Statistical Modeling for Radiation Hardness Assurance

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List of Abbreviations

BJT—Bipolar Junction Transistor
CL—Confidence Level
DSEE—Destructive SEE
JFET—Junction Field Effect Transistor
MOSFET—Metal-Oxide-Semiconductor Field Effect Transistor
RLAT—Radiation Lot Acceptance Test
SEB—Single-Event Burnout
SEE—Single-Event Effect
SEGR—Single-Event Gate Rupture
SEL—Single-Event Latchup
SOI—Silicon on Insulator
TID—Total Ionizing Dose
RPP—Rectangular Parallelepiped
WC—Worst Case
Outline

I. Current RHA and It’s Statistical Models
II. Other Sources of Data and How They Are Used
III. Bayesian Probability and Why It’s Well Suited to RHA
IV. Example: Bayesian SEE
V. A More Complicated Example: Bayesian TID
VI. Flight Heritage as data
VII. Fitting data and Bounding rates in SEE
VIII. Using Statistical Modelling to Meet Challenges of Destructive SEE
IX. Conclusions
Radiation Hardness Assurance (RHA) Assumptions

- Single-Event Effect (SEE) Hardness Assurance Assumptions
  - Poisson errors on event counts dominate
  - Part-to-part and lot-to-lot variation in SEE response (usually) negligible
    - Destructive SEE (DSEE) may be exception
  - Charge collected defines SEE response
  - Mitigation: Large event counts minimize Poisson errors on cross sections
  - Rate constant; can happen any time

- Total Ionizing Dose (TID) Hardness Assurance Assumptions
  - Dominant errors from sampling due to part-to-part and lot-to-lot variation
  - Mitigation: Increase sample size and/or understand failure mechanisms and variation
  - Failures tend to cluster around mean failure dose
How do radiation errors/failures compare to reliability issues?

- **Failure Rate Decreasing**
  - Infant Mortality: SEE Look Like This. Redundancy Works.
  - No Radiation Analogue
- **Constant Failure Rate**
  - Failures Independent
- **Increasing Failure Rate; Failures Cluster**
  - Wearout Region
  - TID Look Like This. Redundancy is ineffective.

**Equivalent (accelerated + actual) Time (yrs)**

- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8

Current RHA and RHA Statistical Models

- Because radiation testing is destructive
  - test a sample representative of flight parts
  - use results to bound flight performance

- Least restrictive model is binomial
  - Assumes only that all parts from same distribution
  - Makes no assumptions about distribution of parts (failures, parametric changes, etc.)
  - Outcome is binary (Pass/Fail),
    - defines success probability, $P_s$ and failure probability $P_f=1-P_s$
  - May be only choice when failure distribution highly uncertain

- Lack of guidance for derating and large sample size requirements provide incentive to adopt more restrictive model
  - Restrict population to a well behaved subset or “Lot”
  - Normal or Lognormal often assumed

- Disadvantages of binomial sampling
  - No guidance on effectiveness of derating/increased margin
  - Requires large samples (e.g. 22 samples for $P_s=90\%$ @90\% confidence level (CL))

Conventional model for TID RHA

- Conventional TID RHA assumes “lot” is well behaved and unimodal
  - Wafer diffusion lot usually (not always) shows homogeneous performance
  - Assuming Normal distribution Sample mean and variance converge to population values rapidly
  - Can use one-sided tolerance limits—KTL(Ps,CL,n)—to bound performance and ensure success probability Ps with confidence CL for a test sample of size n
Distribution Pathologies do Occur

Analog Devices (ADI) OP484 shows bimodal response even in same wafer lot

Similar bimodal response seen in National Semiconductor LM111 (Kreig et al. NSREC 1999)

Some devices show thick tails

NES 2N5019 Junction Field Effect Transistor (JFET) showed high variation in IGSS after 0.3 and 1 Mrad(Si) (x-axis is logarithmic)

