Visible Contrast Energy Metrics for Detection and Discrimination

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Energy Metric for Detection

- Inputs: Luminance image = L(x,y)
  Pixel area = dx dy in deg \(^2\),
  Duration = dt in sec
- Compute visible contrast image = Cv(x,y)
- Visible Energy Metric :
  \[ Ev = dx \, dy \, dt \sum_{x,y} Cv(x,y)^2 \, \text{deg}^2 \, \text{sec} \]
  \[ dBV = 10 \log_{10} \left( \frac{Ev}{10^{-6}} \right) \]
- Modelfest average threshold = 7 \(\pm\) 2 dBV
Visible Contrast Image

- Optic Blur: \( \text{Lo}(x,y) = \text{L}(x,y) \times \text{O}(x,y) \)
- Background Luminance:
  \( \text{Lb}(x,y) = a(dt) \times (\text{Lo}(x,y) \times \text{B}(x,y)) + (1 - a(dt)) \times \text{B}_0 \)

- Contrast: \( \text{C}(x,y) = \frac{\text{Lo}(x,y) - \text{Lb}(x,y)}{\text{Lb}(x,y)} \)
- Eccentricity Sensitivity:
  \( \text{Cv}(x,y) = \text{C}(x,y) \times \text{S}(x,y) \)
Parameters

- Optic Blur: \( F(O(x,y)) = \exp(-f/f_0), \)
  \[ f = \sqrt{(f_x^2 + f_y^2)}, \quad f_0 = 12 \text{ cpd} \]

- Background Luminance:
  \( F(B(x,y)) = \exp(- (f/f_1)^2), \quad f_1 = 2 \text{ cpd} \)
  \[ a(dt) = \exp(-dt/t_0), \quad t_0 = 0.4 \text{ sec} \]

- Eccentricity Sensitivity:
  \[ S(x,y) = 1/(1 + g (1 - \exp(-r/r_0))), \]
  \[ r = \sqrt{x^2 + y^2}, \quad r_0 = 5.7 \text{ deg}, \]
  \[ g = 4.1, \quad 1/(1 + g) = 0.2 \]
Metric-Validating Model

- Visibility Image: \( C_v(x,y) \)
- Additive White Noise with 2-sided power spectral density
  \[ N = \sigma^2 \, dx \, dy \, dt , \]
  Each pixel is independently distributed as
  Normal with mean 0 and standard deviation \( \sigma \)
- Ideal Observer detects presence or absence of signal in a two interval forced choice (2IFC) experiment.
2IFC Model Performance

• Visibility Image: \( C_v(x,y) \) with visible contrast energy \( E_v \) and noise spectral density \( N \)

• Distance between observer output distributions divided by their common standard deviation is

\[
d' = \sqrt{\frac{2 \ E_v}{N}}
\]

• Prob(Correct) = \( P_c = F_z(d') - 0.5 \)

• Estimated \( N = \frac{2 \ E_v}{d'^2} \)

• If \( P_c = 0.84, \ d' = 1, \ N = 2 \ E_v \)

• Modelfest : \( 10 \ \log_{10}(N) + 60 = 10 \pm 2 \ dB \)
Discrimination Model

- Visibility Images: $C_v(x,y,j), j = 1,M$
- Additive White Noise with power spectral density $N = \sigma^2 \, dx \, dy \, dt$
- Ideal Observer responds $k$ if image $j$ is presented and image $k$ has the smallest squared distance $d(k)$ to the noisy image

$$d(k) = \| C_v(j) + N - C_v(k) \|^2$$

$$d(k) = \| C_v(j) + N\|^2 + \| C_v(k) \|^2 - 2 (C_v(j) \cdot C_v(k) + N \cdot C_v(k))$$
Discrimination Model Metric

- Orthogonal Images: \( C_v(j) \cdot C_v(k) = 0, \ j \neq k \)
- Same energy: \( E_v(j) = E_v \)
- Let \( d' = \sqrt{\frac{E_v}{N}} \)
  \[
P_c = \int F(x)^{k-1} f(x-d') \, dx ,
\]
  where \( F \) and \( f \) are the cumulative and density distribution functions of the standard normal.
- Also \( E_v = \int \int \int \left( \sum_j ||C_v(j) - C||^2 / (M-1) \right) \)
  where \( C = \sum_j C_v(j) / M \)
Discrimination Model Performance

\[ d = \left( \frac{E}{N} \right)^{0.5} \]

- \( M = 2 \)
- \( M = 4 \)
- \( M = 10 \)
- \( M = 26 \)
Pedestal invariance of ideal observer allows the orthogonal stimulus model.
Example: Tumbling E's

- Model simulation for $n = 10000$ trials, $d' = 1$.  
  95% confidence interval for $P_c = 0.538 \pm 0.010$
- Metric prediction $P_c = 0.552$
Method Considerations

- When pattern energies are similar, varying the contrast adds little or no uncertainty; varying size or blur contributes significant uncertainty.

- Practice of computing thresholds by averaging reversal endpoints has problems
  1) $P_c$ at threshold is not actually known
  2) No estimate of the slope at threshold is provided
  3) Valuable data is effectively discarded
Summary

- Detection metric: Visible contrast energy
- Approximate Discrimination metric: Average \((M-1)\) squared distance from each visible contrast pattern to the mean visible contrast pattern
- Model simulation is fast
c = s'*s ; \% 4x25 times 25x4
[u , x, v] = svd(c) ;
f = u*(x.^0.5) ;

sn =
ones(n,4)*c(1,1:4)+randn(n,4)*f';
Pc=mean(
    sn(1:n,1)>max(sn(1:n,2:4)')'
);
Watson & Ahumada (2005)
Metric Elements

Figure 4. Elements of the component model.