Successful forecasting of energetic particle events in space weather models require algorithms for correctly predicting the spectrum of ions accelerated from a background population of charged particles. We present preliminary results from a model that diffusively accelerates particles at multiple shocks. Our basic approach is related to box models (Protheroe and Stanev, 1998; Moral and Axford, 1983; Ball and Kirk, 1992; Drury et al., 1999) in which a distribution of particles is diffusively accelerated inside the box while simultaneously experiencing decompression through adiabatic expansion and losses from the convection and diffusion of particles outside the box (Melrose and Pope, 1993; Zank et al., 2000). We adiabatically decompress the accelerated particle distribution between each shock by either the method explored in Melrose and Pope (1993) and Melrose and Pope (1994) or by the approach set forth in Zank et al. (2000) where we solve the transport equation by a method analogous to operator splitting. The second method incorporates the additional loss terms of convection and diffusion and allows for the use of a variable time between shocks. We use a maximum injection energy \( E_{\text{inj}} \) appropriate for quasi-parallel and quasi-perpendicular shocks (Zank et al., 2000, 2006; Dosh and Shalchi, 2010) and provide a preliminary application of the diffusive acceleration of particles by multiple shocks with frequencies appropriate for solar maximum (i.e., a non-Markovian process).

### Diffusive Shock Acceleration (DSA)

- The acceleration of charged particle is due to repeated reflections across a shock. This is seen in the reflection at magnetic mirrors, but is applicable for shocks due to the wave-particle interaction at the shock front.
- The injection energy must be a few times the thermal energy in order to make an initial crossing at the shock boundary.
- It is thought to be the primary mechanism for particle acceleration at shock waves.
- Injection problem – particles must have energies significantly higher than the thermal energy in order to cross the shock boundary.

### Motivation

We want to take the concept of particle acceleration at a single shock and extend it to a system of multiple shocks (non-Markovian process). This would be analogous to solar maximum time period. 1. For \( v_{\text{sh}} = 200 \text{ km/s} \), time to Earth \( \approx 4.3 \text{ days} \)
2. Typical shocks takes 2-3 days to propagate to 1 AU
3. If assume \( x = 6.65 \times 10^{10} \text{ m/s} \) then \( t_{\text{shock}} \approx 4.8 \text{ days} \)
4. During solar max, accelerated particles will still be in system as second shock passes – non-Markovian process

In order to extend the model to a system such as solar maximum, we need information on the time history of actual shock environment for input to the model. We investigated different CME databases (CDW Gopalswamy et al., 2009; SEEDS Olmedo et al., 2008; CACTus Robbrecht and Berghmans, 2004). We find on average that during solar maximum the velocity of the CME is higher and the frequency of occurrence of CMEs is larger.

### Multiple Shock Model
- Related to Box model (Drury et al, 1999)
- Generally assumed five shocks, but model can accommodate any number of shocks
- Use DSA equation to model the time history of accelerated protons at solar maximum
- Need information on the time history of actual shock environment at solar maximum for input to model
- Incorporate maximum injection energy
- Assume the CME expands outward
- Two decomposition methods: Melrose and Pope, 1993 and Transport Equation method

### Methodology

- Total injected distribution
- \( f(p') = \phi(p') \) is the seed particle distribution
- \( \phi(p') \) is the upstream injection distribution

1. Accelerate the injection distribution at an interplanetary or CME driven shock via diffusive shock acceleration.
2. Decompress the accelerated distribution using one of two methods
3. Re-accelerate the newly decompressed distribution at a subsequent shock wave event.

### Decompression Methods

**Melrose and Pope, 1993**
- Based on Liouville's Theorem
- Distribution expands fully in volume, \( R^3 \)
- Repeated acceleration of particles at multiple shocks
- Time between shocks is not considered
- Spectral index is unchanged during decompression, but distribution shifts left

**Transport Equation Method**
- Solve the cosmic ray transport equation via method analogous to operator splitting (Zank et al., 2006)
- Equation:

\[
\frac{d\phi}{dt} = -\frac{2}{\tau} \int_0^1 \phi(\psi) \psi'(x) \frac{d\psi}{dx} dx
\]

**Convection Term**
- Consider inner expanding solar wind
  - \( a \) assume constant background flow velocity
  - Approximately constant diffusion tensor \( w \times \nabla \phi \)
  - Spherical symmetry
- Now incorporates losses due to convection and diffusion, adiabatic decompression, as well as a time (frequency) variability of shocks.

\[
f = f_\text{lof} - \frac{2}{\tau} \int_0^1 \phi(\psi) \psi'(x) \frac{d\psi}{dx} dx
\]

**Energy Term**
- Possible reason for divergence – places where there is no data are days that there were no shocks.
- As delta t increases, decompression and losses increase and there are less available particles to accelerate.
- For lower delta t, crossovers are shifted to the left due to larger number of particles being accelerated.
- With repeated injections there is no cross-over point because of additional upstream distribution. The added distribution ensures the spectral amplitude increases from one shock to another.

### Different shocks:
- Model is able to adequately and appropriately model these changing conditions.
- Implement \( \psi_{\text{inj}} \) appropriate for quasi-parallel or quasi-perpendicular shocks.
- Shock and upstream distribution parameters are important to the acceleration.
- Harder shock produces harder spectrum and it’s captured in the model.
- Distribution of different shock make a difference.

### Time History of Multiple Shocks, Extension to Solar Maximum

**Model Parameters**
- Uppermost distribution constructed using daily averages from SWARM
- 30 day (time period during solar maximum (73 shocks))
- Included shocks on west limb (BIT F & S) (607) meaning
- Propagating shock more likely to interact with accelerated particles from previous shock
- Propagating shock more likely to interact with Earth

**Conclusion**
- Used two methods of decompression
- Melrose & Pope – adiabatically decompress
- Transport method – adds losses due to convection and diffusion, plus a temporal variability
- DSA during solar maximum is a non-Markovian process and previous shocks must be considered
- Spectrum flatten for subsequent accelerations
- Decompression due to the transport method models the physical processes more appropriately as it includes losses due to convection and diffusion, as well as adiabatic decompression and temporal variability. The transport equation method is more appropriate for an expanding solar wind.
- Multiple shock model compares well with observed intensity profile at several energies at 1 AU.

### Table

| Time between shocks | Decompression and losses increase and there are less available particles to accelerate. | For lower delta t, crossovers are shifted to the left due to larger number of particles being accelerated. | With repeated injections there is no cross-over point because of additional upstream distribution. The added distribution ensures the spectral amplitude increases from one shock to another. | Model is able to adequately and appropriately model these changing conditions. | Implement \( \psi_{\text{inj}} \) appropriate for quasi-parallel or quasi-perpendicular shocks. | Shock and upstream distribution parameters are important to the acceleration. | Harder shock produces harder spectrum and it’s captured in the model. | Distribution of different shock make a difference. |