Dissertation Defense

Computational Fluid Dynamics Uncertainty Analysis for Payload Fairing Spacecraft Environmental Control Systems

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  – Research Goals
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Agenda

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Chapter 1: Introduction

Motivation
Research Goals
Motivation

• Spacecraft components may be damaged due to airflow produced by Environmental Control Systems (ECS).

• Spacecraft must survive both pre-launch and launch environments

• ECS Systems supply air to keep spacecraft cool, dry, clean while on the ground

• Delicate spacecraft instruments are sensitive to high velocity flow from ECS system
  – Manufactures set Impingement Requirements

• (2) Methods to Verify Requirements
  – Test vs. CFD
• Environmental Control System
  – Prior to launch, cold air (air conditioning) flows downward around the spacecraft after it has been encapsulated in the Payload Fairing.
  – The cold air is delivered through an air-conditioning (AC) pipe, which intersects the fairing and flows past a diffuser located at the pipe/fairing interface
  – After passing over the spacecraft, it is finally discharged through vents
  – The Payload Fairing air conditioning is cut off at lift off.

Motivation: ECS System Overview

- Example of ECS CFD Analysis

Motivation: ECS System Overview

• Example of an ECS system airflow test

Motivation: Problem

- Computational Fluid Dynamics (CFD) is being used without proper validation

- “There can be no validation without experimental data with which to compare the results of the simulation” – Coleman and Stern

- Experimental Data is expensive
  - Shrinking Budgets

- Pairing experimental data, uncertainty analysis, and analytical predictions provides a comprehensive approach to verification and is the current state of the art. (ASME V&V 20-2009)

- A method is sought to conservatively envelop the exact solution using CFD only
  - Without Experimental Data
Motivation: NASA-STD-7009

STANDARD FOR MODELS AND SIMULATIONS

Requirement 4.4.7 & 4.4.8

• Shall document any uncertainty quantification processes used for:
• Shall document any quantified uncertainties, both physical and numerical, for:
  a. The referent data
  b. The input data
  c. The Modeling and Simulation (M&S) results
  d. The propagation of uncertainties
  e. The quantities derived from M&S results
Research Goals

1. Demonstrate a CFD Uncertainty Analysis for 3-D, low speed, incompressible, highly turbulent, internal flow can be calculated for an entire simulation domain.

2. Investigate a higher order interpolation scheme to be used for grid interpolations and uncertainty quantification.

3. Investigate the applicability of using the ASME 5-Step procedure for the entire computational domain to estimate numerical uncertainties.

4. Calculate the uncertainty in using different turbulent models.

5. Demonstrate this method can contribute to the study of importance of input parameters in CFD.

6. Compile a table for uncertainty estimates by input parameter.

7. Demonstrate the ability to use OPENFOAM to calculate the velocity field of an Environmental Control System.

8. Compare the results of OPENFOAM verses an industry standard CFD software program (ie FLUENT and STARCCM+).
Chapter 2: Literature Review

Summary of Literature Review
Summary of the State of the Art
Uncertainty Analysis
Proposed Methodology without Test Data
Summary of ASME Standard
ASME V&V-20-2009

Standard for Verification and Validation in Computational Fluid Dynamics and Heat Transfer
Approach

• Estimate Interval within which $\delta_{model}$ falls with a given degree of confidence

$$\delta_{model} \in [E - u_{val}, E + u_{val}]$$

$$E = S - D$$

• Error Sources ($U_{num}, U_{input}, U_{D}$), Uncertainty Equation

$$u_{val} = k \left( \sqrt{u_{num}^2 + u_{input}^2 + u_{D}^2} \right)$$
Numerical Uncertainty, $U_{num}$

- 5 Step Procedure for Uncertainty Estimation

  - Step 1: Representative Grid Size

  $$h_1 = \left(\frac{Total \ Volume}{total \ number \ of \ cells \ in \ fine \ grid}\right)^{\frac{1}{3}}$$

  $$h_2 = \left(\frac{Total \ Volume}{total \ number \ of \ cells \ in \ medium \ grid}\right)^{\frac{1}{3}}$$

  $$h_3 = \left(\frac{Total \ Volume}{total \ number \ of \ cells \ in \ coarse \ grid}\right)^{\frac{1}{3}}$$
Numerical Uncertainty, $U_{num}$ (continued)

– Step 2: Select 3 significantly ($r>1.3$) different grid sizes

\[ r_{21} = \frac{h_2}{h_1} \]

\[ r_{32} = \frac{h_3}{h_2} \]

– Use CFD Simulation to analyze key variables, Sk

\[ \varepsilon_{21} = S_{k2} - S_{k1} \]

\[ \varepsilon_{32} = S_{k3} - S_{k2} \]
– Step 3: Calculate observed order, $p$

\[
p = \left[ \frac{1}{\ln(r_{21})} \right] \times \ln \left( \frac{\varepsilon_{32}}{\varepsilon_{21}} \right) + \ln \left( \frac{r_{21}^p - \text{sign} \left( \frac{\varepsilon_{32}}{\varepsilon_{21}} \right)}{r_{32}^p - \text{sign} \left( \frac{\varepsilon_{32}}{\varepsilon_{21}} \right)} \right)
\]

– Step 4: Calculate extrapolated values

\[
S_{ext}^{21} = \frac{(r_{21}^p \times S_{k1} - S_{k2})}{(r_{21}^p - 1)}
\]

\[
e_{a}^{21} = \frac{(S_{k1} - S_{k2})}{(S_{k1})}
\]
Numerical Uncertainty, $U_{num}$ (continued)

– Step 5: Calculate Fine Grid Convergence Index & Numerical Uncertainty, Factor of Safety, $F_s=1.25$

$$GCI_{fine}^{21} = \frac{1.25 \times e_a^{21}}{(r_{21}^p - 1)}$$

– Assumption that the distribution is Gaussian about the fine grid, 90% Confidence

$$\alpha_{num} = \frac{GCI_{fine}^{21}}{2}$$
Input Uncertainty, $U_{input}$

- Input error is based on a Taylor Series expansion in parameter space

$$u_{input} = \sqrt{\sum_{i=1}^{n} \left( \frac{\partial S}{\partial X_i} u_{xi} \right)^2}$$
Proposed Methodology without Test Data
Without Experimental Data??