ADI AD590 also shows thick tail

When There’s No Lot-Specific Data

- **Historical Data**—Test data for the same part type/# as the flight parts, taken under similar application conditions
- **Similarity/Process Data**—Test data for parts with similar function fabricated at the same facility in the same process, taken under comparable application conditions
- **Heritage Data**—Data regarding past flight use of the same part type/#; heritage mission environments must usually be at least as severe as the current mission and application conditions must be comparable
- **When you are really desperate**
  - Physics—What do we know about the part and the radiation effects mechanisms that can place limits on their severity—example: Silicon-on-Insulator (SOI) limits SEE susceptibility
  - Technology trends—How have the susceptibilities changed over previous “similar” generations—e.g. SEL susceptibility for commercial SRAMs with minimum Complementary Metal Oxide Semiconductor (CMOS) feature size from 0.35 down to 0.11 microns
  - Technology generation—Susceptibility and trends thereof for comparable technologies from the same and even other vendors
  - Expert opinion—What do the smart kids think?
- **None of these lines of evidence may restrict susceptibility in a meaningful way**
Structure of Constraining Data

- All relevant data (ARD)
  - Physics
  - Technology Trends
  - Part Generation
  - Expert Opinion

- Similar Parts
  - Same Fabrication Facility
  - Same Process
  - Similar Design Rules

- Historical Data
  - Same Part Type, #

- Flight Lot

- Flight Part

- Less Representative

SEE Risk Data Structure

Flight Part SEE Risk

Flight Part SEE Rate(s)
- Part-to-Part Negligible

Flight Lot SEE Rate(s)
- Lot-to-Lot Negligible

Historical Part SEE Rate(s)
- Part-Type-to-Part-Type Variation

Flight Part SEE Consequence(s)
- Part-to-Part Negligible

Flight Lot SEE Consequence(s)
- Lot-to-Lot Negligible

Historical Part SEE Consequence(s)
- Part-Type-to-Part-Type Variation

Distribution of Rates
- Mean, $\mu_R$
- Std. Dev., $\sigma_R$

Distribution of Consequences
- Mean, $\mu_C$
- Std. Dev., $\sigma_C$
Data Structure for TID RHA

Flight Part TID Performance

Lot mean performance

Lot perf. Std. Dev.

Flight-Lot TID performance

Mean of Historical Lot Means

Std. Dev. of Historical Lot Means

Historical TID performance

Mean and Std. Dev. of mean Hist. Means Across Process

Mean and Std. Dev. Of Std. Devs. of Hist. Means Across Process

Mean and Std. Dev. of mean Hist. Std. Devs.

Mean and Std. Dev. of Std. Devs. of Hist. Std. Devs. Across Process

Other

Process TID performance

Other

Note: TID RHA based on risk avoidance, so we’re interested only in failure probability, not consequences.
Two Types of Probability

• Physical probability
  • Indeterminacy is inherent in process;
  • no amount of additional data will change your odds for betting on outcome

• Subjective probability
  • indeterminacy reflects our limited knowledge
  • probability can change as our knowledge increases

• Analogues to RHA
  • Probability of observing an SEE for a particular ion is physical
  • Probability our lot has a majority of parts that will meet mission requirements is subjective
Bayes’ Theorem

- Bayes Theorem:

\[ P(\text{Model} | \text{Data}) = \frac{P_{\text{pr}}(\text{Model})P(\text{Data} | \text{Model})}{\sum_{\text{Models}} P_{\text{pr}}(\text{Model})P(\text{Data} | \text{Model})} \]

- \( P_{\text{pr}}(\text{Model}) \) = Prior is probability of our model before we consider new data
- \( P(\text{Model} | \text{Data}) \) is updated (posterior) probability—the probability that takes into account data AND prior expectations
- Denominator ensures normalization, but is also a measure of probability for all models

- Bayesian probabilities are subjective
  - Depend on our knowledge at the time of calculation
  - Can change as we add new knowledge
  - May be associated with a “confidence”

- Bayes Theorem is the most efficient way to update probabilities given new data

\[ P(\text{Data} | \text{Model}) \] is likelihood, L

\[ L = P(X_1) \times P(X_2) \times P(X_3) \times P(X_4) \times P(X_5) \]

Data \( \{X_1, \ldots, X_5\} \) drawn at random. Candidate model is \( P(X) \) above.

