Spectral attenuation of sound in dilute suspensions
with nonlinear particle relaxation

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Abstract: Previous studies on the sound attenuation in particle-laden flows under Stokesian drag and conduction-controlled heat transfer have been extended to accommodate the nonlinear drag and heat transfer. It has been shown that for large particle-to-fluid density ratio, the particle Reynolds number bears a cubic relationship with \( \omega r_d \) (where \( \omega \) is the circular frequency and \( r_d \) the Stokesian particle relaxation time). This dependence leads to the existence of a peak value in the linear absorption coefficient occurring at a finite value of \( \omega r_d \). Comparison of the predictions with the test data for the spectral attenuation of sound with water injection in a perfectly expanded supersonic air jet shows a satisfactory trend of the theory accounting for nonlinear particle relaxation processes.

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1. Introduction

Sound attenuation in fluids, representing the dissipation of acoustic energy from a sound wave, occurs through a number of physical processes involving molecular viscosity, thermal conductivity, and other dissipative or relaxation processes.\(^1\)\(^-\)\(^4\) In all these absorption mechanisms, acoustic energy is converted into thermal energy, and other mechanisms deflect or scatter
acoustic energy. Stokes developed the first successful theory of sound absorption due to the effect of molecular viscosity of the fluid (internal friction). Helmholtz theoretically investigated the effect of viscosity on the sound attenuation in a circular tube, considering the effect of friction near solid boundaries. Kirchhoff theoretically treated sound attenuation due to the viscosity and heat conduction effects in an unbounded fluid medium and in narrow tubes. The so-called classical sound absorption in fluids includes Stokes’ viscous contribution and Kirchhoff’s thermal conduction contribution to attenuation in unbounded fluids.

When a fluid contains inhomogeneities such as suspended particles (solid particles, drops and bubbles) additional viscous and heat conduction losses occur in the immediate neighbourhood of the suspended particles. A comprehensive review of the physics and scientific history of acoustic interactions with particulate mixtures is provide by Challis et al. Referring to Fig. 1, the acoustic intensity $I$ of a plane wave propagating through an absorbing medium is expressed by

$$I = I_0 e^{-\alpha x}$$

where $x$ is the distance traversed, $I_0$ the intensity at $x = 0$, and $\alpha$, the intensity attenuation coefficient for the medium. The quantity $\alpha$, depends on viscosity, thermal conductivity, and other factors such as molecular relaxation.

Sound propagation in aerosols and fog has been studied experimentally and theoretically by several investigators. Sewell, in his pioneering work, theoretically considered the attenuation of sound in a viscous medium containing suspended cylindrical or spherical particles (obstacles) with perfectly rough surfaces. With the aid of a scattering formulation (theory), Sewell predicted the attenuation of sound by rigid particles suspended in a gas, assuming that the particles are immovable. Epstein, in a theoretical treatment of the attenuation of sound by spherical particles suspended in liquids or gases, derived and extended Sewell’s result by permitting the particles to
move. Epstein and Carhart\textsuperscript{12} additionally considered heat conduction effects, and found that for fogs the effects of viscosity and heat conduction are both important, and approximately additive. Their formulation was extended by Allegra and Hawley\textsuperscript{13} to include liquid-liquid as well as liquid-solid systems (with a different model for each case).

The effect of transport processes on the attenuation and dispersion of sound in aerosols have also been reported by Soo,\textsuperscript{14} accounting for nonstationary effects including the Basset history (effect of particle acceleration on viscous drag) and added mass terms. Basset history and added mass terms were included in an elegant coupled phase formulation by Harker and Temple.\textsuperscript{15} Coupled phase effects were also treated by Evans and Attenborough\textsuperscript{16,17} in an extension to the work of Harker and Temple\textsuperscript{15} to incorporate thermal conduction. Nonstationary effects, which become important at low particle Reynolds numbers, are further investigated by Gumerov\textsuperscript{18}. Chow\textsuperscript{19} considered the attenuation of sound in dilute emulsions and suspensions with viscous dissipation and thermal conduction, and additionally included the effect of surface tension, which was shown to be important in the presence of bubbles (and negligible for systems with solid particles or liquid droplets).