Proposed Methodology **conservative estimate to envelop true value

If there is no experimental data, D=0, δ_D=0, and u_D=0.

\[ E = S - D = S \]

\[ \delta S = S - T \]

\[ E = S - D = T + \delta S - (T + \delta_D) = \delta_S - \delta_D = \delta_S \]

\[ u_{val} = k \left( \sqrt{u_{num}^2 + u_{input}^2 + u_D^2} \right) \]

\[ u_{val} = k \left( \sqrt{u_{num}^2 + u_{input}^2} \right) \]

Report the simulated result, S as

\[ S_{u_{val}}^{+} \]
Without Experimental Data -continued

- Report $S +/- u_s$
- $k$ – value *(Use Student-t Distribution)*
- Treat all input variables as ‘random’ and run separate CFD case
- Treat as an oscillatory convergence parameter

$$U_{Oscillatory} = \left| \frac{1}{2} (S_U - S_L) \right|$$

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Chapter 3: Applying the State of the Art CFD Uncertainty Analysis to a Backward Facing Step

Grid Refinement Study
CFD Uncertainty Analysis of Backward Facing Step
Results and Discussion
Backward Facing Step Example

AIAA-2013-0258
Velocity Magnitude Prediction – Backward Facing Step

Uniform Velocity Inlet
\[ U = 10 \text{ m/s} \]

Pressure Outlet
\[ P_{gage} = 0 \]
• There are 87 Different Input Parameters for the ke-realizable model in SimpleFoam
  – These include:
    • Boundary Conditions
    • Wall Functions
    • Fluid Properties
    • Turbulence Parameters
    • Solution Schemes
    • Solvers
    • Mesh
    • etc.
Uncertainty Variables Considered

<table>
<thead>
<tr>
<th>Type of Variable</th>
<th>Variables Xi</th>
<th>Value</th>
<th>Bias Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Conditions</td>
<td>$\varepsilon$: turbulent mixing length dissipation rate inlet (m²/s³)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>$k$: turbulent intensity kinetic energy inlet (m²/s²)</td>
<td>0.05</td>
<td>0.05</td>
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<tr>
<td></td>
<td>pressure outlet (Pa)</td>
<td>101325</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>velocity inlet (m/s)</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>Fluid Properties</td>
<td>kinematic viscosity $\nu$ represents air [0.50-100] deg C</td>
<td>1.79E-06</td>
<td>[13.6e-06 -&gt; 23.06e-06]</td>
</tr>
<tr>
<td>Grid Size</td>
<td>Method - Uses Oscillatory Uncertainty</td>
<td>1,192,000</td>
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<tr>
<td></td>
<td></td>
<td>1,862,500</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3,311,689</td>
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</tr>
<tr>
<td>Numerical</td>
<td>Method - Uses Richardson’s Extrapolation (ASME 5 Step Procedure) – Calculated for Velocity at each Cell</td>
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<tr>
<td>Solver</td>
<td>OpenFOAM (SimpleFoam) vs. Fluent</td>
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<tr>
<td>Turbulence Models</td>
<td>$ke$-realizable, kWSST, and SpalartAlmaras</td>
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</tbody>
</table>

Expanding the data reduction equation for the listed variables in order from top to bottom.

$$U_{CFD\_Velocity} = \left( \left( \frac{\partial V}{\partial e} \right)^2 B_e^2 + \left( \frac{\partial V}{\partial k} \right)^2 B_k^2 + \left( \frac{\partial V}{\partial p} \right)^2 B_p^2 + \left( \frac{\partial V}{\partial U} \right)^2 B_u^2 + \left( \frac{\partial V}{\partial \nu} \right)^2 B_{\nu u}^2 + \left( \frac{\partial V}{\partial g} \right)^2 B_g^2 \right)$$

$$+ \left( \left( \frac{\partial V}{\partial \text{num}} \right)^2 B_{\text{num}}^2 \right) + \left( \left( \frac{\partial V}{\partial \text{solver}} \right)^2 B_{\text{solver}}^2 \right) + \left( \left( \frac{\partial V}{\partial \text{turb}} \right)^2 B_{\text{turb}}^2 \right) \right)^{1/2}$$
The uncertainty for each of the following was calculated for each cell using the following method outlined by Stern, Wilson, Coleman, and Paterson. $S$ is the simulated result. For this case it is the upper velocity $S_U$ and the lower velocity $S_L$.

- **Epsilon turbulent mixing length dissipation rate inlet** $(m^2/s^3)$
- **k Turbulent Intensity kinetic energy inlet** $(m^2/s^2)$
- **Pressure outlet (Pa)**
- **Velocity Inlet (m/s)**
- **Kinematic viscosity $\nu=17.06e-06$ [13.6e-06 -> 23.06e-06] (m$^2$/s) represents air [0-50-100] degrees C**
- **Grid size**
- **Turbulence Models**
- **Solver**

$$U_{Oscillatory} = \frac{1}{2} (S_U - S_L)$$
Results (Oscillatory Variables)

**Epsilon Turbulent Mixing Length Dissipation Rate Inlet (m²/s³)**
0 – 1.155 percent

**Pressure Outlet (Pa)**
0 – 20 percent

**Kinematic Viscosity**
\( \text{nu}=17.066e-06 \text{ [13.6e-06 -> 23.06e-06] (m}^2/\text{s}) \text{ represents air [0-50-100] degrees C} \n0 – 27.727 percent

**Turbulence Models**
> 100 %

**K Turbulent Intensity Kinetic Energy Inlet (m²/s²)**
0 – 0.785 percent

**Velocity Inlet (m/s)**
0 – 6.558 percent

**Grid Size**
0 – 698 percent

**Solver**
> 30 %

Percent – is the percentage change in local velocity
Results (Monotonic Convergence)