Bayesian Approach to RHA

1) Must relate input data to SEE/TID failure rates or consequences
2) Input data must produce fairly compact, well behaved distributions

Effect of Prior and Likelihood

3 Heads In 10 Tosses

30 Heads In 100 Tosses

Probability (P(Heads)) for different Priors as data increases

Minimizing Subjectivity

- Uninformative Priors—Broad, slowly varying (flat) Priors give rise to Posterior distributions that reflect the data (likelihood)
  - If Prior is 0 anywhere, Posterior also 0 there (impossible means impossible)
  - Some Uninformative priors (for discrete distributions) are called Maximum Entropy priors

- Empirical Bayes allows looking at the data before developing Prior
  - Can locate Prior at the maximum of the likelihood and make it very broad
    - This means you think the likelihood probably gives the best guess for the best fit, but allow for significant sampling error
    - Usually yields good results unless there are serious sampling or systematic errors
    - Some Bayesians refer to this as “cheating”

- Maximize the data to update the Prior and dilute its influence.

- Also, you don’t have to try only a single Prior—you could even attach prior probabilities to your priors if you want to go really meta. Or you could try several Priors just to gauge the dependence of results on the Prior.
Data Sources

Table I: SET Data for ADI and Linear Technology (LTC) Op Amps

<table>
<thead>
<tr>
<th>Vendor/ Process</th>
<th>SET Rate (day⁻¹)</th>
<th>WC duration</th>
<th>WC amplitude</th>
<th>Supply—Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>RH108[2]</td>
<td>0.011</td>
<td>7 μs</td>
<td>4.5 V</td>
<td>0/12V—6V</td>
</tr>
<tr>
<td>RH118[2]</td>
<td>0.046</td>
<td>&gt;5 μs</td>
<td>2 V</td>
<td>-8V/+8V—0V</td>
</tr>
<tr>
<td>RH1014[2]</td>
<td>0.029</td>
<td>&gt;35 μs</td>
<td>rail-to-rail</td>
<td>-12V/+12V—6V</td>
</tr>
<tr>
<td>RH1499[3]</td>
<td>0.0082</td>
<td>13 μs</td>
<td>4.4 V</td>
<td>-15V/+15V—4V</td>
</tr>
<tr>
<td>RH1078[4]</td>
<td>0.04</td>
<td>30 μs</td>
<td>2 V</td>
<td>0/10V—2V</td>
</tr>
<tr>
<td>OP27[5]</td>
<td>ADI—Bipolar&gt;2.5 μm</td>
<td>0.036</td>
<td>10 μs</td>
<td>10V</td>
</tr>
<tr>
<td>OP113[2]</td>
<td>ADI—Bipolar&gt;2.5 μm</td>
<td>0.01</td>
<td>&gt;2 μs</td>
<td>1.7 V</td>
</tr>
<tr>
<td>OP270[2]</td>
<td>ADI—Bipolar&gt;2.5 μm</td>
<td>0.35</td>
<td>1.2 μs</td>
<td>3 V</td>
</tr>
<tr>
<td>OP400[2]</td>
<td>ADI—Bipolar&gt;2.5 μm</td>
<td>0.56</td>
<td>10 μs</td>
<td>3 V</td>
</tr>
<tr>
<td>OP05[5]</td>
<td>ADI—Bipolar&gt;2.5 μm</td>
<td>--</td>
<td>12 μs</td>
<td>--</td>
</tr>
<tr>
<td>OP15[5]</td>
<td>ADI—Bipolar&gt;2.5 μm</td>
<td>--</td>
<td>15 μs</td>
<td>--</td>
</tr>
</tbody>
</table>

