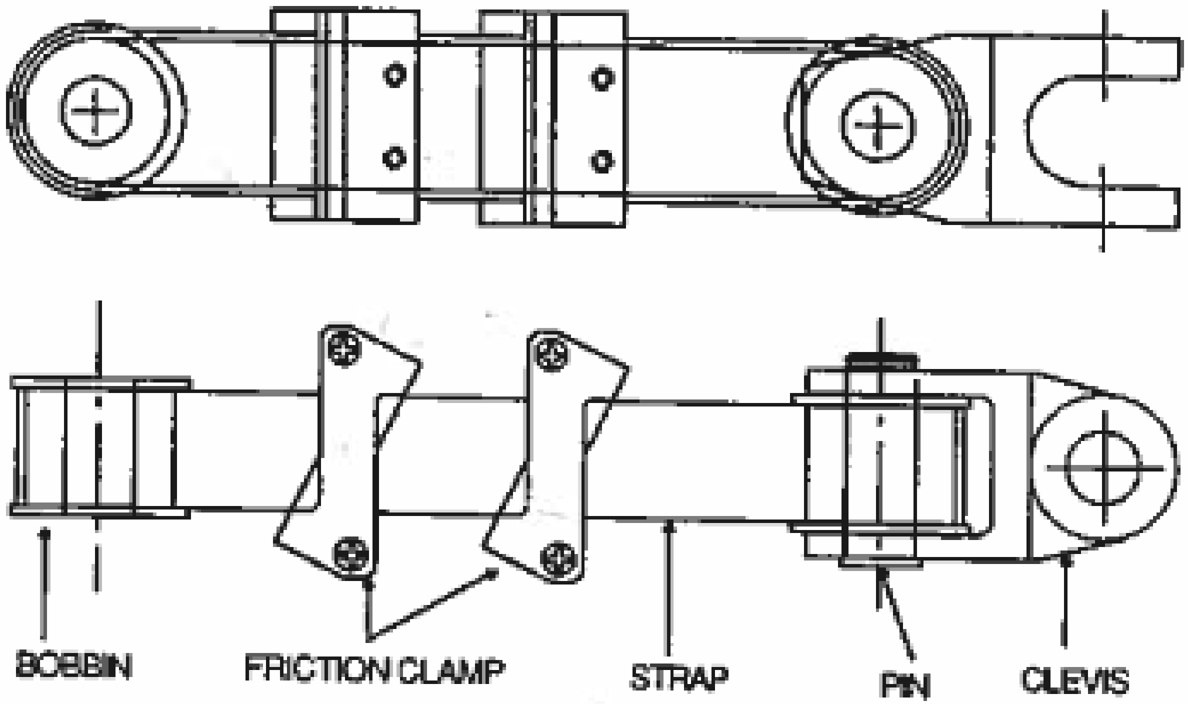