Numerical – ASME V&V-20-2009 5 Step Procedure

For a grid size of 1,192,000 cells [grid 2 -1,862,500 cells], [grid3 - 3,311,689 cells], the uncertainty in the velocity prediction was 0 – 5300 percent as shown in Figure 11 as estimated by Richardson’s extrapolation method.
The highest uncertainty is +/- 4.85 m/s.
Conclusion of AIAA-2013-0258

• This paper outlines an uncertainty analysis for the ke realizable turbulence model for a backward facing step.
• The velocity magnitude was predicted using CFD.
• The uncertainty parameters listed in the Table were analyzed using an oscillatory convergence calculation or a monotonic convergence calculation.
• Plots of the velocity magnitude can be combined with a corresponding uncertainty plot for an accurate velocity prediction.
• Numerical Uncertainty using ASME 5 Step Procedure produced un-realistic results
Conclusion /Recommendation

- The following input uncertainty's are recommended

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<thead>
<tr>
<th>Type of Variable</th>
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<th>Uncertainty</th>
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</thead>
<tbody>
<tr>
<td>Boundary Conditions</td>
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<td>1.2% of local velocity</td>
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<td>k turbulent intensity kinetic energy inlet (m2/s2)</td>
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<td>0.05</td>
<td>0.8 % of local velocity</td>
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<tr>
<td></td>
<td>pressure outlet (Pa)</td>
<td>101325</td>
<td>2%</td>
<td>10x the variation</td>
</tr>
<tr>
<td></td>
<td>velocity inlet (m/s)</td>
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<td>0.5</td>
<td>1.3x the variation</td>
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<td>kinematic viscosity nu represents air [0-50-100] deg C</td>
<td>1.79E-06</td>
<td>[13.6e-06 -&gt; 23.06e-06]</td>
<td>28% of the local velocity</td>
</tr>
<tr>
<td>Grid Size</td>
<td>Method - Uses Oscillatory Uncertainty</td>
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<td>30% of the local velocity</td>
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<tr>
<td>Solver</td>
<td>OpenFOAM (SimpleFoam) vs. Fluent</td>
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<td>Future work will consider more turbulence models</td>
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</tbody>
</table>
Chapter 4: Spacecraft ECS System Overview and Modeling
Publicly Available Information on EELV ECS Systems Modeling and CFD Analysis of (3) Generic Non-proprietary

Environmental Control Systems and Spacecraft Configurations
Spacecraft / ECS System

AIAA-2014-0440
• The Rockets Behind the Missions:
  – Delta II
  – Delta IV
  – Atlas V
  – Pegasus
  – Taurus
  – Falcon 9

• http://www.nasa.gov/centers/kennedy/launchingrockets/
Each of these vehicles have a Payload Planners Guide or Users Guide


Delta II

- Air-conditioning is supplied to the spacecraft via an umbilical after the payload fairing is mated to the launch vehicle.
- The payload air-distribution system provides air at the required temperature, relative humidity, and flow rate as measured.
- The air-distribution system uses a diffuser on the inlet air-conditioning duct at the fairing interface.
- If required, a deflector can be installed on the inlet to direct the airflow away from sensitive spacecraft components.
- The air can be supplied to the payload between a rate of
  - 1300 to 1700 scfm.
- Diameter of Fairing is 3 meters.

Delta IV

• The air is supplied to the payload at a maximum flow rate of 36.3 kg/min to 72.6 kg/min (80 to 160 lb/min) for 4-m fairing launch vehicles and 90.7 kg/min to 136.0 kg/min (200 to 300 lb/min) for 5-m fairing launch vehicles.

• Air flows around the payload and is discharged through vents in the aft end of the fairing.

• Fairing sizes 4 meter and 5 meters in diameter

• Internal ducting defectors in the PLF direct the gas upward to prevent direct impingement on the spacecraft.
• The conditioning gas is vented to the atmosphere through one-way flapper doors below the spacecraft.
• The PLF air distribution system will provide a maximum air flow velocity in all directions of no more than 9.75 mps (32 fps) for the Atlas V 400 and 10.67 mps (35 fps) for the Atlas V 500.
• There will be localized areas of higher flow velocity at, near, or associated with the air conditioning outlet.
• Maximum air flow velocities correspond to maximum inlet mass flow rates.
• Reduced flow velocities are achievable using lower inlet mass flow rates.

• Flow Rates
  A) Atlas V 400: 0.38–1.21 kg/s ±0.038 kg/s (50–160 lb/min ±5 lb/min),
  B) Atlas V 500: 0.38–2.27 kg/s ±0.095 kg/s (50–300 lb/min ±12.5 lb/min)

• Fairing sizes are 4 meters and 5 meters in diameter

Pegasus

- The fairing is continuously purged with filtered air.
- The flowrate of air through the fairing is maintained between 50 and 200 cfm.
- The air flow enters the fairing forward of the payload and exits aft of the payload. There are baffles on the inlet that minimize the impingement velocity of the air on the payload.
- Fairing diameter is 0.97 meters

http://www.orbital.com/NewsInfo/Publications/Pegasus_UG.pdf
• Upon encapsulation within the fairing and for the remainder of ground operations, the payload environment will be maintained by the Taurus Environmental Control System (ECS).

• Fairing inlet conditions are selected by the Customer

• Fairing diameters are 63 inches and 92 inches

Falcon 9

- Once fully encapsulated and horizontal, the Environmental Control System (ECS) is connected
- Payload environments during various processing phases are:
  - In hanger, encapsulated – Flow Rate: 1,000 cfm
  - During rollout: 1,000 cfm
  - On pad: Variable from 1000 to 4500 cfm
- Fairing diameter is 5.2 meters
(3) Configurations

- Fairing Sizes are approximately 1m, 1.6m, 2.3m, 3m, 4m, 5m in diameter.
- (3) generic fairing diameters are selected to envelop the EELV fairing configurations
  - 0.75m
  - 3.5 m
  - 5.5 m
- Inlet Conditions range from 1000 cfm to 4500 cfm
- Spacecraft diameters range with fairing sizes, a generic spacecraft was drawn and scaled accordingly
(3) Configurations

- CAD model of the spacecraft was created in Pro/ENGINEER, 0.75m
(3) Configurations

