Improved Data Reduction Algorithm for the Needle Probe Method Applied to In-Situ Thermal Conductivity Measurements of Lunar and Planetary Regoliths

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Abstract

The needle probe method (also known as the ‘hot wire’ or ‘line heat source’ method) is widely used for in-situ thermal conductivity measurements on soils and marine sediments on the earth. Variants of this method have also been used (or planned) for measuring regolith on the surfaces of extra-terrestrial bodies (e.g., the Moon, Mars, and comets). In the near-vacuum condition on the lunar and planetary surfaces, the measurement method used on the earth cannot be simply duplicated, because thermal conductivity of the regolith can be ~2 orders of magnitude lower. In addition, the planetary probes have much greater diameters, due to engineering requirements associated with the robotic deployment on extra-terrestrial bodies. All of these factors contribute to the planetary probes requiring much longer time of measurement, several tens of (if not over a hundred) hours, while a conventional terrestrial needle probe needs only 1 to 2 minutes. The long measurement time complicates the surface operation logistics of the lander. It also negatively affects accuracy of the thermal conductivity measurement, because the cumulative heat loss along the probe is no longer negligible. The present study improves the data reduction algorithm of the needle probe method by shortening the measurement time on planetary surfaces by an order of magnitude. The main difference between the new scheme and the conventional one is that the former uses the exact mathematical solution to the thermal model on which the needle probe measurement theory is based, while the latter uses an approximate solution that is valid only for large times. The present study demonstrates the benefit of the new data reduction technique by applying it to data from a series of needle probe experiments carried out in a vacuum chamber on JSC-1A lunar regolith stimulant. The use of the exact solution has some disadvantage, however, in requiring three additional parameters, but two of them (the diameter and the volumetric heat capacity of the probe) can be measured and the other (the volumetric heat capacity of the regolith/stimulant) may be estimated from the surface geologic observation and temperature measurements. Therefore, overall, the new data reduction scheme would make in-situ thermal conductivity measurement more practical on planetary missions.
Introduction

Measurement of heat released from the interior of an extra-terrestrial body is important in understanding the body’s internal structure, composition, and origin (e.g., NRC 2011). The Apollo 15 and 17 missions were the first to measure heat flow on an extra-terrestrial body (Langseth et al., 1976). ESA’s Rosetta spacecraft bound to Comet 67P/Churyumov-Gerasimenko carries a heat flow probe (Marczewski et al., 2004). NASA’s InSight mission, expected to be launched for Mars in 2016, will include a heat flow probe in its science payload (Spohn et al., 2012). Heat flow is obtained as a product of the thermal gradient and thermal conductivities measurements made beneath the surface. The present study focuses on thermal conductivity measurement.

Regoliths of the Moon, Mars, and comets are much less thermally conductive than soils on the earth, mainly because the atmospheric pressures of these extra-terrestrial bodies are much lower (Presley and Craddock, 2006). For example, thermal conductivity of lunar regolith is roughly two orders of magnitude less in the lunar vacuum than in the pressure of the earth’s atmospheric (Horai, 1981). Thermal conductivity instruments for planetary missions must be able to measure this value to less than 0.01 W/m·K. Pre-flight development of such instruments involves laboratory tests in a vacuum chamber using regolith simulants. In calibrating the instruments, researchers must be able to independently determine thermal conductivity of the simulant, and its pressure dependency.

Methods used for the thermal conductivity instruments on the aforementioned planetary missions and those used in vacuum chambers are variants of the so-called needle probe method (Presley and Christensen, 1997a; Kömle et al., 2011). A cylindrical probe containing electric heaters and temperature sensors is inserted into the regolith (or simulant). When the heater is activated, temperature of the probe rises (Fig. 1). Thermal conductivity of the regolith has direct relation with the time-rate of the temperature increase seen by the probe. If the regolith is thermally insulating (i.e., of low thermal conductivity), temperature of the probe rises rapidly as the heat builds up in the vicinity of the probe. If the regolith is highly conductive, the heat dissipates away from the probe quickly, and temperature of the probe rises more slowly.