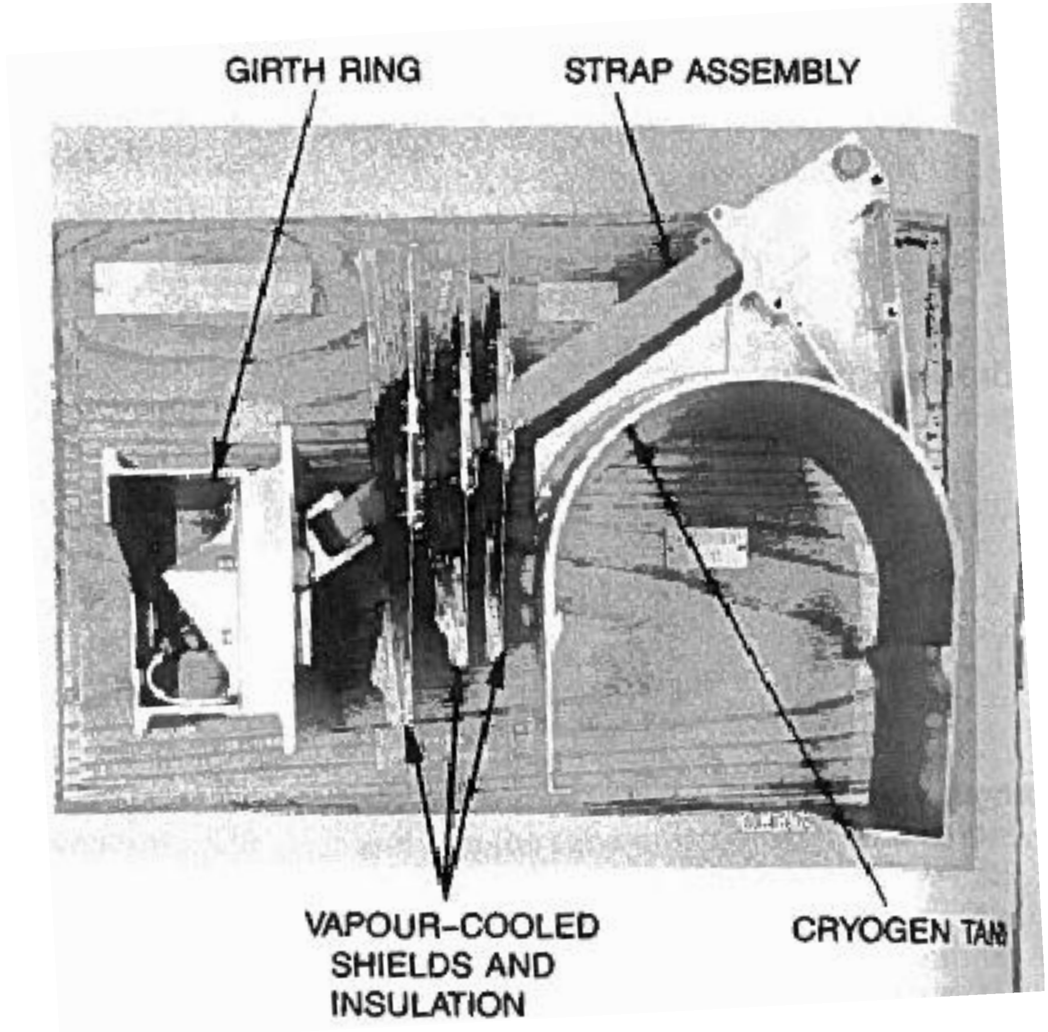


