Introduction to radiative transfer theory and models (Optical Domain)

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Passive Optical Remote Sensing

RADAR: radiation generated next to sensor

Atmospheric Correction of Earth Observation Data for Environmental Monitoring: Theory
Solar Energy Paths
atmospheric contribution
direct + direct
diffuse + direct
direct + diffuse
multiple scattering

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Solar (reflective) spectral domain

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Observation Geometry

Solar zenith angle

View zenith angle

Relative azimuth angle

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Solution of the Radiative Transfer in the reflective domain for non absorbing atmosphere and lambertian ground

\[ \rho_{app}(\theta_s, \theta_v, \phi) = \rho_{atm}(\theta_s, \theta_v, \phi) + T_{atm}(\theta_s)T_{atm}(\theta_v) \frac{\rho_{ground}}{1 - \rho_{ground}S_{atm}} \]

- **Atmospheric reflectance**
- **Ground reflectance** (\(=\) albedo for lambertian)
- **Atmospheric Transmissions**
- **Apparent reflectance at satellite level**
- **Atmosphere spherical albedo**

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Perfect Lambertian Reflector

Radiance of the Perfect Lambertian Reflector

\[
\int_0^{2\pi} \int_0^\pi RPLF(\theta_S, \theta, \phi) \cos(\theta) \sin(\theta) d\theta d\phi = E_s \cos(\theta_s)
\]

Isotropic radiation

\[
\rho_{\text{Perfect Lambertian reflector}} (\theta_S, \theta_V, \phi) = 1
\]

\[
\rho_{\text{Lambertian reflector}} (\theta_S, \theta_V, \phi) = \rho
\]
Simple Radiative Transfer Equation
SRTE (cont.)

\[ \rho_{\text{atm}}(\theta_s, \theta_v, \phi) \]

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SRTE (cont.)

\[ T_{\text{atm}}(\theta_s) = \frac{E_t}{E_i} \]

\[ \theta_s \]
\[ E_r = \rho_{\text{ground}} E_t \]
SRTE (cont.)

\[ T_{atm}(\theta_v) = \frac{E_o}{E_r} \]
SRTE 1 interaction (cont.)

\[ \rho_{app} = \rho_{atm} + \frac{E_o}{E_i} \]

\[ \frac{E_o}{E_i} = \frac{T(\theta_v)E_r}{E_i} = \frac{T(\theta_v)\rho_{ground}E_t}{E_i} = T(\theta_v)\rho_{ground}T(\theta_s) \]

\[ \rho_{app} = \rho_{atm} + T(\theta_v)\rho_{ground}T(\theta_s) \]
SRTE 2 interactions

$E_i T(\theta_s) \rho_{\text{ground}} S_{\text{atm}} \rho_{\text{ground}} T(\theta_v)$

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SRTE Multiple Interactions

\[ \rho_{\text{app}} = \rho_{\text{atm}} + T(\theta_s)T(\theta_v)\rho_{\text{ground}} \left[ 1 + \rho_{\text{ground}} S_{\text{atm}} + \left( \rho_{\text{ground}} S_{\text{atm}} \right)^2 + \left( \rho_{\text{ground}} S_{\text{atm}} \right)^3 \ldots \right] \]

\[ 1 + r + r^2 + r^3 + \ldots r^{n-1} = \frac{1 - r^n}{1 - r} \]

\[ \rho_{\text{ground}} S_{\text{atm}} < 1 \text{ so when } n \to \infty \text{ then } (\rho_{\text{ground}} S_{\text{atm}})^n \to 0 \]

Therefore \[ \left[ 1 + \rho_{\text{ground}} S + \left( \rho_{\text{ground}} S \right)^2 + \left( \rho_{\text{ground}} S \right)^3 \ldots \right] = \frac{1}{1 - \rho_{\text{ground}} S} \]

\[ \rho_{\text{app}} = \rho_{\text{atm}} + T_{\text{atm}}(\theta_s)T_{\text{atm}}(\theta_v) \frac{\rho_{\text{ground}}}{1 - \rho_{\text{ground}} S_{\text{atm}}} \]
STRE for non absorbing atmosphere and lambertian ground

\[ \rho_{\text{app}}(\theta_s, \theta_v, \phi) = \rho_{\text{atm}}(\theta_s, \theta_v, \phi) + T_{\text{atm}}(\theta_s)T_{\text{atm}}(\theta_v) \frac{\rho_{\text{ground}}}{1 - \rho_{\text{ground}} S_{\text{atm}}} \]