- 3.5m
(3) Configurations

• 5.5m
CFD Modeling

• Snappy Hex – Mesher

0.75m Configuration (6762865 number of cells)

3.5m Configuration (8594480 number of cells)

5.5m Configuration (6980673 number of cells)
CFD Modeling

• OpenFoam - SimpleFoam

0.75m Configuration (6762865 number of cells)

3.5m Configuration (8594480 number of cells)

5.5m Configuration (6980673 number of cells)
Chapter 5: Computational Fluid Dynamics Uncertainty Analysis

Interpolation Scheme needed for CFD Uncertainty Analysis
Feasibility of using Richardson’s Extrapolation for Entire Computational Domain
Proposed CFD Uncertainty Method Compared to Exact Solution – Laminar Flow Between Parallel Plates
Proposed CFD Uncertainty Method Applied to Heat Transfer over a Flat Plate
Interpolation Method needed for Numerical Uncertainty Analysis of Computational Fluid Dynamics

AIAA-2014-1433
Summary of Richardson’s Extrapolation

• Navier Stokes Equations
  – 2\textsuperscript{nd} order, non-homogeneous, non-linear partial differential equations

• Richardson’s Extrapolation is used to produce 4\textsuperscript{th} order accurate solution from separate 2\textsuperscript{nd} order accurate Navier Stokes Solutions
Summary of Richardson’s Extrapolation


- Assumptions
  1. Three discrete solutions are in the asymptotic range
  2. Meshes have a uniform spacing over the domain
  3. Meshes are related through systematic refinement
  4. Solutions are smooth
  5. Other sources of numerical error are small
Solver Interpolation

- **FLUENT**
  - Includes a Mesh-to-Mesh Interpolation
  - Performs a zeroth-order (nearest neighbor) interpolation
  - Designed for initial conditions from a previous solution

- **OPENFOAM**
  - Mapfields function interpolation
  - Used for initialization of a solution from a previous model

- Using these ‘zeroth-order’ interpolation schemes is not sufficient for comparing errors from the mesh
Matlab Interpolation Schemes

- Matlab
  - High level language used for numerical computations
- CFD data is in various forms
  - 1D, 2D, 3D, uniform, non-uniform
  - Generic Scheme is sought for all CFD data

<table>
<thead>
<tr>
<th>Interpolation Method</th>
<th>interp1</th>
<th>interp2</th>
<th>interp3</th>
</tr>
</thead>
<tbody>
<tr>
<td>nearest' - Nearest neighbor interpolation</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>linear' - Linear interpolation (default)</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>spline' - Cubic spline interpolation</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>pchip' - Piecewise cubic Hermite interpolation</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>cubic'</td>
<td>X</td>
<td>X (uniformly-spaced only)</td>
<td>X (uniformly-spaced only)</td>
</tr>
<tr>
<td>v5cubic' - cubic interpolation used in Matlab 5</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example Problem

– Fully developed flow between parallel plates
  • Exact Solution to Navier Stokes
  • Provide a good example of errors that can be induced from interpolation

\[
\mathbf{V} = -\frac{1}{12\mu} \left( \frac{\partial P}{\partial x} \right) a^2
\]

\[
u = \frac{a^2}{2\mu} \left( \frac{\partial P}{\partial x} \right) \left[ \left( \frac{y}{a} \right)^2 - \left( \frac{y}{a} \right) \right]
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>(\text{rho}) (kg/m(^3))</td>
<td>1.225</td>
</tr>
<tr>
<td>(\mu) (Ns/m(^2))</td>
<td>0.00001789</td>
</tr>
<tr>
<td>(d\rho/dx) (N/m(^3))</td>
<td>-0.004</td>
</tr>
</tbody>
</table>
Example Problem

- Constructed a CFD Model in FLUENT
  - 3 Grids
    - Coarse, 7,140 Cells
    - Medium, 14,186 Cells
    - Fine, 24,780 Cells
Example Problem

- Interpolation Direction?
  1. Interpolate Coarse and Medium Mesh -> Fine

\[ \varepsilon_{21} = \varphi_2 - \varphi_1 \]
\[ \varepsilon_{32} = \varphi_3 - \varphi_2 \]

2. Interpolate Medium and Fine Mesh -> Coarse

\[ \varphi_1 \]
\[ \varphi_2 \]
\[ \varphi_3 \]
1. Linearly Interpolate Coarse and Medium Mesh -> Fine

Max % Error Extrapolated Values | Average % Error Extrapolated Values
--- | ---
0.8950 | 0.0596
Example Problem

2. Linearly Interpolate Fine and Medium Mesh -> Coarse

![Figure showing interpolation of fine, medium, and coarse meshes]

<table>
<thead>
<tr>
<th>Max % Error Extrapolated Values</th>
<th>Average % Error Extrapolated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0792</td>
<td>0.0175</td>
</tr>
</tbody>
</table>
Example Problem

- Interpolation Direction?

1. Interpolate Coarse and Medium Mesh -> Fine

<table>
<thead>
<tr>
<th>Extrapolated Solution on Fine Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position (m)</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
<tr>
<td>0.0100</td>
</tr>
<tr>
<td>0.0200</td>
</tr>
<tr>
<td>0.0300</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Max % Error Extrapolated Values</th>
<th>Average % Error Extrapolated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8950</td>
<td>0.0596</td>
</tr>
</tbody>
</table>

2. Interpolate Medium and Fine Mesh -> Coarse

<table>
<thead>
<tr>
<th>Extrapolated Solution on Coarse Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position (m)</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
<tr>
<td>0.0100</td>
</tr>
<tr>
<td>0.0200</td>
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<td>0.0300</td>
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</thead>
<tbody>
<tr>
<td>0.0792</td>
<td>0.0175</td>
</tr>
</tbody>
</table>
## Example Problem

<table>
<thead>
<tr>
<th>Grids</th>
<th>Max % Error</th>
<th>Average % Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Coarse vs Exact)</td>
<td>0.1910</td>
<td>0.1265</td>
</tr>
<tr>
<td>(Medium vs Exact)</td>
<td>0.0969</td>
<td>0.0367</td>
</tr>
<tr>
<td>(Fine vs Exact)</td>
<td>0.0289</td>
<td>0.0121</td>
</tr>
</tbody>
</table>