Figure 2 SHOOT support strap with friction clamp

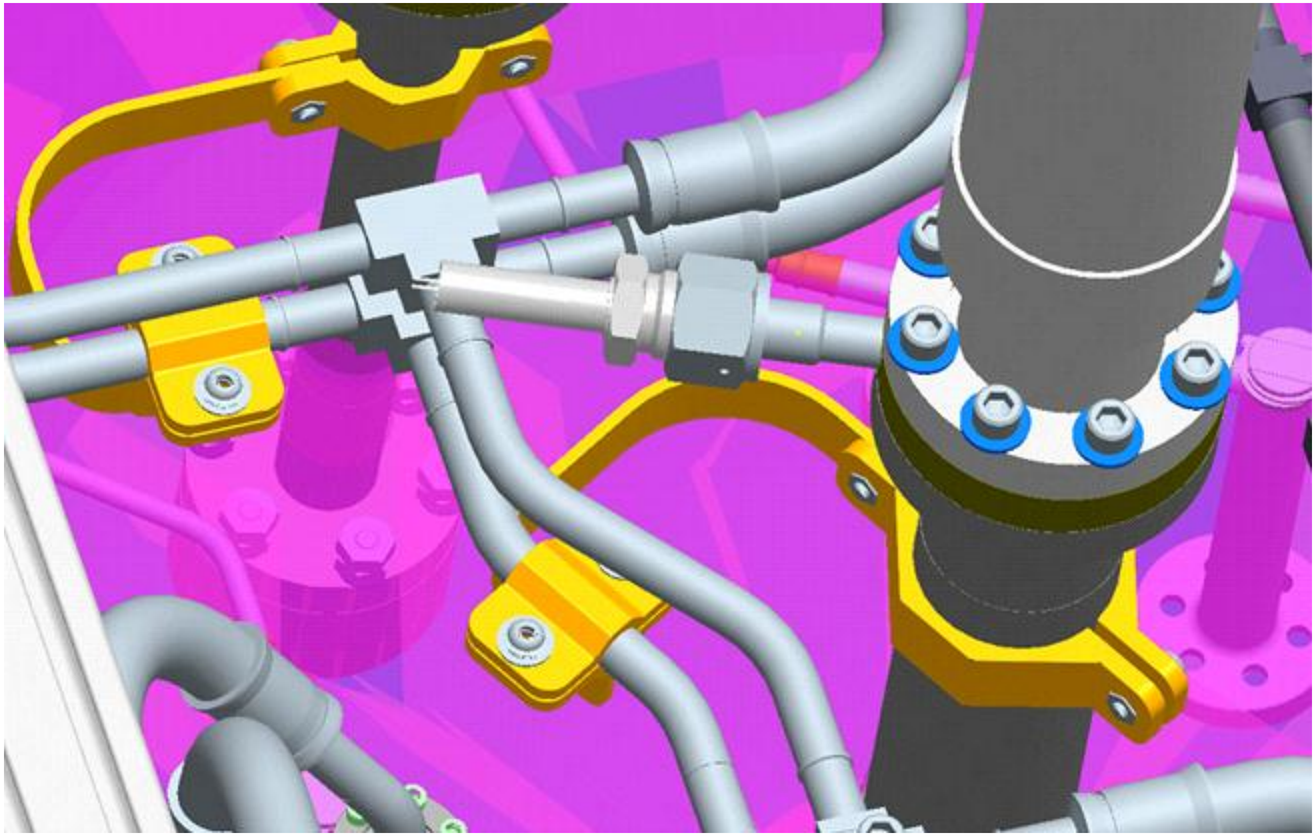
K.F. Weintz et al. "SHOOT Dewar Support Strap Design"