Atmospheric reflectance

Ground reflectance (= albedo for lambertian)

Apparent reflectance at satellite level

Atmospheric Transmissions

Atmosphere spherical albedo

Atmospheric Correction of Earth Observation Data for Environmental Monitoring: Theory and Best Practises
The composition of the atmosphere

<table>
<thead>
<tr>
<th>Permanent Constituents</th>
<th>% by volume</th>
<th>Variable constituents</th>
<th>% by volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen (N₂)</td>
<td>78.084</td>
<td>Water Vapor (H₂O)</td>
<td>0.04</td>
</tr>
<tr>
<td>Oxygen (O₂)</td>
<td>20.948</td>
<td>Ozone (O₃)</td>
<td>12 x 10⁻⁴</td>
</tr>
<tr>
<td>Argon (Ar)</td>
<td>0.934</td>
<td>Sulfur dioxide (SO₂)</td>
<td>0.001 x 10⁻⁴</td>
</tr>
<tr>
<td>Carbon dioxide (CO₂)</td>
<td>0.033</td>
<td>Nitrogen dioxide (NO₂)</td>
<td>0.001 x 10⁻⁴</td>
</tr>
<tr>
<td>Neon (Ne)</td>
<td>18.18 x 10⁻⁴</td>
<td>Ammonia (NH₃)</td>
<td>0.001 x 10⁻⁴</td>
</tr>
<tr>
<td>Helium (He)</td>
<td>5.24 x 10⁻⁴</td>
<td>Nitric oxide (NO)</td>
<td>0.0005 x 10⁻⁴</td>
</tr>
<tr>
<td>Krypton (Kr)</td>
<td>1.14 x 10⁻⁴</td>
<td>Hydrogen sulfide (H₂S)</td>
<td>0.00005 x 10⁻⁴</td>
</tr>
<tr>
<td>Xenon (Xe)</td>
<td>0.089 x 10⁻⁴</td>
<td>Nitric acid vapor</td>
<td>trace</td>
</tr>
<tr>
<td>Hydrogen (H₂)</td>
<td>0.5 x 10⁻⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Methane (CH₄)</td>
<td>1.5 x 10⁻⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrous Oxide (N₂O)</td>
<td>0.27 x 10⁻⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carbon Monoxide (CO)</td>
<td>0.19 x 10⁻⁴</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Gaseous Absorption (H$_2$O)
Modified SRTE to account for absorption

In case of a pure molecular atmosphere (no aerosol) we can write:

\[
\rho_{\text{app}}(\theta_s, \theta_v, \phi) = T_g^{\text{othergases}}(m, U_{\text{gaz}}) \left[ \rho_{\text{atm}}(\theta_s, \theta_v, \phi) + T_g^{H_2O}(m, U_{H_2O})T_{\text{atm}}(\theta_s)T_{\text{atm}}(\theta_v) \frac{\rho_{\text{ground}}}{1 - S_{\text{atm}}\rho_{\text{ground}}} \right]
\]

\[m\text{ is the air mass } = \frac{1}{\cos(\theta_s)} + \frac{1}{\cos(\theta_v)}\]

\[U_{\text{gaz}}\text{ is the gaz concentration}\]
\[
\rho_{\text{app}}(\theta_s, \theta_v, \phi) \sim T_g^{\text{other gases}}(m, U_{\text{gaz}}) \left[ \rho_R(\theta_s, \theta_v, \phi) + T_g^{H_2O}(m, U_{H_2O} / 2) \rho_A(\theta_s, \theta_v, \phi) + T_g^{H_2O}(m, U_{H_2O}) T_A(\theta_s) T_A(\theta_v) T_R(\theta_s) T_R(\theta_v) \frac{\rho_{\text{ground}}}{1 - S_{R+A} \rho_{\text{ground}}} \right]
\]
Water vapor effect for different sensors in the near infrared

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Scattering angle, \( \Theta \), is the relative angle between the incident and the scattered radiation.

\[ \Theta \]

- The scattering angle, \( \Theta \), is the relative angle between the incident and the scattered radiation.
Phase function