Table II: SEL Data for ADI 0.6 μm CMOS ADCs and DACs

<table>
<thead>
<tr>
<th>Part #</th>
<th>Process</th>
<th>Onset LET (MeVcm²/mg)</th>
<th>Test Facility</th>
<th>Çlim (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD5305</td>
<td>0.6 um</td>
<td>LET&lt;11</td>
<td>BNL</td>
<td>??</td>
</tr>
<tr>
<td>AD7472</td>
<td>0.6 um</td>
<td>11&lt;LET&lt;15</td>
<td>BNL</td>
<td>5.00E-04</td>
</tr>
<tr>
<td>AD7476</td>
<td>0.6 um</td>
<td>LET&lt;60</td>
<td>BNL</td>
<td>??</td>
</tr>
<tr>
<td>AD7664</td>
<td>0.6 um</td>
<td>6</td>
<td>BNL</td>
<td>2.50E-04</td>
</tr>
<tr>
<td>AD7714</td>
<td>0.6 um</td>
<td>15&lt;LET&lt;24</td>
<td>BNL</td>
<td>6.00E-04</td>
</tr>
<tr>
<td>AD9260</td>
<td>0.6 um</td>
<td>8</td>
<td>BNL</td>
<td>??</td>
</tr>
<tr>
<td>AD5334</td>
<td>0.6 um</td>
<td>5</td>
<td>TAMU</td>
<td>5.00E-04</td>
</tr>
<tr>
<td>AD7664</td>
<td>0.6 um</td>
<td>4</td>
<td>TAMU</td>
<td>1.00E-03</td>
</tr>
<tr>
<td>AD7675</td>
<td>0.6 um</td>
<td>20&lt;LET&lt;22</td>
<td>TAMU</td>
<td>3.00E-05</td>
</tr>
<tr>
<td>AD7714</td>
<td>0.6 um</td>
<td>20&lt;LET&lt;22</td>
<td>TAMU</td>
<td>1.00E-04</td>
</tr>
</tbody>
</table>

Single-event transients (SETs) depend on application conditions. Data here correspond to worst-case transients for the device. Worst-case SET rate and duration can be modeled as lognormal distributions for op amps across the ADI bipolar>2.5 micron and LTC RH process. WC amplitude can be modeled as a beta distribution, but assuming rail-to-rail transients is not overly conservative.

SEL saturated cross sections vary too widely to model SEL rate as a compact distribution. Onset Linear Energy Transfer (LET) appears to be bimodal, but we can model the lower mode as a lognormal to determine worst case (WC) onset LET for SEL in a process at a given confidence level.

SETs rates for ADI OP series bipolar feature size > 2.5 μm op amps are distributed roughly lognormally across the family with lognormal mean = -3 and standard deviation = 0.45. The 90% softest part would have ~0.089 SETs/day, or with 90% confidence <0.19 SETs/day.

The best-fit lognormal to SET rates across LTC RH Series op amps had lognormal mean = -3.8 and lognormal standard deviation = 0.65. The 90% softest part would show ~0.027 SETs/day or with 90% confidence <0.13 SETs/day.
SET durations in ADI and LTC op amps can also be modeled as a lognormal, yielding best fit mean and standard deviation and the results used to bound SET duration to any desired confidence level (See Table III.) These results can be used to select parts or for purposes of filtering transients in a design.

**Table III: Bounding SET Widths for ADI and LTC Op Amps**

<table>
<thead>
<tr>
<th>Process</th>
<th>Typical</th>
<th>90% WC</th>
<th>Typical@90% CL</th>
<th>90%WC@90%CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADI</td>
<td>15 µs</td>
<td>&lt;35 µs</td>
<td>143 µs</td>
<td>&lt;218 µs</td>
</tr>
<tr>
<td>LTC</td>
<td>22 µs</td>
<td>&lt;35 µs</td>
<td>75 µs</td>
<td>&lt;170 µs</td>
</tr>
</tbody>
</table>

SEL cross sections vary too much across parts in a process to model rates. However, onset LET for SEL for ADI’s 0.6 μm CMOS ADCs and DACs seems to fall into two compact modes. To bound the onset LET, we model the lower mode as a lognormal. With 90% confidence, onset LET for parts in this process should be >2.7 Mev-cm$^2$/mg.

Heritage is often proposed en lieu of testing to qualify a component/system. If heritage environment differs from mission environment, each heritage year can be weighted by the ratio of the heritage SEE rate to the mission SEE rate. However, this ratio depends strongly on device geometry if the heritage and mission environment differ significantly.
TID Degradation of Bipolar Junction Transistor (BJT) Gain