The needle probe method was developed over 50 years ago for measurements on soils, marine sediments, and other materials of similar texture on the earth’s surface (DeVries and Peck, 1958; Von Herzen and Maxwell, 1959), and it is still widely used by terrestrial researchers (Beardsmore and Cull, 2001). The method is also known as the ‘hot-wire’ or ‘line heat source’ method. A typical needle probe used in terrestrial soil measurements consists of a cylindrical metal tube of 3- to 10-cm length and 1- to 3-mm diameter. It contains an electric heater wire (e.g., nichrome), which runs along the length of the probe, and a temperature sensor (either thermistor or thermocouple) positioned at the mid-point of the probe. The needle probe method is popular among terrestrial researchers mainly for three reasons. First, the experimental hardware is simple, easily transportable, and relatively inexpensive. Second, a needle probe experiment can be carried out very quickly. It takes only 1-2 minutes to measure a terrestrial soil sample. Third, in this method, the thermal conductivity is obtained as a simple algebraic function of the heater power and the rate of the temperature increase. Knowledge of any additional thermal parameters, such as heat capacity of the soil, is not necessary.
Some of the advantages of the needle probe method are not applicable to planetary missions and vacuum chamber tests. Because of engineering requirements driven by the robotic deployment on distant planetary bodies, the thermal conductivity instruments on planetary missions deviate from the well established configuration of the needle probe used on the earth. Most notably, these probes are of greater diameter (1 to 2.5 cm) and length (30 to 50 cm). In addition, these probes are used for measuring materials that are ~100 times less thermally conductive than the earth’s soils, for which the method was originally intended.

As we explain in detail in the next section, when a large-diameter probe is used for measuring materials of very low thermal conductivity, it requires a much longer measuring time, several tens (if not over a hundred) of hours. In case of the Apollo heat flow experiments, they used a 2.5-cm diameter probe and heated it for 40 hours (Langseth et al., 1972). The long duration of experiment also causes a large portion of the heat emitted by the probe to travel along the probe itself and escape into the mechanism that deploys it. That negatively impacts the accuracy of the thermal conductivity determination (Blackwell 1954; Presley and Christensen 1997a). Because of these additional complexities, it is nearly impossible for the instruments on the aforementioned planetary missions to obtain the thermal conductivity as simple algebraic function of the heater power and the rate of temperature increase. Sophisticated computer simulations of the heating experiments on the surfaces of the extra-terrestrial bodies, which involve a number of loosely-constrained parameters, are necessary for determination of the thermal conductivity of the regolith on these planetary missions (e.g., Grott et al., 2010).

The present study proposes improvement in the data reduction scheme of the needle probe method so that much less time is required for measurements, even for large-diameter probes and low-conductivity (< 0.01 W/m·K) materials. Because of this improvement, in-situ thermal measurement of planetary regolith would become more practical for missions in which time management on the surface is critical. The improved data reduction algorithm would also help researchers expedite and simplify thermal conductivity measurements in vacuum chambers. We have carried out a suite of needle probe experiments on JSC-1A lunar regolith simulant (Schrader et al., 2010) in a vacuum chamber, and reduced the data using this new data reduction algorithm.

**Theoretical Basis for the Needle Probe Method**

Here we summarize the mathematical heat conduction model which serves as the theoretical basis for the thermal conductivity measurement using a needle probe (e.g., Jaeger, 1956; DeVries and Peck, 1958; Von Herzen and Maxwell, 1959). We also describe how it is applied to measurements on terrestrial soils.