Temkin and Dobbins,\textsuperscript{20} in their classical work, theoretically considered particle attenuation and dispersion of sound in a manner which illustrates explicitly the relaxation character of the problem. For rigid particles, the linear droplet drag and heat transfer are respectively obtained from

\begin{align}
F_p &= 6\pi \mu g \left( u_p' - u_g' \right) \quad \text{(2a)} \\
Q_p &= 2 \pi d_p k_g \left( T_p' - T_g' \right) \quad \text{(2b)}
\end{align}

which correspond to the zero droplet Reynolds number limit ($\text{Re}_p \to 0$), where

\begin{equation}
\text{Re}_p = \frac{\rho_g \left| u_g - u_p \right| d_p}{\mu_g} \quad \text{(2c)}
\end{equation}
In the above, \( u \) is the velocity, \( T \) the temperature, \( \rho \) the density, \( \mu \) the dynamic viscosity, \( k \) the thermal conductivity, and \( d \) the particle diameter. The subscripts \( g \) and \( p \) stand respectively for the gas and the particle, and the primes denote fluctuations from the mean. The properties \( \rho_g, \mu_g, k_g \) and \( \rho_p \) refer to the mean values. According to Temkin and Dobbins,\(^{20} \) Stokes linear drag law can be justified for \( 0 \leq \omega \tau_d \approx 1 \), provided that \( \rho_g / \rho_p \ll 1 \) and \( \left( \frac{\alpha d^2}{8 \nu_g} \right)^{1/2} \ll 1 \).

Recent data\(^{21} \) on sound attenuation in supersonic air jets containing suspended water droplets reveal that the linear absorption coefficient displays a spectral peak. The particulate relaxation models for the sound attenuation are all based on Stokes drag (linear drag law) and pure conduction limit (linear heat transfer), and do not account or explain this attenuation behaviour. This article, primarily based on Ref. 22, attempts to investigate this attenuation behaviour by considering nonlinear drag and heat transfer laws applicable to relatively large-sized droplets.

2. Analysis for sound attenuation

The present analysis extends the work of Temkin and Dobbins\(^{20} \) for dilute suspensions to accommodate the nonlinear drag and heat transfer laws, which become important at high particle Reynolds numbers \( \Re_p \) and at high frequencies. Only sound attenuation is considered here, with sound dispersion excluded from consideration.

2.1 Nonlinear particle drag and heat transfer

The theory of Temkin and Dobbins\(^{20} \) or dilute solutions is applicable at low mass concentrations \( C_m = n_0 m_p / \rho_g \ll 1 \), where \( n_0 \) refers to the mean number of particles per unit volume of mixture, and \( m_p \) the mass of one particle. Without any loss of generality the
attenuation of sound for large particle Reynolds numbers with nonlinear particle relaxation may be expressed as follows:

$$
\bar{\alpha} = \left( \frac{c_0 \alpha}{C_m \omega} \right) = \frac{\omega \tau_{d1}}{1 + \omega^2 \tau_{d1}^2} + \left( \gamma - 1 \right) \left( \frac{c_{pp}}{c_{pg}} \right) \frac{\omega \tau_{t1}}{1 + \omega^2 \tau_{t1}^2}
$$

(3a)

In the above the quantity $\bar{\alpha}$ refers the attenuation per unit frequency per unit mass fraction, $\alpha$ the linear absorption coefficient or the amplitude attenuation coefficient (imaginary part of the wavenumber), $c_p$ the specific heat, $c_0$ the speed of sound in the gas phase, and $\gamma$ the isentropic exponent (specific-heat ratio). Note that $\alpha = \alpha_i / 2$.

The relaxation times $\tau_{d1}$ and $\tau_{t1}$ correspond to those under nonlinear drag conditions (generally representative of large-sized particles). They are related to the relaxation times $\tau_d$ and $\tau_t$ by the relations

$$
\tau_{d1} = \tau_d \psi_1 \left( \text{Re}_p \right), \quad \tau_{t1} = \tau_t \psi_2 \left( \text{Re}_p \right)
$$

(3b)

where

$$
\psi_1 \left( \text{Re}_p \right) = \frac{C_{D1}}{C_D}, \quad \psi_2 \left( \text{Re}_p, \text{Pr} \right) = \frac{\text{Nu}_1}{\text{Nu}}
$$

(3c)

with $C_{D1}$ standing for the nonlinear drag coefficient, and $\text{Nu}_1$ for the nonlinear heat transfer. In the above,

$$
\tau_d = \frac{d_p^2 \rho_p}{18 \mu_x}
$$

(4a)

and

$$
\tau_t = \frac{m_p c_{pp}}{2 \pi d_p \rho_p k_g} = \frac{\text{Pr} c_{pp} d_p^2 \rho_p}{12 \mu_g c_{pg}} = \left( \frac{3}{2} \right) \left( \frac{c_{pp}}{c_{pg}} \right) \text{Pr} \tau_d
$$

