Metallic Rotor Sizing and Performance Model for Flywheel Systems

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Abstract

The NASA Glenn Research Center (GRC) is developing flywheel system requirements and designs for terrestrial and spacecraft applications. Several generations of flywheels have been designed and tested at GRC using in-house expertise in motors, magnetic bearings, controls, materials and power electronics. The maturation of a flywheel system from the concept phase to the preliminary design phase is accompanied by maturation of the Integrated Systems Performance model, where estimating relationships are replaced by physics based analytical techniques. The modeling can incorporate results from engineering model testing and emerging detail from the design process.

Introduction

GRC is designing and building a new flywheel system for a terrestrial application. As part of this effort, an integrated performance model of a flywheel system is being developed to evaluate evolving concepts and to provide performance predictions of future flywheel designs. Of the subsystems in the flywheel, the rotor dictates much of the design. For this reason, developing a rotor sizing and performance prediction tool is of the most importance.

Flywheel Design

For terrestrial applications of flywheel systems, maintaining low cost is typically the driver, whereas for space applications, reductions in system mass typically drive the design. Therefore, for the current terrestrial application, a metallic rotor was selected over composite designs to reduce cost. The vertically oriented rotor is supported radially and axially by an active magnetic bearing system. In the event of a magnetic bearing system fault, a set of rolling element mechanical (back-up) bearings are mounted on each shaft end. The entire flywheel system is housed within a vacuum chamber to minimize rotor drag losses.

Rotor Analysis Code Development

The rotor analysis code is written in MATLAB (The Mathworks, Inc., Natick, MA) and organized in modular form, dividing the calculations of the rotor subsystem into more specific categories. The first modular section of the code displays a graphical user interface that allows the user to select a rotor material, specify the required energy and then choose between parametric or point design. A small material database was generated to allow the user to select from a group of potential metallic rotor materials. Material and fatigue properties were taken from Metallic Materials Properties Development and Standardization (MMPDS), sixth edition. The fatigue analysis also references the NASA Rotating Machinery Design Guide.

\textsuperscript{1} Camille J. Moore, NASA Glenn Research Center, Cooperative Education (Co-op) student.
For the flywheel design, the fatigue stress ratio was chosen to be 0.10. This means that the alternating, or minimum, stress of a stress cycle is 10 percent of the maximum stress. This ratio is then used in the fatigue equations given by MMPDS-06:

\[
\log(N) = B(1) + B(2) \cdot \log(\sigma_{eq}) \tag{1}
\]

\[
\sigma_{eq} = \sigma_{max} \cdot (1 - R)^m - B(3) \tag{2}
\]

Where
- \( \sigma_{eq} \) Equivalent Stress, ksi
- \( \sigma_{max} \) Maximum stress, ksi
- \( N \) Fatigue life, number of cycles to failure
- \( m \) optimized constant obtained from MMPDS-06
- \( B \) material constants obtained from MMPDS-06

Solving Equations (1) and (2) for \( \sigma_{max} \) will warrant the maximum allowable stress for the rotor due to fatigue.

Once the material is chosen, the user inputs a rotor inner diameter and outer diameter, or a range of outer diameters for the parametric case. For the selected rotor material, based on the user inputs, the maximum allowable stress for the rotor is calculated by comparing the ultimate and yield stress, with safety factors applied, to the maximum stress over a specified life due to fatigue. After each is calculated, the most conservative stress value is selected as the maximum allowable stress for the rotor. For each outer diameter in the specified range, a maximum allowable rotational speed is calculated (based on allowable maximum stress).

Next, the model determines the necessary length of the rotor for the required energy output of the flywheel. This is calculated using relationships between the maximum allowable stress, angular velocity of the rotor, and required energy output. These relationships are different depending on if the user specifies a solid rotor (inner diameter = 0) versus a hollow rotor (inner diameter > 0). The basic equations for radial and hoop stress for a solid disk are:

\[
\sigma_r = \frac{(3 + v)}{8} \cdot \rho \cdot \omega^2 \cdot (b^2 - r^2) \tag{3}
\]

\[
\sigma_\theta = \frac{(3 + v)}{8} \cdot \rho \cdot \omega^2 \cdot b^2 - \frac{(1 + 3 \cdot v)}{8} \cdot \rho \cdot \omega^2 \cdot r^2 \tag{4}
\]

Where
- \( \sigma_r \) = radial stress, ksi
- \( \sigma_\theta \) = hoop stress, ksi
- \( v \) = Poisson’s ratio
- \( \rho \) = material density, lbm/in.\(^3\)
- \( \omega \) = angular velocity, rad/s
- \( b \) = maximum outer radius of rotor, in.
- \( r \) = radius at which the stress is to be determined, in.
Because the maximum radial and hoop stress occurs at the center of the disk,

\[ \sigma_{r\text{MAX}} = \sigma_{0\text{MAX}} = \frac{(3 + \nu)}{8} \cdot \rho \cdot \omega^2 \cdot b^2 \]  \hspace{1cm} (5)

Since the maximum allowable stress has already been determined based on material properties, the Equation (5) is solved for angular velocity, yielding the maximum rate at which the flywheel rotor can spin.

\[ \omega_{\text{MAX}} = \frac{1}{b} \sqrt{\frac{8 \cdot \sigma_{0\text{MAX}}}{\rho \cdot (\nu + 3)}} \]  \hspace{1cm} (6)

From this, the maximum tip speed is calculated by multiplying Equation (6) by the rotor radius and is then used to determine the required length of the rotor.