- Rad. Performance (RP=Gain(Prerad)/Gain(Postrad)) for every part (1<RP)
- Part-to-part variation: Each lot has a mean and a standard deviation, usually determined by Radiation Lot Acceptance Test (RLAT)
- Lot-to-lot variation: Look at how lot means and standard deviations vary
  - A lognormal distributions of means with a lognormal mean and standard deviation
  - Lognormal distribution of standard deviations (two parameters)
- Part-type-to-part-type: each lot-to-lot parameter now a distribution
  - 8 parameters describing part-to-part, lot-to-lot and part-type-to-part-type variation

\[
\text{RLAT Data for lot } j \text{ of part type } i \\{\text{RP}_1, \text{RP}_2, ..., \text{RP}_l \} \sim N(\text{RP}_{ij}, m_{ij}, s_{ij})
\]

\[
\text{Historical Data, Part-Type } i (k \geq 3 \text{ lots)}
\]
1) \{m_{i1}, m_{i2}, m_{ik}\} \sim N(m_i, m_{m_{i}}, s_{mi})
2) \{s_{i1}, s_{i2}, s_{ik}\} \sim LN(s_i, \mu_{si}, \sigma_{si})

\[
\text{Similarity Data summarizes variation over part types}
\]
\[
\{m_{mi}\} \sim N(m_m, m_{m_{mi}}, s_{mm}) = \text{Variation of avg. } m_m \text{ over part types}
\]
\[
\{\mu_{si}\} \sim N(\mu_s, m_{\mu_{si}}, s_{\mu}) = \text{Variation of avg. } s_\mu \text{ over part types}
\]
\[
\{s_{mi}\} \sim LN(s_m, \mu_{sm}, \sigma_{sm}) = \text{Variation of avg. } s_m \text{ over part types}
\]
\[
\{\sigma_{si}\} \sim LN(\sigma_s, \mu_{\sigma_s}, \sigma_{\sigma_s}) = \text{Variation of avg. } \sigma_s \text{ over part types}
\]

a) Variability of radiation performance (RP) for a lot \(j\) of part type \(i\) is assumed to follow a Normal distribution (N) with mean \(m_{ij}\) and standard deviation \(s_{ij}\). b) We summarize variability across lots for part type \(i\) by fitting the lot means to a Normal distribution with \(m_{mi}\) and \(s_{mi}\) as parameters and the standard deviations to a lognormal (LN) with parameters \(\mu_{si}\) and \(\sigma_{si}\) since standard deviations are positive definite. c) Likewise, we can summarize variability over similar part types in the same process by fitting the historical parameters to Normal or lognormal distributions as indicated.
Fig. 3. Prior probability distributions summarize (a) the expected gain degradation and the standard deviation about that mean for an arbitrary Semicoa NPN BJT as well as (b) the variability of those quantities from lot to lot.
Fig. 4 GDF Priors for a generic Microsemi-Lawrence (MS-L) PNP after 300 krad(Si) for operation in the “middle” of the device’s current range. Despite the fact that the transistors for the Prior have different voltage and current characteristics, the Priors place meaningful constraints on expected gain degradation. However, lot-to-lot variation is less well constrained, especially for part-to-part standard deviation.
What If There’s No Data to Update Priors?

- Consider probability distribution for expected mean degradation
  - Have \( P(m, m_m, s_{mm}) \) and \( P_{mm}(m_m, s_{mm}) \)
  - Can select most probable value of \( m_{mm} \) and \( s_{mm} \) based on \( P_{mm}(m_m, s_{mm}) \)
  - Alternatively, can treat \( m_{mm} \) and \( s_{mm} \) as nuisance variables and integrate over them to find an average or expected probability distribution for \( P(m_m) \)—the expected mean for a generic mean degradation of a generic transistor type in this process
  \[
  \langle P(m_m) \rangle = \int P(m, m_m, s_{mm}) P_{mm}(m_m, s_{mm}) dm_{mm} ds_{mm}
  \]
  - Similar operations possible for all lot-to-lot parameters and can be repeated to yield expected degradation for a generic xster in a generic lot for a generic xster in process

Over 90% of PNP xstrs from MS-Lawrence would have mean lot gain degradation factor < 2.1. Repeating for other parameters: Gain of the 99% WC part from the the 90% WCPNP in the MS-L process should degrade < 3x after 300 krad(Si).
It’s Not Bayesian if You Don’t Update the Prior

- For SEE, weight of heritage mission depends on environment
  - 1st order: Ratio of Figure-of-Merit coefficients in heritage and mission environments
  - Also depends on details of charge collection volume in device—and difference can be much larger if heritage and mission radiation environments differ dramatically