(4b)

where $\text{Pr} = c_{pg} \mu_g / k_g$ stands for the Prandtl number of gas. Physically the quantity $\tau_{d1}$ is a measure of the inability of the particles to follow (respond to) the fluctuations in the fluid motion. Likewise, the quantity $\tau_{t1}$ is a measure of the thermal response of the particles to follow the fluctuations in the temperature of the fluid.
The drag coefficient and the Nusselt number in Eq. (3c) are defined by

\[ C_D = 2F_D \left( \frac{\rho_g \pi}{4} d_p^2 \frac{d^2}{\mu_g} \right), \quad Nu = h_g d_p / k_g \]  

(5)

where \( F_D \) is the particle drag force, and \( h_g \) the gas-droplet convective heat transfer coefficient. A good approximation (curve fit) for the drag coefficient \( C_D \) is recommended as:

\[ C_D = \frac{24}{Re_p} + \frac{6}{1+\sqrt{Re_p}} + 0.4 \]  

(6)

The first term on the RHS of Eq. (6) is the Stokesian drag coefficient defined by

\[ C_D = \frac{24}{Re_p} \]  

(7)

which is valid for particle Reynolds numbers less than about one. Eqs. (5) and (6) yield an expression for \( \psi_1 \) as:

\[ \psi_1(Re_p) = 1 + \frac{Re_p}{24} \left( \frac{6}{1+\sqrt{Re_p}} + 0.4 \right) \]  

(8)

With regard to the particle heat transfer, a good correlation for heat transfer (by conduction and convection) is expressed by the well-known Ranz-Marshall correlation:

\[ Nu_1 = 2 + 0.6 Re_p^{0.5} Pr^{0.33} \]  

(9)

In the pure conduction limit, we have \( Nu = 2 \), so that the function \( \psi_2 \) in Eq. (3c) becomes:

\[ \psi_2 = \frac{Nu_1}{Nu} = 1 + 0.3 Re_p^{0.5} Pr^{0.33} \]  

(10)

It now remains to find a relation for the droplet Reynolds number \( Re_p \) in terms of the density ratio and the particle relaxation time.
2.2 Droplet Reynolds number

The determination of particle Reynolds number required in the evaluation of the functions $\psi_1$ and $\psi_2$ in Eqs. (8) and (10) respectively is exceedingly complex. There exists relatively little information on the dependence of particle Reynolds number on the particle characteristics in two-phase flows. A study of turbulent diffusion of droplets in a gaseous medium\cite{25,26} indicates a plausible relationship of the form

$$Re_p = f\left(\rho_g / \rho_p, \omega \tau_d\right) \quad (11)$$

Since Eq. (3a) is applicable to large particle to fluid density ratios, we postulate here that the particle Reynolds number depends only on the particle relaxation time, and is independent of the particle to fluid density ratio: that is

$$Re_p = f(\omega \tau_d) \quad (12)$$

The physical basis for this supposition may be justified as follows. Particles with a smaller relaxation time are able to respond better to the fluctuations in the fluid motion, which leads to reduced particle relative velocity, and thus a reduced particle Reynolds number.

In the present investigation, the following power law relation is proposed such that a peak in the linear absorption coefficient (as indicated by the measurements) is realized (an exponent of 2.5 or greater for the relaxation time ensures this peak):

$$Re_p = c(\omega \tau_d)^3 \quad (13)$$

The adjustable constant $c$ is determined from a correlation of the theory with the test data. A value of $c = 10$ is found to be satisfactory based on the data of Norum\cite{21} for water droplets in a supersonic air jet.
3. Results and comparison

3.1 Effect of Nonlinear Particle Relaxation

The effect of nonlinear particle relaxation on the absorption coefficient per unit frequency is demonstrated in Fig. 2a for comparison with the theory of Temkin and Dobbins\textsuperscript{20} for dilute concentrations. The results suggest that below \( \omega \tau_d \approx 0.7 \) the nonlinear particle relaxation effects are seen to be unimportant. Beyond \( \omega \tau_d \approx 0.7 \), the nonlinear theory departs from the linear theory.

Fig. 2b shows a comparison of the predicted linear spectral attenuation of sound under nonlinear particle relaxation. The theoretical result by Temkin and Dobbins\textsuperscript{20} for dilute concentrations is also presented for a comparison. The theory suggests that at high frequencies the linear absorption coefficient predicted by the nonlinear theory decreases with increasing frequency in accordance with experimental trend for large particle sizes.