1. **Linearly Interpolated Coarse, Medium to Fine**

| (Interpolated Coarse vs Exact) | 1.9760 | 0.2528 |
| (Interpolated Medium vs Exact) | 0.6322 | 0.0679 |

2. **Linearly Interpolated Medium, Fine to Coarse**

| (Interpolated Medium vs Exact) | 0.0728 | 0.0362 |
| (Interpolated Fine vs Exact)   | 0.0787 | 0.0223 |

**Extrapolated**

<table>
<thead>
<tr>
<th></th>
<th>Max % Error</th>
<th>Average % Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Linear Interpolation Coarse and Medium to Fine (Extrapolated vs Exact)</td>
<td>0.8950</td>
<td>0.0596</td>
</tr>
<tr>
<td>2. Linear Interpolation Medium and Fine to Coarse (Extrapolated vs Exact)</td>
<td>0.0792</td>
<td>0.0175</td>
</tr>
</tbody>
</table>
Example Problem

- Interpolating to the coarse grid was selected
- Other interpolation methods
  - “nearest”  – Fluent’s Mesh-to-Mesh

  - “linear”  – Matlab
    \[
    y_{fi} = \text{interp1}(\text{fine}(:,2),\text{fine}(:,1),\text{coarse}(:,2),'linear')
    \]

  - “cubic”  – Matlab
    \[
    y_{fi} = \text{interp1}(\text{fine}(:,2),\text{fine}(:,1),\text{coarse}(:,2),'cubic')
    \]
Example Problem

- “nearest” – Fluent’s Mesh-to-Mesh

Zeroth Order Interpolation
Example Problem

- "linear"

\[
yfi = \text{interp1}(\text{fine}(::2), \text{fine}(::1), \text{coarse}(::2), 'linear')
\]

1st Order Interpolation
Example Problem

- "cubic"
- Matlab

\[ y_{fi} = \text{interp1}(\text{fine}(:,2), \text{fine}(:,1), \text{coarse}(:,2), 'cubic') \]
Matlab Interpolation Schemes

• Extending the Interpolation Schemes to 2D and 3D
  • Interp2 and Interp3 Matlab Functions
    • Require use of MeshGrid
      • Transforms the domain of vectors into arrays
      • For Meshes in the 4 million to 8 million Cell Range
        • Error “Maximum variable size allowed by program is exceeded”
  • Griddata Function
    • Nearest, Linear, Natural, Cubic, and v4
    • Nearest, Linear, and Natural are the only options available in 2D and 3D

• The only options available for 1D, 2D, and 3D
  • Interp1 and Griddata – ‘nearest’ and ‘linear’
3D Example

- Airflow around encapsulated spacecraft
  - Matlab griddata ‘linear’ option used
  - Interpolating Fine and Medium Grid onto Coarse Grid
3D Example

• Comparing using a Line Plot
3D Example

• Comparing using a Line Plot
• By comparing the interpolation schemes in one, two, and three dimensions and investigating the options that are readily available in Matlab
  • Recommend the “linear” option be used when comparing the error or uncertainty due to the grid
    • interp1 or griddata Matlab commands
• If coarse grid has the level of detail required
  • Recommend interpolating from the fine and medium grids onto the coarse grid
    • Lower Error in the Extrapolated Solution
    • Smaller Data Set
  – Future Work include higher order interpolation schemes in 3D (Radial Basis Function Interpolation, 4th order)
Feasibility of using Richardson’s Extrapolation for Entire Computational Domain
Summary of Method

- Following method outlined by Stern, Wilson, Coleman, and Paterson
- Convergence studies require a minimum of three solutions to evaluate convergence with respect to an input parameter. Consider the situation for 3 solutions corresponding to fine $S_{k1}$, medium $S_{k2}$, and coarse $S_{k3}$ values for the $kth$ input parameter. Solution changes $\varepsilon$ for medium-fine and coarse-medium solutions and their ratio $R_k$ are defined by:

\[
\varepsilon_{21} = S_{k2} - S_{k1} \\
\varepsilon_{32} = S_{k3} - S_{k2} \\
R_k = \frac{\varepsilon_{21}}{\varepsilon_{32}}
\]

- Three convergence conditions are possible:
  - Monotonic convergence: $0 < R_k < 1$
  - Oscillatory convergence: $R_k < 0$
  - Divergence: $R_k > 1$
Results (Monotonic Convergence?)  
Backward Facing Step

Three convergence conditions are possible:

Monotonic convergence: $0 < R_k < 1$

Oscillatory convergence: $R_k < 0$

Divergence: $R_k > 1$

(i) – Cells with Monotonic Convergence $0 < R_k < 1$

(ii) – Cells with Oscillatory Convergence $R_k < 0$

(iii) – Cells with Divergence $R_k > 1$
Results (Monotonic Convergence?)
Spacecraft / ECS System

Three convergence conditions are possible:
Monotonic convergence: \( 0 < R_k < 1 \)
Oscillatory convergence: \( R_k < 0 \)
Divergence: \( R_k > 1 \)
Proposed CFD Uncertainty Method Compared to Exact Solution – Laminar Flow Between Parallel Plates
Example Fully Developed Laminar Flow between Parallel Plates

- Heat Transfer Correlations, Traditional
- The uncertainty analysis will follow the methodology laid out by Coleman and Steele (Experimentation and Uncertainty Analysis for Engineers, 2nd ed, J. Wiley and Sons, 1999). This methodology is in line with the ISO Guide to the Expression of Uncertainty in Measurement (1993).

\[
U = \left( \sum_{i=1}^{J} \left( \frac{\partial r}{\partial x_i} \right)^2 B_i^2 \right) + 2 \sum_{i=1}^{J} \sum_{k=i+1}^{J} \left( \frac{\partial r}{\partial x_i} \frac{\partial r}{\partial x_k} [B_i B_k]_{\text{correlated}} \right) + \sum_{i=1}^{J} \left( \frac{\partial r}{\partial x_i} \right)^2 p_i^2 \right)^{1/2}
\]

