Visible Contrast Energy Metrics for Detection and Discrimination

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Energy Metric for Detection

- Inputs: Luminance image = \( L(x,y) \)
  
  Pixel area = \( dx \ dy \) in deg \(^2\),
  
  Duration = \( dt \) in sec

- Compute visible contrast image = \( C_v(x,y) \)

- Visible Energy Metric :
  
  \[ Ev = dx \ dy \ dt \sum_{x,y} C_v(x,y)^2 \text{ deg}^2 \text{ sec} \]

  \[ dBV = 10 \log_{10}( \frac{Ev}{10^{-6}} ) \]

- Modelfest average threshold = 7 \( \pm \) 2 dBV
Visible Contrast Image

- Optic Blur: \( L_o(x,y) = L(x,y) \times O(x,y) \)
- Background Luminance:
  \( L_b(x,y) = a(dt) \times (L_o(x,y) \times B(x,y)) + (1 - a(dt)) \times B_0 \)

- Contrast: \( C(x,y) = \frac{L_o(x,y) - L_b(x,y)}{L_b(x,y)} \)
- Eccentricity Sensitivity:
  \( C_v(x,y) = C(x,y) \times S(x,y) \)
Parameters

- **Optic Blur**: \( F(O(x,y)) = \exp(- f / f_0) \),
  \[ f = \sqrt{(f_x^2 + f_y^2)}, \quad f_0 = 12 \text{ cpd} \]

- **Background Luminance**: \( F(B(x,y)) = \exp(- (f / f_1)^2), \quad f_1 = 2 \text{ cpd} \)
  \[ a(dt) = \exp(- dt / t_0), \quad t_0 = 0.4 \text{ sec} \]

- **Eccentricity Sensitivity**: \( S(x,y) = 1/(1 + g (1 - \exp(- r / r_0))) \),
  \[ r = \sqrt{x^2 + y^2}, \quad r_0 = 5.7 \text{ deg}, \quad g = 4.1, \quad 1 / (1 + g) = 0.2 \]
Metric-Validating Model

- Visibility Image: \( C_v(x,y) \)
- Additive White Noise with 2-sided power spectral density

\[
N = \sigma^2 \, dx \, dy \, dt ,
\]

Each pixel is independently distributed as

Normal with mean 0 and standard deviation \( \sigma \)

- Ideal Observer detects presence or absence of signal in a two interval forced choice (2IFC) experiment.
2IFC Model Performance

- Visibility Image: \( \text{Cv}(x,y) \) with visible contrast energy \( \text{Ev} \) and noise spectral density \( \text{N} \)
- Distance between observer output distributions divided by their common standard deviation is
  \[
  d' = \sqrt{\frac{2 \ \text{Ev}}{\text{N}}}
  \]
- \( \text{Prob(Correct)} = \text{Pc} = Fz(d') - 0.5 \)
- Estimated \( \text{N} = \frac{2 \ \text{Ev}}{d'^2} \)
- If \( \text{Pc} = 0.84, \ d' = 1, \ \text{N} = 2 \ \text{Ev} \)
- Modelfest: \( 10 \ \log_{10}(\text{N}) + 60 = 10 \pm 2 \ \text{dB} \)
Discrimination Model

- Visibility Images: $C_v(x,y,j)$, $j = 1,M$
- Additive White Noise with power spectral density
  \[ N = \sigma^2 \, dx \, dy \, dt \]
- Ideal Observer responds $k$ if image $j$ is presented and image $k$ has the smallest squared distance $d(k)$ to the noisy image
  \[ d(k) = \| C_v(j) + N - C_v(k) \|^2 \]
  \[ d(k) = \| C_v(j) + N \|^2 + \| C_v(k) \|^2 - 2 \left( C_v(j) \cdot C_v(k) + N \cdot C_v(k) \right) \]
Discrimination Model Metric

- Orthogonal Images: \( Cv(j) \cdot Cv(k) = 0, \ j \neq k \)
- Same energy: \( Ev(j) = Ev \)
- Let \( d' = \sqrt{\frac{Ev}{N}} \)

\[
Pc = \int F(x)^{k-1} f(x-d') \, dx ,
\]
where \( F \) and \( f \) are the cumulative and density distribution functions of the standard normal.

- Also \( Ev = \int dx \int dy \int dt \sum_j ||Cv(j) - C||^2 / (M-1) \)

where \( C = \sum_j Cv(j) / M \)
Discrimination Model Performance

$$d = \left( \frac{E_v}{N} \right)^{0.5}$$

$P_{\text{Prob Correct}}$ is plotted against $d$ for different values of $M$: $M = 2$, $M = 4$, $M = 10$, and $M = 26$. The graph shows the probability of correct discrimination as a function of $d$. The higher the value of $M$, the higher the probability of correct discrimination for a given $d$. The horizontal axis represents $d$, and the vertical axis represents the probability of correct discrimination.
Example: Landolt C

Pedestal invariance of ideal observer allows the orthogonal stimulus model.
Example: Tumbling E's

- Model simulation for $n = 10000$ trials, $d' = 1$.
  - 95% confidence interval for $P_c = 0.538 \pm 0.010$
- Metric prediction $P_c = 0.552$
Method Considerations

- When pattern energies are similar, varying the contrast adds little or no uncertainty; varying size or blur contributes significant uncertainty.

- Practice of computing thresholds by averaging reversal endpoints has problems
  1) $P_c$ at threshold is not actually known
  2) No estimate of the slope at threshold is provided
  3) Valuable data is effectively discarded
Summary

- Detection metric: Visible contrast energy
- Approximate Discrimination metric:
  Average (M-1) squared distance from each visible contrast pattern to the mean visible contrast pattern
- Model simulation is fast
Tumbling E Model Matlab Code

c = s'*s ; \% 4x25 times 25x4
[u , x, v] = svd(c) ;
f = u*(x.^0.5) ;

sn =
ones(n,4)*c(1,1:4)+randn(n,4)*f';
Pc=mean(
    sn(1:n,1)>max(sn(1:n,2:4)')'
    );
Figure 4. Elements of the component model.