Cooling scheme application: shield mounted

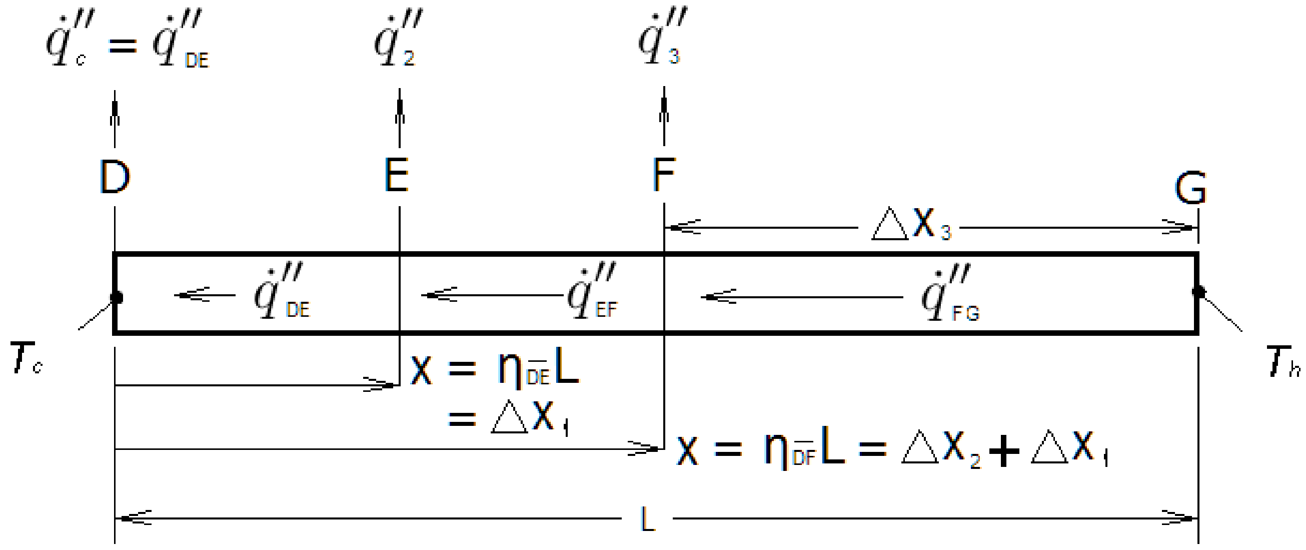


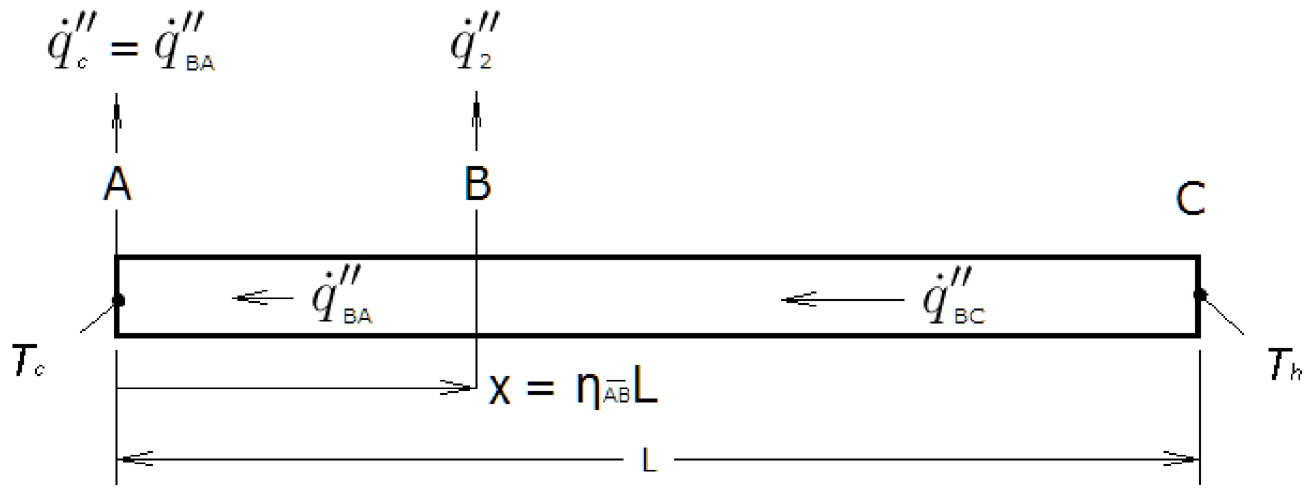
T. Hirokawa et al., "Design of support strap with advanced composite for cryogenic application"

Another cooling scheme: thermal links



Discrete, two stage, perfect heat transfer





Hilal and Boom (1977)

230 M. A. Hilal and R. W. Boom

Table II. Optimum Temperatures, Locations, and Refrigeration Power for Finite Number of Shields

Number of shields	Cycle*	T_1, K	T_2, K	T_3, K	$\Delta x_1/L$	$\Delta x_2/L$	$\Delta x_3/L$	$\Delta x_4/L$	PL/A, W/cm
				304 Stainless steel ($T_c = 4.2$ K)					
1	C	39.7	—	—	0.338	0.662	—	—	445
1	A	39.7	—	—	0.338	0.662	—	—	1781
2	C	21.6	81.7	—	0.189	0.334	0.477	—	316

* C, Carnot cycle efficiencies; A, actual cycle efficiencies.

In my estimate, the second row is actually correct. Because solving the Carnot case gives $T_2 = 40$K and $\eta_{\overline{AB}} = 0.351$ and when I tested a sample real coefficient of performance, I found roughly $T_2 = 39$K and $\eta\overline{AB} = 0.33$. So the first row–the Carnot case–was misprinted. Hilal and Boom actually had solutions up to four stages and also studied the same solutions for Narmco 570 cloth, but that information, being irrelevant, was omitted.