Bias \hspace{2cm} Correlated \hspace{2cm} Random

\[
u = \frac{a^2}{2\mu} \left( \frac{\partial P}{\partial x} \right) \left[ \left( \frac{y}{a} \right)^2 - \left( \frac{y}{a} \right) \right]
\]

\[
\frac{\partial u}{\partial \mu} = -\frac{\partial P}{\partial x} \cdot y \cdot (y - a)/(2\mu)
\]

\[
\frac{\partial u}{\partial \frac{\partial P}{\partial x}} = \frac{a^2}{2\mu} \left[ \left( \frac{y}{a} \right)^2 - \left( \frac{y}{a} \right) \right]
\]
Example Fully Developed Laminar Flow between Parallel Plates

- Uncertainty for 5% Bias in pressure gradient and viscosity

\[ u_u = \left( \left( -\frac{\delta P}{\delta x} \cdot y \cdot (y - a)/(2\mu) \right)^2 B^2_{\mu} \right)^{1/2} + \left( \frac{a^2}{2\mu} \left[ \left( \frac{y}{a} \right)^2 - \left( \frac{y}{a} \right) \right]^2 B_{\delta P}^2 \right)^{1/2} \]
Example Fully Developed Laminar Flow between Parallel Plates

- Numerically Evaluating (Traditional):

**Exact Solution with Uncertainty**

**Uncertainty**
**CFD Uncertainty Cases**

1. Coarse Grid
2. Medium Grid
3. Fine Grid
4. Velocity Low
5. Velocity High
6. Density Low
7. Density High
8. Outlet Pressure Low
9. Outlet Pressure High
10. Solver

<table>
<thead>
<tr>
<th>Number of Cases</th>
<th>Degrees of Freedom</th>
<th>Confidence 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>6.314</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2.92</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2.353</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>2.132</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>2.015</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>1.943</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>1.895</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>1.836</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>1.833</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>1.796</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>1.782</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
<td>1.771</td>
</tr>
</tbody>
</table>

\[
u_{val} = 1.833 \ast \left( \left( \left( \frac{\partial V}{\partial \text{num}} \right)^2 B_{num}^2 \right) + \left( \left( \frac{\partial V}{\partial \text{velocity}} \right)^2 B_{velocity}^2 \right) \right. \\
\left. + \left( \left( \frac{\partial V}{\partial \text{pressure}} \right)^2 B_{pressure}^2 \right) + \left( \left( \frac{\partial V}{\partial \text{rho}} \right)^2 B_{rho}^2 \right) + \left( \left( \frac{\partial V}{\partial \text{solver}} \right)^2 B_{solver}^2 \right)^{1/2} \right) / 2 (S_U - S_L) \]
Proposed Methodology using CFD Only

- Results for proposed methodology
Comparison of Methods

- Traditional vs. Proposed

**Velocity**

**Uncertainty**

Uncertainty in Proposed Method

Uncertainty in Traditional Method

CFD Uncert High

CFD Uncert Low
Comparison of Methods

- Traditional vs. Proposed
• Proposed Method Envelops the True value and uses only CFD Data to Estimate the Uncertainty for Laminar Flow between Parallel Plates
  – No Testing

• Proposed methodology can be used to conservatively estimate the uncertainty in CFD models
Computational Fluid Dynamics Uncertainty Analysis applied to Heat Transfer over a Flat Plate

APS-DFD13-2013-000087
Example Heat Transfer over Flat Plate

• Heat Transfer Correlations, Traditional
• The uncertainty analysis will follow the methodology laid out by Coleman and Steele (Experimentation and Uncertainty Analysis for Engineers, 2nd ed, J. Wiley and Sons, 1999). This methodology is in line with the ISO Guide to the Expression of Uncertainty in Measurement (1993).

\[ U = \left( \sum_{i=1}^{J} \left( \frac{\partial r}{\partial X_i} \right)^2 B_i^2 \right) + 2 \sum_{i=1}^{J} \sum_{k=i+1}^{J} \left( \frac{\partial r}{\partial X_i} \frac{\partial r}{\partial X_k} [B_i B_k]_{correlated} \right) + \sum_{i=1}^{J} \left( \frac{\partial r}{\partial X_i} \right)^2 P_i^2 \right)^{1/2} \]

Bias

Correlated

Random

\[ \frac{dh}{dV} = \frac{4ck\rho}{5\mu \left( \frac{LV\rho}{\mu} \right)^{1/5}} \]

\[ \frac{dh}{d\mu} = -\frac{4CVk\rho}{5\mu^2 \left( \frac{LV\rho}{\mu} \right)^{1/5}} \]

\[ \frac{dh}{d\rho} = \frac{4ckV}{5\mu \left( \frac{LV\rho}{\mu} \right)^{1/5}} \]

\[ \frac{dh}{dL} = \frac{4CVk\rho}{5L\mu \left( \frac{LV\rho}{\mu} \right)^{1/5}} - \frac{Ck \left( \frac{LV\rho}{\mu} \right)^{4/5}}{L^2} \]

\[ \frac{dh}{dk} = \frac{c}{L} \left( \frac{LV\rho}{\mu} \right)^{4/5} \]

\[ \frac{dh}{dC} = \frac{k}{L} \left( \frac{LV\rho}{\mu} \right)^{4/5} \]
Example Heat Transfer over Flat Plate

• Heat Transfer Correlation Uncertainty

\[ U_h = \left( \left( \left( \frac{\partial h}{\partial v} \right)^2 B_v^2 \right) + \left( \left( \frac{\partial h}{\partial \rho} \right)^2 B_{\rho}^2 \right) + \left( \left( \frac{\partial h}{\partial k} \right)^2 B_k^2 \right) + \left( \left( \frac{\partial h}{\partial \mu} \right)^2 B_{\mu}^2 \right) + \left( \left( \frac{\partial h}{\partial L} \right)^2 B_L^2 \right) + \right) + \]

\[ \left( \left( \frac{\partial h}{\partial C} \right)^2 P_C^2 \right) + 2 \left( \frac{\partial h}{\partial \rho} \right) \left( \frac{\partial h}{\partial k} \right) B_\rho B_k + 2 \left( \frac{\partial h}{\partial \rho} \right) \left( \frac{\partial h}{\partial \mu} \right) B_\rho B_\mu + \]

\[ 2 \left( \frac{\partial h}{\partial k} \right) \left( \frac{\partial h}{\partial \mu} \right) B_k B_\mu \right)^{1/2} \]
Example Heat Transfer over Flat Plate