The required energy output of the flywheel dictates the length of the rotor. For the purposes of this parametric flywheel model, any energy inefficiencies are calculated externally. This is because the model only takes into account the mass of the metallic rotor and not that of the motor laminates or other subsystems. As the Integrated Systems Performance Model is expanded, other energy inefficiencies can be introduced by each additional subsystem. The energy of a rotating body is defined as:

\[ E = \frac{1}{2} \cdot I \cdot \omega^2 \]  \hspace{1cm} (7)

Where

\[ I = \text{Moment of inertia about the rotating axis, in}^4 \]

By substituting equations for the moment of inertia (Eq. (8)) and mass of the rotor (Eq. (9)), (Eq. (7)) becomes (Eq. (10)):

\[ I = \frac{1}{2} \cdot m \cdot b^2 \]  \hspace{1cm} (8)

\[ m = \rho \cdot \pi \cdot b^2 \cdot L \]  \hspace{1cm} (9)

\[ E = \frac{1}{2} \left[ \frac{1}{2} \left( \rho \cdot \pi \cdot b^2 \cdot L \right) \cdot b^2 \right] \cdot \omega^2 \]  \hspace{1cm} (10)

Where

\[ m = \text{mass of the metallic rotor, lbm} \]
\[ L = \text{length of the rotor, in.} \]

Then solving for length and using required energy and maximum angular velocity yields the required rotor length:

\[ L_{\text{req}} = \frac{4 \cdot E_{\text{req}}}{\pi \cdot b^4 \cdot \rho \cdot \omega_{\text{MAX}}^2} \]  \hspace{1cm} (11)

If the user wishes to design a hollow rotor then Equations (6) and (11) become:
\[ \omega_{\text{MAX}} = \sqrt{\frac{\sigma_{\text{0MAX}}}{\rho \cdot \left( \frac{(v+3)}{4} \right) \cdot \left( b^2 + \frac{(4-v)}{3+v} \cdot a^2 \right)}} \]  

(12)

\[ L_{\text{req}} = \frac{4 \cdot E_{\text{req}}}{\left( \pi \cdot a^4 \cdot \rho \cdot \omega_{\text{MAX}}^2 \right) - \left( \pi \cdot b^4 \cdot \rho \cdot \omega_{\text{MAX}}^2 \right)} \]  

(13)

Where

\[ a = \text{inner diameter, in.} \]

The process outlined above for solid rotors calculations is the same for hollow rotors.

**Graphical User Interfaces**

An important aspect of the parametric model is ease of use. The model is being designed to be able to be run by personnel inexperienced with stress analysis, as well as provide customizable control for an experienced design engineer. To accommodate this need, several graphical user interfaces (GUIs) were developed. This allows for quick changes to governing variables as well as detailed control over the model. Designed using the MATLAB Guide feature, the main GUI (shown in Fig. 1) is executed like any other m-file from the MATLAB command window. In the main GUI, the user controls the entire model. The model can be used in parametric analysis over a specified range of outer diameters, or for a point design. This is regulated by two mutually exclusive radio buttons found in the upper right hand corner. By employing a drop-down menu, the user can select a material to use for the rotor from an established materials database. This materials database was created using data from the most recent version of the Metallic Materials Properties Development and Standardization (MMPDS-06). Within the database a more detailed description can be found for how to locate the data within MMPDS-06. If the user wishes to use a material not included in the database the GUI provides the option to manually input material properties by clicking on “Material Input.” Doing so opens another GUI, as shown in Figure 2.

If the user selects a material from the drop-down menu the “Input Material” button on Figure 1 changes to read “View Properties.” Figure 2 will then display the material properties, of the selected material, from the database.

The main GUI also features a button that when clicked, reveals information on how to change the fatigue properties of a materials. This was included because a design engineer may want to have more control over the equations, from MMPDS-06, used for fatigue analysis.

Another feature of the main GUI is the ability to include cost analysis. When the GUI is opened, the cost analysis section default is to be disabled. Because cost of materials is constantly changing with the market, it is important for cost analysis to be easily controlled by the user. Enabling cost analysis will warrant a basic assessment of the material cost of the rotor; this does not include manufacturing or shipping costs. Also incorporated into the main GUI is a series of pop-up warnings to alert the user if a user has not entered proper values into each field.
Figure 1.—Flywheel rotor design analysis.

Figure 2.—Input material properties.
Sample Results

Depending on whether the user has selected parametric or point design, the results of the model are displayed in different forms. If the user specifies parametric design, the results are shown in multiple graphs (Fig. 3), plotted against the range of specified outer diameters. The plots are rotor length, rotational speed, cost per joule and inertia ratio. These are results that a design engineer would be most interested in. Important for rotor dynamic analysis, the inertia ratio is a design check related to the dynamic control of the rotor. If the user specifies point design, individual results are displayed in a pop-up GUI, as shown in Figure 4. A stress profile plot (Fig. 5), is also created for the point design case to highlight the stress distribution over the entire radius of the rotor.
Figure 4.—Results.

Figure 5.—Stress plot.
Conclusions

With this parametric rotor design tool, the GRC Flywheel project will be able to quickly perform rotor sizing studies, for metallic flywheels. It is important for the project to assess the performance of new concepts in a timely manner. This model will be used as stepping stone for further development of an Integrated Systems Performance model for the GRC Flywheel system. Ultimately, a flywheel Integrated Systems Performance model could be used to size flywheels for both terrestrial and spacecraft applications.

References

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