Hilal and Boom (1977)

230 M. A. Hilal and R. W. Boom

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304 Stainless steel ($T_c = 4.2$ K)									
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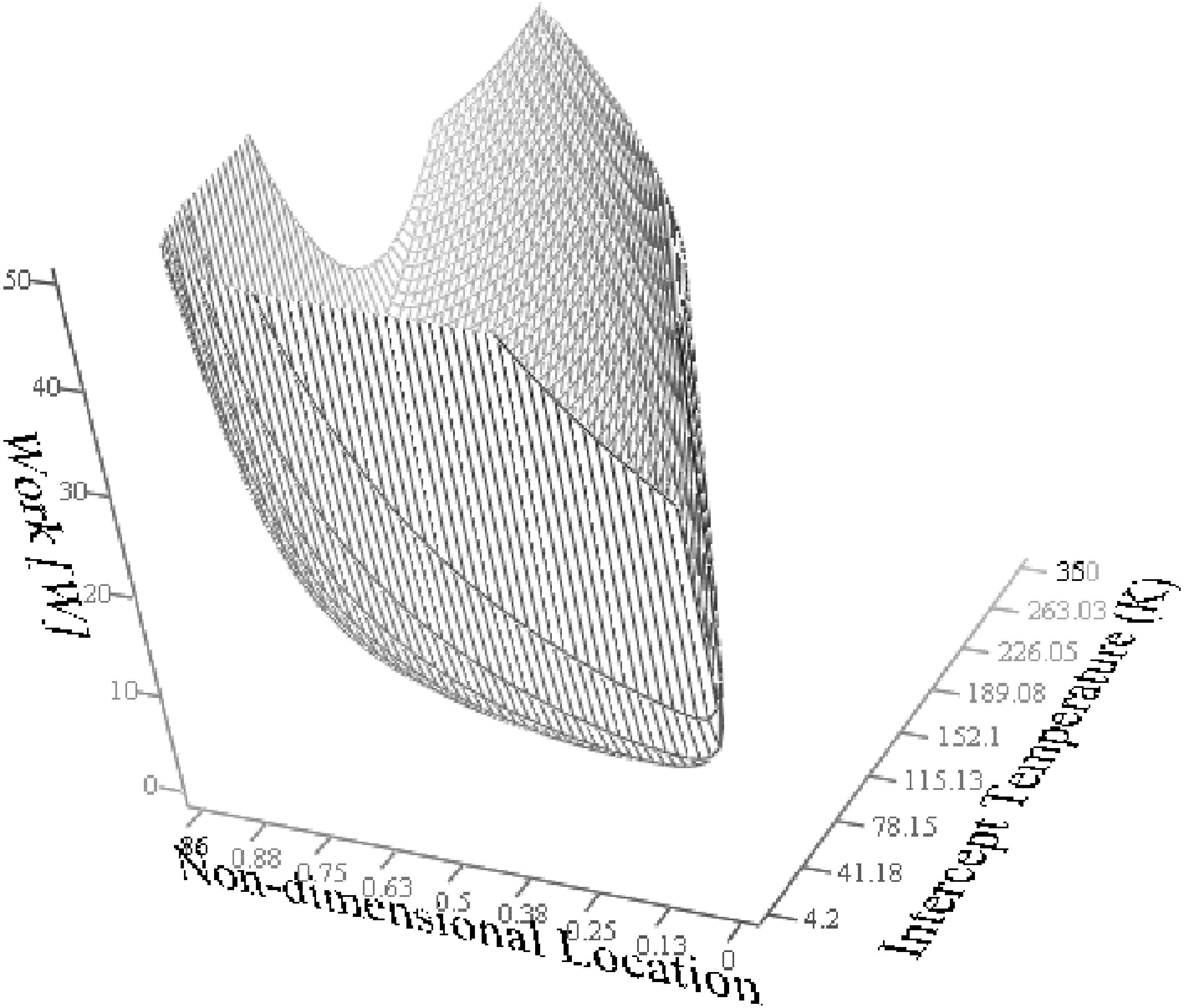
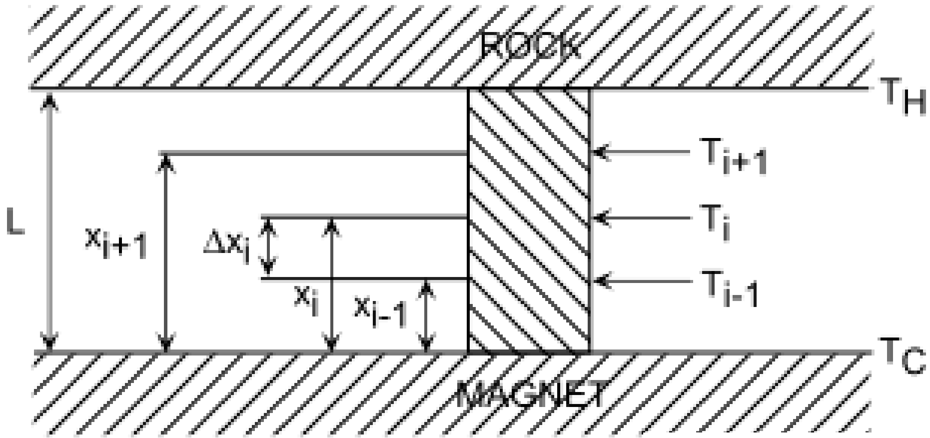
* C, Carnot cycle efficiences; A, actual cycle efficiencies.