- Plug in Partial Derivatives

\[
U_h = \left( \left( \frac{4ck\rho}{5\mu \left( \frac{LV\rho}{\mu} \right)^{1/5}} \right)^2 B_V^2 \right) + \left( \left( \frac{4ckV}{5\mu \left( \frac{LV\rho}{\mu} \right)^{1/5}} \right)^2 B_{\rho}^2 \right) + \left( \left( \frac{c}{L \left( \frac{LV\rho}{\mu} \right)^{4/5}} \right)^2 B_k^2 \right) + \\
\left( \left( \frac{\partial h}{\partial \mu} \right)^2 B_{\mu}^2 \right) + \left( \left( \frac{4CVk\rho}{5L\mu \left( \frac{LV\rho}{\mu} \right)^{1/5}} - \frac{ck \left( \frac{LV\rho}{\mu} \right)^{4/5}}{L^2} \right)^2 B_L^2 \right) + \left( \left( \frac{k}{L \left( \frac{LV\rho}{\mu} \right)^{4/5}} \right)^2 p_c^2 \right) + \\
2 \left( \frac{4ckV}{5\mu \left( \frac{LV\rho}{\mu} \right)^{1/5}} \right) \left( \frac{c}{L \left( \frac{LV\rho}{\mu} \right)^{4/5}} \right) B_{\rho} B_k + 2 \left( \frac{4ckV}{5\mu \left( \frac{LV\rho}{\mu} \right)^{1/5}} \right) \left( - \frac{4CVk\rho}{5\mu^2 \left( \frac{LV\rho}{\mu} \right)^{3/5}} \right) B_{\rho} B_{\mu} + \\
2 \left( \frac{c}{L \left( \frac{LV\rho}{\mu} \right)^{4/5}} \right) \left( - \frac{4CVk\rho}{5\mu^2 \left( \frac{LV\rho}{\mu} \right)^{3/5}} \right) B_k B_{\mu} \right)^{1/2}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity, V</td>
<td>3%</td>
</tr>
<tr>
<td>Density, rho</td>
<td>3%</td>
</tr>
<tr>
<td>Thermal Conductivity, k</td>
<td>3%</td>
</tr>
<tr>
<td>Viscosity, ( \mu )</td>
<td>3%</td>
</tr>
</tbody>
</table>
Example Heat Transfer over Flat Plate

- Numerically Evaluating (Traditional):

**Heat Transfer Coefficient**

**Uncertainty in Heat Transfer Coefficient**
Proposed Methodology using CFD Only

<table>
<thead>
<tr>
<th>CFD Uncertainty Cases</th>
<th>Number of Cases</th>
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<th>Confidence 90%</th>
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<tbody>
<tr>
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<td>2</td>
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<td>8</td>
<td>7</td>
<td>1.895</td>
</tr>
<tr>
<td>Thermal Conductivity High</td>
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<td>1.86</td>
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<td>Thermal Conductivity Low</td>
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<td>1.812</td>
</tr>
<tr>
<td>Viscosity High</td>
<td>12</td>
<td>11</td>
<td>1.778</td>
</tr>
<tr>
<td>SA Turbulence Model</td>
<td>13</td>
<td>12</td>
<td>1.782</td>
</tr>
</tbody>
</table>

\[
U_h = \left(\left(\frac{\partial h}{\partial v}\right)^2 B_v^2 + \left(\frac{\partial h}{\partial p}\right)^2 B_p^2 + \left(\frac{\partial h}{\partial k}\right)^2 B_k^2 + \left(\frac{\partial h}{\partial \mu}\right)^2 B_\mu^2 + \left(\frac{\partial h}{\partial c}\right)^2 P_c^2 \right) + 2 \left(\frac{\partial h}{\partial p}\right) \left(\frac{\partial h}{\partial k}\right) B_p B_k + 2 \left(\frac{\partial h}{\partial p}\right) \left(\frac{\partial h}{\partial \mu}\right) B_p B_\mu + 2 \left(\frac{\partial h}{\partial k}\right) \left(\frac{\partial h}{\partial \mu}\right) B_k B_\mu \right)^{1/2}
\]

\[u_{val} = 1.782 \times \left| \frac{1}{2} (S_U - S_L) \right|\]
Proposed Methodology using CFD Only

- Results for proposed methodology

### Heat Transfer Coefficient

- **Nominal CFD**
- **Medium grid CFD**
- **High grid CFD**
- **Low CFD**
- **High CFD**
- **Baseline CFD**
- **Grid CFD**
- **Velocity CFD**
- **Density CFD**
- **Thermal Conductivity CFD**

### Uncertainty in Heat Transfer Coefficient

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average Difference in htc (W/m²K)</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulence</td>
<td>0.693102673</td>
<td>1</td>
</tr>
<tr>
<td>Grid</td>
<td>0.130514851</td>
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Comparison of Methods

• Traditional vs. Proposed

Heat Transfer Coefficient

Uncertainty in Heat Transfer Coefficient

CFD Uncert High

Uncertainty in Proposed Method

CFD Uncert Low

Uncertainty in Traditional Method
Comparison of Methods

• Traditional vs. Proposed Average Heat Transfer Coefficient over Flat Plate

  – Traditional

  \[ h_{avg} = 2.66 +/- 0.74 \text{ [W/m}^2\text{K]} \]

  – Proposed CFD,

  \[ h_{avg} = 2.66 +/- 1.39 \text{ [W/m}^2\text{K]} \]
Conclusion

• Proposed Method Envelops the True value and uses only CFD Data to Estimate the Uncertainty for Heat Transfer over a Flat Plate
  – No Testing

• Proposed methodology can be used to conservatively estimate the uncertainty in CFD models
Chapter 6: Demonstration and Implementation of the Proposed CFD Uncertainty Method for Spacecraft ECS Systems

