Fractal-based lightning channel length estimation from convex-hull flash areas for DC3 Lightning Mapping Array Data

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Motivation
Use VHF Lightning Mapping Array data to estimate NOx per flash and per unit channel length, including the vertical distribution of channel length. What’s the best way to find channel length from VHF sources? 

Challenges
• Channel is fractal. Infinite length.
• LMA detection variability with range, individual network noise floor, number of stations, etc.

Strategy
Investigate geometric properties of lightning flashes to infer length estimates, while also seeking to discover fundamental characteristics of the channel geometry.

In the adjacent poster (Thomas et al., a comparison of three methods of calculating flash length is presented: - connect-the-dots - box-covering - fractal estimate

This paper presents the rationale for the fractal method, which is closely related to the box-covering method.

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• Use box-counting principles to estimate flash length from flash area

A power-law relationship between count and box size indicates the dimension D of an object, including objects with fractal dimension (Theiler, 1990, J. Optical Soc. Am.)

Use D to estimate flash length from flash area

Use box-counting principles

• - Flash-spanning A_i is largest box
• - Area of the convex hull is the minimum convex measure
• - Channel step length L_i is smallest sensible box

Lower bound to box size prevents length from growing infinitely, i.e., only part of the flash obeys a scaling law.

Fractal length estimate L_i is:

L_i = N_i L_0 = b_i \left( \frac{\sqrt{N_i}}{b_0} \right)^D = \left( \frac{b_i}{b_0} \right)^D

where N_i is number of boxes of size b_i

The method above is easily extended into three dimensions by using the cube root of the volume of 3D convex hull.

Fractal length from flash area and volume

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Vertical distribution of length

To determine the channel length distribution with height, partition the whole-flash total length estimate as follows:

- The convex hull volume V_H is calculated from an underlying Delaunay triangulation
- Use the Delaunay sub-volumes V_i (tetrahedra) that give the natural-neighbor connectivity between VHF sources
- The local estimate is a volume-weighted partitioning of the global flash length estimate

S_i = \sqrt{V_i}

Simple length from sub-volume Predicted fractal length from sub-volume

Hull volume acts as large box size

Sub-volume V_i acts as local b_i

Estimating the total length using this method, we find that the vertical length partitioning is nearly identical to the VHF source histogram

Application of this method and the addition of ICGC discrimination is planned as part of a delivery of flash-level analysis products

Summary

Two-step approach accounts for the intrinsic geometry of a lightning flash

- Global per-flash length estimation from D_i, V_i, b_i
- Nearly independent of detection efficiency (only needs accurate volume)
- Appropriate for whole-storm estimates
- Local flash length weighted by local natural-neighbor volume
- Appropriate for vertical or horizontal distributions of flash length

With tuning, various flash length estimation methods give comparable estimates

- There are highly sensitive to changes in global parameters, as expected for a fractal object
- Requires input from lightning physics and chemistry specialists to inform physically-based choices for D_i, V_i, b_i

Future work

Incorporate polarity-specific D

Positive and more brush-like => larger D

Account for step length variation with altitude

200 m at 10 km, smaller at lower altitude


See also Wirth et al. (2011) and Klages and Bross (1990)

Does the 3D spatial geometry and additional information in the time coordinate relate to the (re)distribution of electrostatic potential in some way? Can the flash area product (top right) be explored in this role?

Preliminary storm-total-length total estimate

We have conducted a preliminary fractal length estimate using flashes identified in the gridding process for the time period shown above. For the cell near the center network, a preliminary fractal total length of 16249 m can be obtained using D = 1.5, b_i = 130 m.

With a significant caveat about the robustness of the flash-matching process between datasets, the LNOM-estimated total length for this storm was estimated at 16512 m. (Preliminary data courtesy Koshak and Peterson.)

Other D, b_i combinations are possible, but this preliminary analysis shows the possibility of correspondence between the LNOM and fractal methodologies.

DC3 gridded flash products

Example of preliminary data above is for the hour beginning 2300 UTC on 21 May 2012 in North Alabama, and has also been produced for West Texas using this method.

Grids are defined with constant lat-lon increments equal to 3km at the network center location. Grids are produced for each five minute window in the hour. Flash sorting is the McCaul et al. (2009) method, with 0.15 s and 3 km VHF source spacing thresholds and 10 point minimum per valid flash. VHF source criteria are chi-squared less than 5.0 and at least six contributing stations.

Code (Python) to produce these grids may be found in a public repository at http://bitbucket.org/deeplycloudy/lmatools

Below, comparison of TTU gridding with an independent flash sorting and gridding implementation in IDL by Thomas. Data from Colorado on 22 June 2012.