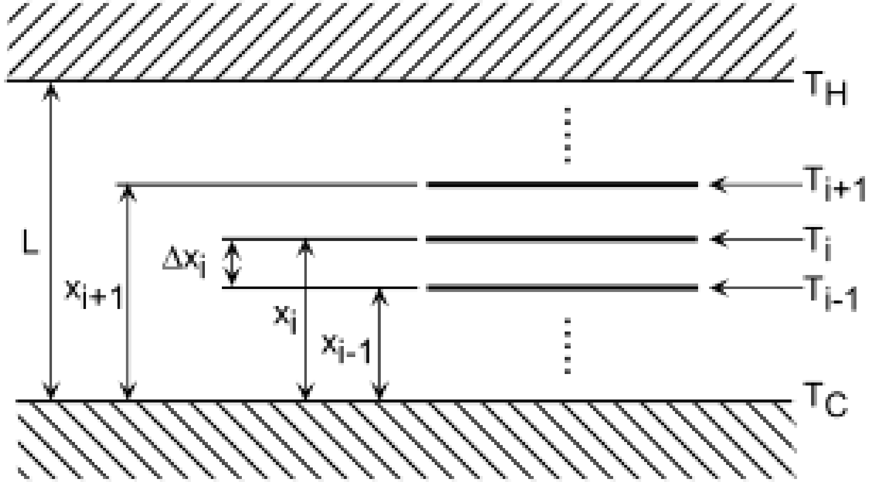
Number of Stages	T_c (K)	T_2 (K)	T_3(K)	η_{AB}	η_{DE}	η_{DF}
2	4.2	40.0	-	0.351	-	-
3	4.2	21.1	80.7	-	0.194	0.524