3.2 Comparison with experimental data

A direct comparison of the present theory with the measured spectral attenuation by Norum\textsuperscript{21} for water droplets in a perfectly expanded (jet exit pressure equals the ambient pressure; thus shock-free) supersonic jet is displayed in Fig. 3. The data correspond to hot supersonic jet of air from a convergent-divergent (CD) nozzle operation at a jet total temperature \( T_t = 867 \text{ K} \), and a jet exit Mach number \( M_j = 1.45 \). The jet Mach number is defined as \( M_j = u_j / c_j \), where the subscript \( j \) refers to the nozzle exit conditions. The mass flow rate (maximum considered) of water to that of the jet is about 0.85. The angle \( \theta \) is measured from the jet inlet axis. The jet exit Reynolds number \( \text{Re}_j = u_j d_j / \mu_g \) is about \( 1.3 \times 10^6 \), where \( d_j \) is the jet exit diameter. At this condition, supersonic turbulent jet mixing noise dominates upstream noise radiation, and Mach wave radiation dominates the downstream noise radiation. In the data, water is injected at 45 deg. The
data include spectra measured at angles of 45 deg, 90 deg and 135 deg, thus highlighting
directivity effects.

From the measured ΔSPL (reduction in Sound Pressure Level) at a given frequency due to
water injection, the linear absorption coefficient is deduced as

\[ \frac{\alpha}{\alpha_{\text{max}}} = \frac{\Delta\text{SPL}}{\Delta\text{SPL}_{\text{max}}} \]  

where the subscript refers to the peak spectral reduction. The comparisons suggest that the
proposed theory based on the nonlinear particle relaxation processes satisfactorily describes the
measurements for the spectral attenuation of sound, indicating a spectral peak. The inclusion of
directionality effect (dependence on the angle of observation) on the spectral absorption is
beyond the scope of the present work.

It should be remarked, however, that the data on supersonic jet noise considered here for
comparison are not directly pertinent to plane waves of sound for which the theory has been
developed. Since noise from turbulent jets may be regarded as a superposition of plane waves of
differing frequency, the absolute values of the absorption coefficient in the test data could be
different from that expressed by the theory. In this connection, the reduction of turbulent jet
mixing noise reduction by water injection was theoretically considered by the author in Ref. 27 in
conjunction with the scaling laws proposed for jet noise\textsuperscript{28}. Finally, the effect of droplet
evaporation on the spectral attenuation of sound, as indicated by the theory of Marble et al.\textsuperscript{29} is
left out of account in the present analysis.

4. Conclusion

The theory proposed here for sound attenuation in dilute suspensions with nonlinear particle drag
and heat transfer is shown to satisfactorily represent the test data for noise reduction with water
droplets suspended in a supersonic jet. It is found that the nonlinear particle relaxation processes
are primarily responsible for reduction in the linear absorption coefficient at high frequencies.
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References and links


Figure Captions

Fig. 1 Attenuation of a plane sound wave in a gas-droplet mixture.

Fig. 2a (color online) Predicted absorption coefficient per unit frequency with nonlinear particle relaxation processes.

Fig. 2b (color online) Predicted linear absorption coefficient with nonlinear particle relaxation processes.

Fig. 3 (color online) Comparison of the predictions for the linear absorption coefficient with test data of Norum.\textsuperscript{21}
Fig. 1 Attenuation of a plane sound wave in a gas-droplet mixture.
Fig. 2a Predicted absorption coefficient per unit frequency with nonlinear particle relaxation processes.
Fig. 2b Predicted linear absorption coefficient with nonlinear particle relaxation processes.
Fig. 3 Comparison of the predictions for the linear absorption coefficient with test data of Norum\textsuperscript{21}. 

\( \frac{\alpha}{\alpha_{\text{max}}} \) vs. \( \frac{\omega}{\omega_{\text{max}}} \) for different angles: 
- Present theory
- Data (\( \theta = 45 \text{ deg} \))
- Data (\( \theta = 90 \text{ deg} \))
- Data (\( \theta = 135 \text{ deg} \))

\( \theta \) measured from jet inlet axis.
## 4. TITLE AND SUBTITLE
Spectral Attenuation of Sound in Dilute Suspensions with Nonlinear Particle Relaxation

## 14. ABSTRACT
Previous studies on the sound attenuation in particle-laden flows under Stokesian drag and conduction-controlled heat transfer have been extended to accommodate the nonlinear drag and heat transfer. It has been shown that for large particle-to-fluid density ratio, the particle Reynolds number bears a cubic relationship with $wtd$ (where $w$ is the circular frequency and $td$ the Stokesian particle relaxation time). This dependence leads to the existence of a peak value in the linear absorption coefficient occurring at a finite value of $wtd$. Comparison of the predictions with the test data for the spectral attenuation of sound with water injection in a perfectly expanded supersonic air jet shows a satisfactory trend of the theory accounting for nonlinear particle relaxation processes.

## 15. SUBJECT TERMS
spectral attenuation, nonlinear particle relaxation, sound