• The phase function, \( P(\Theta) \), describes the distribution of scattered radiation for one or an set of particles. It is normalized such as:

\[
\int_0^{2\pi} \int_0^{\pi} P(\Theta) d\omega = 4\pi
\]

since

\[
\int_0^{2\pi} \int_0^{\pi} P(\Theta) \sin(\theta) d\theta d\phi = 2\pi \int_0^{\pi} P(\Theta) \sin(\theta) d\theta
\]

we have

\[
\int_0^{\pi} P(\Theta) \sin(\theta) d\theta = 2
\]
Rayleigh/molecular scattering

1/4

- Rayleigh or molecular scattering refers to scattering by atmospheric gases, in that case:

\[ P(\Theta) = \frac{3}{4} \left( 1 + \cos^2(\Theta) \right) \]

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Rayleigh/molecular scattering 2/4

• The concentration in scatterer is better described by the efficiency they scatter at a certain wavelength or the proportion of direct transmission which is related to the spectral optical thickness $\tau(\lambda)$

\[
\frac{E_t(\lambda)}{E_0(\lambda)} = e^{-\tau(\lambda)}
\]

• For Rayleigh $\tau(\lambda)$ is proportional to $\lambda^{-4}$ and for standard pressure is $\sim 0.235$ at 0.45 $\mu$m
Rayleigh/molecular scattering

3/4

- The rayleigh reflectance, $\rho_R$, could be crudely approximated by:

$$
\rho^\lambda_R(\theta_s, \theta_v, \phi) \sim \frac{\tau^\lambda_R P(\Theta)}{4 \cos(\theta_s) \cos(\theta_v)}
$$
• Compute the reflectance of the sky (assumed clear no aerosol) at solar noon at 45 degree latitude at vernal equinox looking straight up at 0.45\(\mu\text{m}\), 0.55\(\mu\text{m}\), 0.65\(\mu\text{m}\)

\[
\rho_R^{0.45\mu m}(45^\circ, 0^\circ, \phi) = \frac{0.235P(45^\circ)}{4 \cos(45^\circ) \cos(0^\circ)}
\]

\[
= \frac{0.235 \times 0.75(1 + \cos^2(45^\circ))}{4 \cos(45^\circ) \cos(0^\circ)} = \frac{0.265}{2.8} \sim 0.1
\]

\[
\rho_R^{0.55\mu m} = 0.1 \times (0.45/0.55)^4 \sim 0.1 \times 0.4 = 0.04
\]

\[
\rho_R^{0.65\mu m} = 0.04 \times (0.55/0.65)^4 \sim 0.04 \times 0.5 = 0.02
\]
Aerosol scattering refers to scattering by particles in suspension in the atmosphere (not molecules). The MIE scattering theory could be applied to compute the aerosol phase function and spectral optical depth, based on size distribution, real and imaginary index.
Aerosol scattering 2/5

Continental aerosol phase function

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Aerosol scattering 3/5

\[ \rho_A^\lambda(\theta_s, \theta_v, \phi) \sim \frac{\omega_A^\lambda \tau_A^\lambda P(\Theta)}{4 \cos(\theta_s) \cos(\theta_v)} \]

single scattering albedo (0.2-1.0) to account for absorbing particles

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Aerosol scattering 4/5

Continental aerosol optical thickness spectral variation

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Continental aerosol single scattering albedo spectral variation
Atmospheric effect: Vegetation 1/3

Surface reflectance (Kuusk’s model)

Leaf Area Index
Atmospheric effect: Vegetation 2/3

No absorption, Continental aerosol

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Absorption tropical atmosphere, Continental aerosol

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Atmospheric effect: Ocean 2/2

Top of the atmosphere reflectance (Ocean model)

Pigment concentration (mg/m³)

Wavelength (µm)

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Perfect Lambertian Reflector

\[
\int_0^{2\pi} \int_0^{\pi} \! RPLF(\theta_s, \theta, \phi) \cos(\theta) \sin(\theta) \, d\theta \, d\phi = \frac{E_s \cos(\theta_s)}{\pi}
\]

\[
RPLF(\theta_s, \theta_v, \phi) = \frac{E_s \cos(\theta_s)}{\pi}
\]

Isotropic radiation
Different Types of Reflectors

Specular reflector (mirror)

Nearly Specular reflector (water)

Diffuse reflector (lambertian)

Nearly diffuse reflector

Hot spot reflection
Sun glint as seen by MODIS

Gray level temperature image

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MODIS data illustrating the hot-spot over dense vegetation