- Without test data, must update Prior w/ heritage data
- For TID, we only know mission $i$ parts worked after exposure to dose $D_i$
  $$L = \prod_{i=1}^{n} (1 - P(D_i))$$
  - $D_i$'s must be lower bounds

Fitting SEE Data

• For small event counts, use fit that considers Poisson errors
• Generalized linear model:
  \[ L = \prod_{i=1}^{n} \text{Poisson}(n_{i}^{\text{obs}}, \mu_{i}) \]
  \[ \mu_{i} = \sigma_{\text{sat}} \times \text{Weibull}(\text{LET}_{i} - \text{LET}_{0}, W, s) \]
• Solve for \( \sigma_{\text{sat}}, \text{LET}_{0}, W \) and \( s \) that give best fit or WC rate at desired confidence level
• Can use any model to predict \( \mu_{i} \)

• If Poisson errors negligible, use exact fit
• Least squares not optimal @ low LET
• Least log-squares (LLS) may be better
  \[ \text{LLS} = \sum_{i=1}^{n} F_{i}^{\text{pred}} \times (\log F_{i}^{\text{pred}} - \log F_{i}^{\text{obs}})^{2} \]
• LLS gives better fit near-threshold

Statistical Models for Destructive SEE

• Single-event latchup (SEL)
  – parasitic bipolar effect in CMOS
  – Can sometimes get several SELs per device before failure
  – Recovery very disruptive
  – Also seen at cryogenic temp.
  – Latent damage possible

• Rate estimation trouble:
  – poor statistics
  – Dataset may include many devices w/ varying susceptibility
  – depends on Energy + LET

• Single-event burnout (SEB)
  – parasitic bipolar effect
  – Similar events seen in FLASH, bipolar ICs, Schottky diodes
  – Can get several SEBs per device for discrete xstrs, but not for other device types

• Rate estimation difficult:
  – poor statistics
  – Dataset may include many devices w/ varying susceptibility
  – Complicated dependence on energy, angle and ion species

• Single-event Gate Rupture (SEGR) depends on gate oxide and charge transport
  – Always destructive
  – Precursor events damage oxide
  – Single-event dielectric rupture (SEDR) somewhat analogous

• Rate est. complicated by
  – poor statistics
  – Dataset may include many devices w/ varying susceptibility
  – Complicated dependence on energy, angle and ion species

GLM Approach Applied to LTC RAD1419

Can estimate rate for a given confidence if statistics adequate for each part.

Limitations of GLM for Destructive SEE

- GLM technique works well when statistics accumulated for each test device
  - Allows bounding of rate for a given confidence level
  - Flexible and can be adapted to include other sources of error
  - Model need not be standard Weibull rectangular parallelepiped
    - Could even be Monte Carlo output for several geometric models of sensitive volume
- Unfortunately, for some device types, every event kills a part
  - SEGR is always destructive to power Metal-Oxide-Semiconductor Field Effect Transistor (MOSFET) as are failures in FLASH, bipolar microcircuits and diodes
  - Accumulating statistics done over several parts
    - How do you detect/treat part-to-part variation
- Even if statistics gathered for each device, susceptibility can change w/ time
  - TID can alter susceptibility
  - Latent damage due to overcurrent (SEL, SEB, etc.) or bus contention due to SEE
  - Complex devices may have different susceptibilities during different
- How do we deal with Poisson error, part-to-part variation and time dependent susceptibility—possibly all at the same time?
What Can We Tell from Statistics of DSEE?