Carnot, two stage, ideal; graphical solution

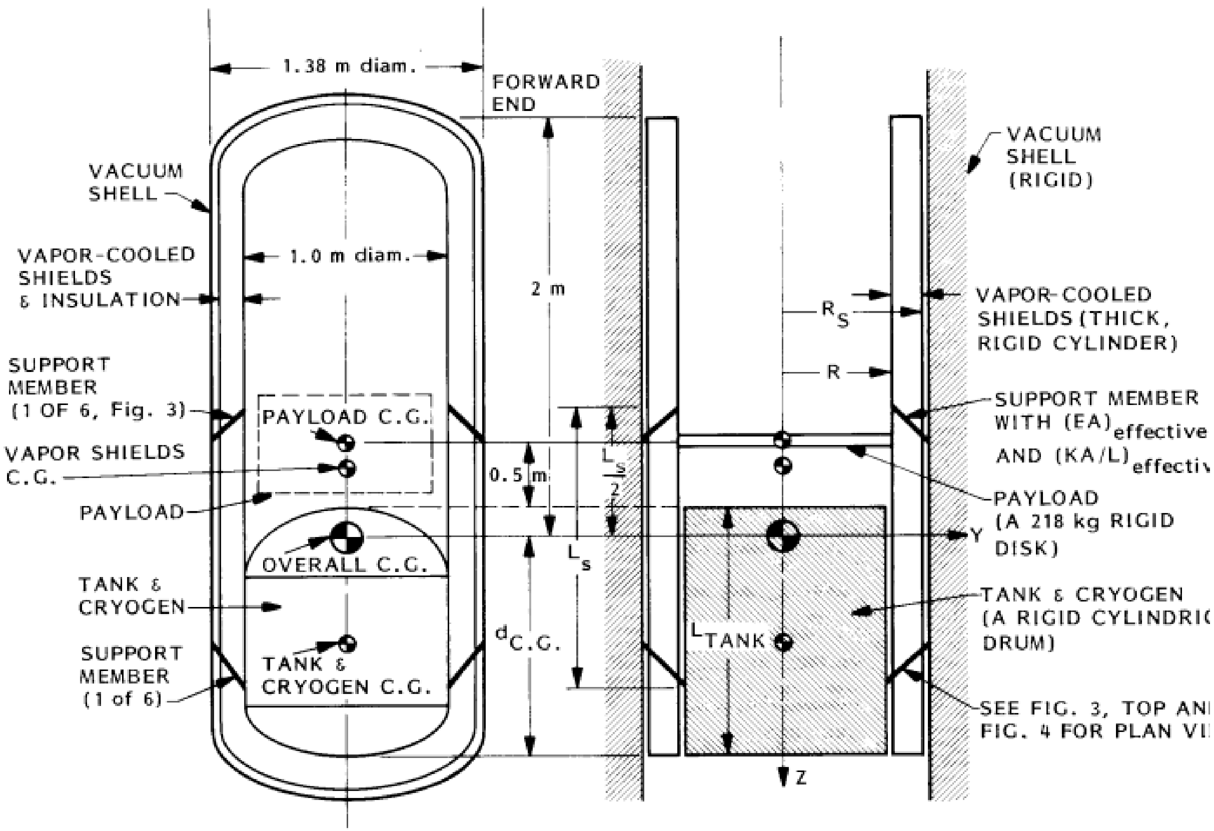


Kokkolaras, Audet, John E. Dennis, Jr. (2001)