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BRDF atmosphere coupling correction

Lambertian infinite target approximation

\[
\rho_{\text{app}}(\theta_s, \theta_v, \phi) = \rho_{\text{atm}}(\theta_s, \theta_v, \phi) + T_{\text{atm}}(\theta_s)T_{\text{atm}}(\theta_v) \frac{\rho_{\text{ground}}}{1 - \rho_{\text{ground}}S_{\text{atm}}}
\]

BDRF atmosphere coupling approximation

\[
\rho_{\text{app}}(\theta_s, \theta_v, \phi) = \rho_{\text{atm}}(\theta_s, \theta_v, \phi) + e^{-\tau/\mu_s} e^{-\tau/\mu_v} \rho_s(\theta_s, \theta_v, \phi)
\]
\[
+ e^{-\tau/\mu_v} t_d(\theta_s) \bar{\rho}_s + e^{-\tau/\mu_s} t_d(\theta_v) \bar{\rho'}_s + t_d(\theta_v) t_d(\theta_s) \bar{\rho}_s
\]
\[
+ \frac{T_{\text{atm}}(\theta_s)T_{R+A}(\theta_v)S_{\text{atm}}(\bar{\rho}_s)^2}{1 - S_{\text{atm}} \bar{\rho}_s}
\]

\[
\bar{\rho}_s(\mu_s, \mu_v, \phi) = \frac{\int_0^{2\pi} \int_0^1 \mu L_{\text{atm}}(\mu_s, \mu, \phi') \rho_s(\mu, \mu_v, \phi'-\phi) d\mu d\phi'}{\int_0^{2\pi} \int_0^1 \mu L_{\text{atm}}(\mu_s, \mu, \phi') d\mu d\phi'}
\]

\[
\bar{\rho'}_s(\mu_s, \mu_v, \phi) = \bar{\rho'}_s(\mu_s, \mu_v, \phi) = \bar{\rho}_s(\mu_s, \mu_v, \phi)
\]
Adjacency effect correction

Lambertian infinite target approximation

\[ \rho_{app}(\theta_s, \theta_v, \phi) = \rho_{atm}(\theta_s, \theta_v, \phi) + T_{atm}(\theta_s)T_{atm}(\theta_v) \frac{\rho_{ground}}{1 - \rho_{ground} S_{atm}} \]

adjacency effect approximation

\[ \rho_{app} = \rho_{atm} + \frac{T_{atm}(\theta_s)}{1 - S_{atm}\rho_e} \left( e^{-\tau/\mu_v} \rho_s + t^d_{atm}(\theta_v) \rho_e \right) \]

\[ \rho_e = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \rho(r,\psi) \frac{dF(r)}{dr} dr d\psi \]

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Adjacency effect correction (practical implementation)

\[ \rho_i = \frac{\rho_{app} - \rho_{atm}}{T_{atm}(\theta_s)T_{atm}(\theta_v)} \]

\[ \rho_{s}^{inf} = \frac{\rho_i}{1 + S_{atm}\rho_i} \]

\[ \rho_i = \frac{\rho_s e^{-\tau/\mu_v} + t_{atm}^d(\theta_v)\rho_e}{T_{atm}(\theta_v)(1 - S_{atm}\rho_e)} \]

\[ \rho_s = \frac{\rho_i T_{atm}(\theta_v)(1 - S_{atm}\rho_e) - t_{atm}^d(\theta_v)\rho_e}{e^{-\tau/\mu_v}} \]

\[ \rho_e = \sum_{j=-n}^{n} \sum_{i=-n}^{n} \frac{dF(r(i, j))}{dr} \rho_{s}^{inf}(i, j) \]

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Adjacency effect correction (testing)

Synthetic data set for surface reflectance in the blue (a), green (b) and red (c), and in RGB (d) corresponding to bright soil (yellow squares) and dense vegetation (green squares).

Typical atmospheric effect on the synthetic surface reflectance shown above
Reflectance’s observed over a horizontal transect on the checkerboard. The red bars are the “true” surface reflectance, the blue bars correspond to the top of the atmosphere signal including adjacency effect. The green bars correspond to the corrected data using the infinite target assumption. The open square correspond to the data corrected for the adjacency effect using the operational method developed.
Adjacency effect correction (validation)