- Poisson DSEE → Failure fluence, \( F_{\text{FAIL}} \) follows exponential distribution
  - Variability insignificant → single mean for all parts, so \( F_{\text{FAIL}} \) follows single exponential
  - Significant variation → \( F_{\text{FAIL}} \) follows mixture of exponentials, or hyperexponential distribution
    - Hidden Markov process— \( F_{\text{FAIL}} \) distribution depends on part selected

- Compare distributions characteristics of \( F_{\text{FAIL}} \) across parts
  - Mean/Std. Deviation, skew, kurtosis, Range
  - For details, see presentation from 2012 SEE Symposium, “Assessing Part-to-Part Variation for Destructive Single-Event Effects”

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**Poisson Distribution**

**Exponential and Hyperexponential**

**Closest Exponential Fit to hyperexponential**
Detecting variability in MOSFETs*

2SK4219, Low-E Kr ions, VDS=100, VGS=-10

- 2SK4219 MOSFET shows more failures at low fluence than expected for exponential
  - Comparison of distribution characteristics: mean/Std. Dev, skew, range, kurtosis
  - Shows 6% probability of purely exponential behavior

HG0K, High-E Kr ions, VDS=100, VGS=-10

- $F_{\text{FAIL}}$ for HG0K MOSFET shows better fit to exponential
  - Comparison of distribution characteristics shows significance only at 71% level

*Data courtesy of Veronique Ferlet-Cavrois, See IEEE TNS vol. 59, pp. 2920-2929

For MOSFETS Failing During PIGS Test

- Often for VGS=0, MOSFETs do not fail during irradiation, but during Post-Irradiation Gate Stress (PIGS) test
  - Evidence of latent damage, and failure imminent
- $F_{\text{FAIL}}$ unknown, but can be estimated if some but not all parts fail at fluence $F$
  - Assume failure probability binomial
  - $P_{\text{FAIL}} \approx N_{\text{FAIL}}/N_{\text{TEST}} \approx 1 - \exp(-F/\langle F \rangle)$
  - For 2SK4219, $\langle F_{\text{Fail}} \rangle \approx 3140 \text{ cm}^{-2}$, with 90% CL from 1599 to 5735 Xe ions per cm²
  - For HG0K, $\langle F_{\text{Fail}} \rangle \approx 15500 \text{ cm}^{-2}$, with 90% CL from 9000 to 25000 Kr ions per cm²
  - Note that this is not necessarily the same as the SEGR $\langle F \rangle$
If We Can Model Fluence to Failure

• Assuming we accumulate $m_i$ events at $\text{LET}_i$, (example model: RPP Weibull)

\[
\mathcal{L} = \prod_{i=1}^{n} \prod_{j=1}^{m_i} \frac{1}{F_i(\text{LET}_i)} \exp\left(-F_i / F_i(\text{LET}_i)\right), \quad F_i(\text{LET}_i) = \frac{1}{\sigma_{\text{lim}}} \times \left[\text{Weibull}(\text{LET} - \text{LET}_0, W, s)\right]^{-1}
\]

– Initially assume $F_i$ the same for each event $j$ accumulated for $\text{LET}_i$,
  • Compare likelihood for each LET—a dramatically poor fit may indicate that expected fluence to failure has changed from event to event
    – May be due to multiple parts, latent damage, change in vulnerability due to new operation…
  • Can introduce models of variability to better match results given enough data.

• Models for $F_{\text{Fail}}$ much more complicated for
  – SEL—$F_{\text{Fail}}(\text{LET},\text{Range})$
  – SEGR and SEB—$F_{\text{Fail}}(Z,\text{Energy},\text{Angle})$
  – May be possible to simplify radiation environment in terms of Equivalent LET
  – $F_{\text{Fail}}$ could be mean values of $F_{\text{Fail}}$ for Monte Carlo simulations done over different candidate models of the sensitive volume
Conclusions

- Statistical models are inherent to current RHA methods; some examples:
  - Use of one-sided tolerance limits for TID
  - Guidance for event counts in SEE testing

- Models to date tend to concentrate on “ideal” or most representative data
  - Assumption that TID performance in a wafer lot follows well behaved distribution
  - Assumption that part-to-part and lot-to-lot variation of SEE response is negligible
  - SEE counts sufficiently high that Poisson errors don’t affect SEE rates

- Much more data less than ideal (similarity, heritage, historical…)
  - Often used for a “warm fuzzy”
  - Bayesian methods here allow quantitative bounds to be placed on radiation response

- Statistical techniques very promising for bounding destructive SEE
  - May be very important if commercial hardware being used
  - Destructive SEE must be rare, or hardware will not meet requirements
    - Rare events are inherently difficult to bound

- Proper use of statistics essential for reliable economical space systems
  - Any fool can lie with statistics—Experts use them to expose the truth