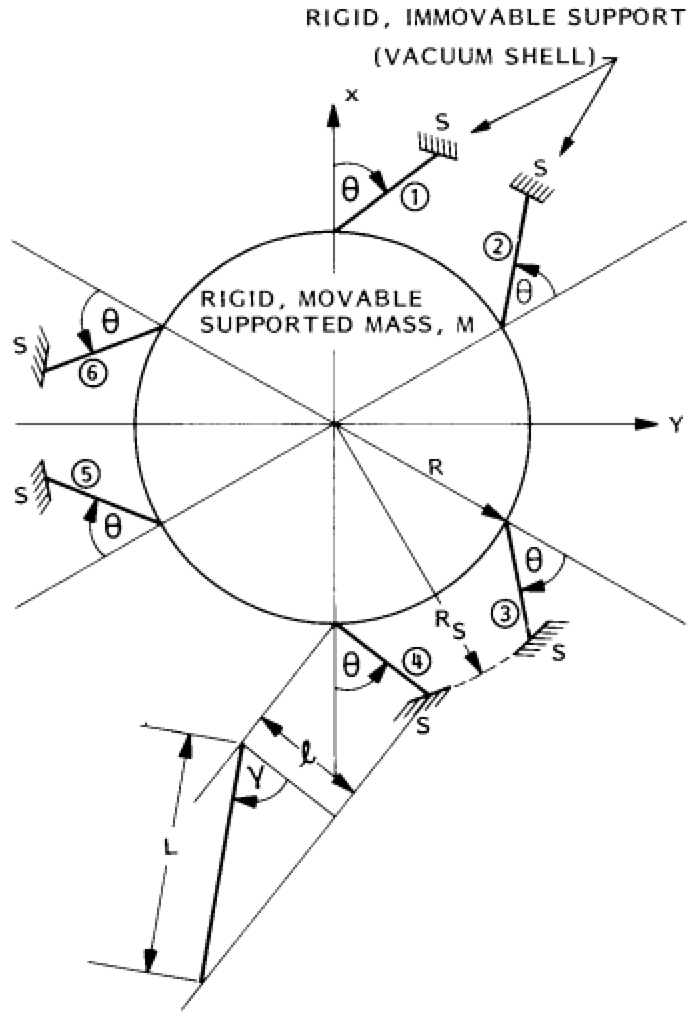


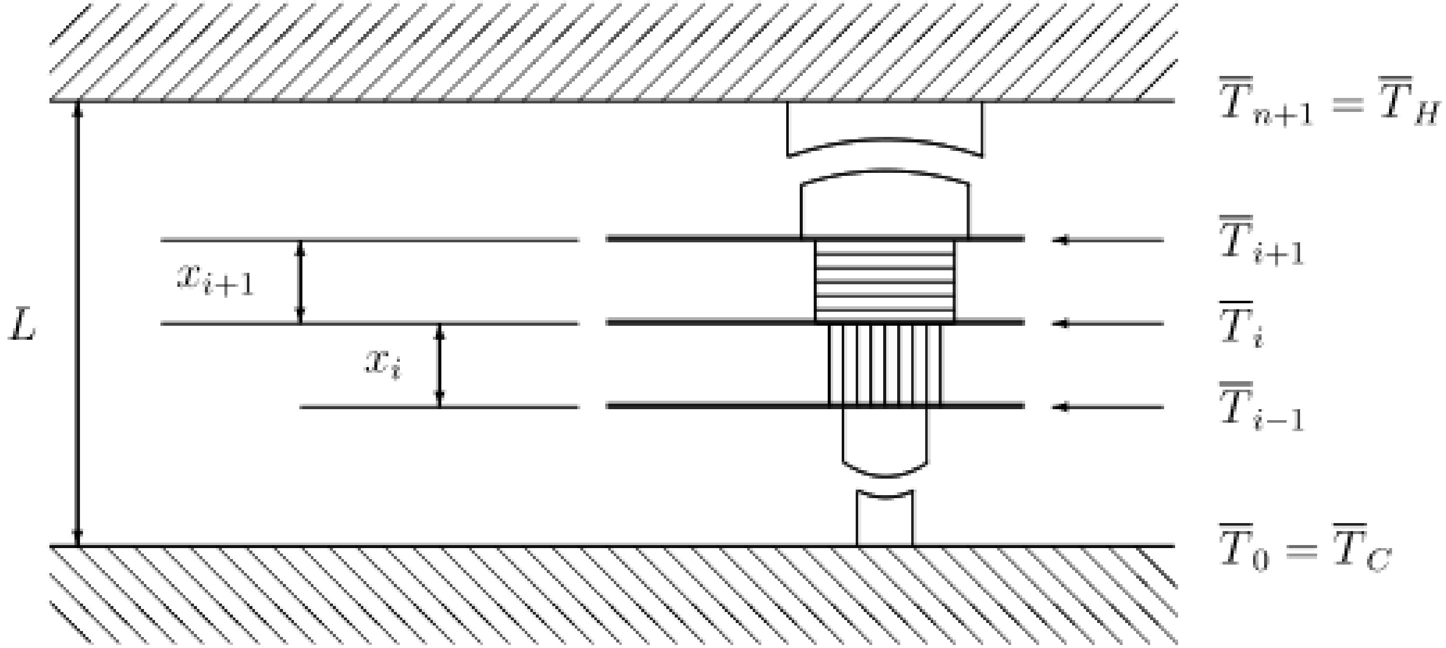


Strut vibration optimization, David Bushnell (1984)



(a) ACTUAL CONFIGURATION (b) IDEALIZED CONFIGURATION





$\overline{T}_{n+1} = \overline{T}_H$
$\overline{T}_{i+1}$
$\overline{T}_i$
$\overline{T}_{i-1}$
$\overline{T}_0 = \overline{T}_C$
x_{i+1}
x_i
L

Vibrations, thermal, thermodynamic, with cooling attachment and radiation

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- All opinions are those of the presenter and not necessarily those of NASA
- Kingsley, Wilson, Kirtley, Keim, Smith, Thullen, "Steady-state electrical tests on the MIT-ERPI 3-MVA Superconducting Generator", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-95, no. 3, May/June 1976

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