0.75m Configuration
3.5m Configuration
5.5m Configuration
ECS System Experimental Comparison
Spacecraft / ECS System

AIAA-2014-0440
Uncertainty Calculation

• Proposed Methodology

\[ S_{+u_{\text{val}}} \]

\[ u_{\text{val}} = k \left( \sqrt{u_{\text{num}}^2 + u_{\text{input}}^2} \right) \]

• Expanding

\[ u_{\text{val}} = k \left( \left( \frac{\partial V}{\partial \text{grid}} \right)^2 B_{\text{grid}}^2 + \left( \frac{\partial V}{\partial \text{pressure}} \right)^2 B_{\text{pressure}}^2 \right) + \left( \frac{\partial V}{\partial \text{velocity}} \right)^2 B_{\text{velocity}}^2 + \left( \frac{\partial V}{\partial \rho} \right)^2 B_{\rho}^2 \]

\[ + \left( \frac{\partial V}{\partial \text{wall functions}} \right)^2 B_{\text{wall functions}}^2 + \left( \frac{\partial V}{\partial \text{surface roughness}} \right)^2 B_{\text{surface roughness}}^2 \]

\[ + \left( \frac{\partial V}{\partial \text{compressibility}} \right)^2 B_{\text{compressibility}}^2 + \left( \frac{\partial V}{\partial \text{solver}} \right)^2 B_{\text{solver}}^2 \]

\[ + \left( \frac{\partial V}{\partial \text{turbulence}} \right)^2 B_{\text{turbulence}}^2 \right)^{1/2} \]
Uncertainty Calculation

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\[ u_{val} = 1.746 \times \left| \frac{1}{2} (S_U - S_L) \right| \]
Results 0.75m Configuration

- Solution and Uncertainty Contour Plots

`one_coarsegrid (m/s)`

- `one_coarsegrid (m/s)`

- `uncert (m/s)`

- `uncert (m/s)`
Results 0.75m Configuration

• Solution Line Plot
Results 0.75m Configuration

- Uncertainty Line Plot
Results 0.75m Configuration

- Uncertainty Ranking

- The uncertainty for each of the input variables were ranked by the non-dimensionalizing the difference in the results by the freestream value and ranking from greatest uncertainty to least uncertainty.

<table>
<thead>
<tr>
<th>Input Variable</th>
<th>Description</th>
<th>Bias</th>
<th>Mean Velocity Uncertainty (m/s)</th>
<th>Mean Non-Dimensionalized Uncertainty</th>
<th>Normalized Ranking %</th>
<th>Numbered Ranking</th>
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<td>1.3115</td>
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Results 3.5m Configuration

- Solution and Uncertainty Contour Plots

one_coarsegrid (m/s)

uncert (m/s)
Results 3.5m Configuration

- Solution Line Plot
Results 3.5m Configuration

- Uncertainty Line Plot
# Results 3.5m Configuration

## Uncertainty Ranking

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<thead>
<tr>
<th>Input Variable</th>
<th>Description</th>
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<th>Mean Velocity Uncertainty (m/s)</th>
<th>Mean Non-Dimensionalized Uncertainty</th>
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Results 5.5m Configuration

- Solution and Uncertainty Contour Plots
Results 5.5m Configuration

• Solution Line Plot
Results 5.5m Configuration

- Uncertainty Line Plot
## Results 3.5m Configuration

### Uncertainty Ranking

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<th>Mean Non-Dimensionalized Uncertainty</th>
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Comparison to Previous LDV Test

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<td>Kinematic Viscosity [0-100] Deg C</td>
<td>[1.36, 1.50, 2.306] e-5 m2/s</td>
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<td>ke-realiziable, kwsst, SA</td>
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Comparison to Previous LDV Test
Comparison to Previous LDV Test
Comparison to Previous LDV Test
Comparison to Previous LDV Test

Uz (z=2in)

Uy (z=2in)

Uz (z=0in)

Uy (z=0in)
Comparison to Previous LDV Test

• Assumed Confidence Interval from Student T distribution and CFD Uncertainty Prediction
  – 90%

• LDV Data
  – Total of 1085 Points Measured
  – 977 Points were inside the 90% CFD Uncertainty Methodology
  – 108 Points Outside CFD Uncertainty Prediction
    • $\frac{977}{1085} = 0.90046$
    • ~90%
AIAA-2014-0440 Conclusion & Recommendation

- Proper validation with experimental data should be used to verify ECS impingement requirements
- This research proposes a CFD uncertainty methodology when experimental data is unavailable and unobtainable
  - Couples Student-T Distribution to the number of CFD models and input parameters
  - All input parameters considered had the same order of magnitude uncertainty
Chapter 7: Conclusions and Future Work
Conclusion & Recommendation

• Proper validation with experimental data should be used when possible

• This research proposes a CFD uncertainty methodology when experimental data is unavailable and unobtainable
  • Couples Student-T Distribution to the number of CFD models and input parameters
  • Methodology proved accurate for:
    • Fully Developed Laminar Flow between Parallel Plates
    • Heat Transfer over a Flat Plate
    • Spacecraft / Fairing Environmental Control Systems
Future Work

• Run Experimental Configurations to further prove methodology
  • Evaluate which models are most realistic
  • Try to reduce conservatism in proposed methodology (if possible)

• Expand Method beyond internal, low-speed, incompressible
  • Other Flow Regimes: External, Compressible, Unsteady
  • Expand Beyond Fluid Dynamics and Heat Transfer
Summary

• “Uncertainty (of measurement) – parameter, associated with the result of a measurement, that characterizes the dispersion of values that could be reasonably attributed to the quantity intended to be measured” – International Vocabulary of Basic and General Terms in Metrology

• Replace
  – Measurement -> Simulation
  – Measured -> Simulated
Thank You

• Chair: Dr. Alain Kassab

• Committee Members
  • Dr. Tuhin Das
  • Dr. Jeffrey Kauffman
  • Dr. Brian Moore

• NASA, Launch Services Program
  • Dr. Paul Schallhorn
  • Lennie Duncil
  • Larry Craig