A needle probe with an outside radius \( r \) is inserted to a homogeneous solid material of thermal conductivity \( K \). For mathematical simplicity, we assume that contact resistance between the probe and the material is negligible. We also assume that the length of the probe is much greater than the radius so that heat conduction is one-dimensional in the radial direction. Initial temperatures of the material and the probe are the same at \( T_0 \). At time \( t = 0 \), the probe is heated uniformly along its length at the constant rate of \( Q \) per unit time per unit length. If the probe is a perfect conductor, its temperature \( T \) is given by:
\[ T = \frac{Q}{K} G(\alpha, \tau) + T_0 \]  

(1)

The non-dimensional parameter \( \alpha \) is the twice the ratio of the heat capacity of the material to that of the bulk heat capacity of the probe:

\[ \alpha = 2\pi a^2 \rho c / S, \]  

(2)

where \( \rho \) and \( c \) are the density and the specific heat, respectively, of the material. \( S \) is the heat capacity of the probe per unit length. Parameter \( \tau \) is the thermal time scale:

\[ \tau = \frac{\kappa}{\rho c} t, \]  

(3)

where \( \kappa \) is the thermal diffusivity of the material:

\[ \kappa = \frac{K}{\rho c}. \]  

(4)

The function \( G(\alpha, \tau) \) of Eq. (1) is given by:

\[ G(\alpha, \tau) = \frac{2\alpha^2}{\pi^3} \int_0^{\infty} \left[ 1 - \exp \left( -\tau u^2 \right) \right] \frac{du}{u^3 \Delta(u)}, \]  

(5)

where \( \Delta(u) \) is given by:

\[ \Delta(u) = \left[ uJ_0(u) - \alpha J_1(u) \right]^2 + \left[ uY_0(u) - \alpha Y_1(u) \right]^2. \]  

(6)

\( J_n(u) \) and \( Y_n(u) \) are Bessel functions of order \( n \) of the first and second kinds. Some examples of \( G \) values for \( \alpha = 1, 2, \) and \( 3 \) and \( \tau = 1 \) through 10 are shown in Fig. 2.

For \( \tau \approx 3 \), \( G(\alpha, \tau) \) can be approximated by the series (Blackwell, 1954):

\[ G(\alpha, \tau) = \frac{1}{4\pi} \left\{ \ln \frac{4\tau}{C} + \frac{1}{2\tau} + \frac{\alpha - 2}{2\alpha\tau} \ln \frac{4\tau}{C} + \ldots \right\}, \]  

(7)

where \( C \) is a dimensionless constant 1.7811.

For even greater values of \( \tau \) (> 14) and for \( \alpha \) values close to 2, this can be truncated to (Jemsek et al, 1985):

\[ G(\alpha, \tau) \approx \frac{1}{4\pi} \cdot \ln \frac{4\tau}{C}. \]  

(8)

Combining Eqs. (1) and (8) yields:
\[ T \approx \frac{Q}{4\pi K} \cdot \ln \frac{4\tau}{C} + T_0 = \frac{Q}{4\pi K} \cdot \ln \frac{4\kappa}{Cr^2} t + T_0 \]

(9)

This can be further modified to:

\[ T \approx \frac{Q}{4\pi K} \cdot \ln t + A, \quad (10) \]

where \( A \) is a constant.

In thermal conductivity measurements using a needle probe, the heater is turned on for a long enough time to achieve a linear relationship between temperature and the natural logarithm of time (Eq. 10). Then, the slope of the linear trend is inversely proportional to the thermal conductivity of the material (Fig. 1B):

\[ K = \frac{Q}{4\pi} \cdot \frac{d(\ln t)}{dT} . \quad (11) \]

Eq. (11) eliminates \( \rho \) and \( c \), and makes it possible to obtain \( K \) based on the knowledge of the heat input \( (Q) \) and the increase in temperature with time.

Needle probes commercially available for terrestrial soil measurements come in 1- to 3-mm diameter. Such probes need only 1 to 2 minutes of heating for use of Eq. (11), and yield an accuracy of a few percent (e.g., Von Herzen and Maxwell, 1959; Beardmore and Cull, 2001).

A much longer heating time may have an adversary effect to the accuracy of measurement, because heat loss in the axial direction becomes significant over time and the problem can no longer be adequately represented by the one-dimensional heat conduction in the cylindrical coordinate. The time limit for 1-D approximation depends on the physical dimension of the probe as well as the thermal properties of the material measured. Previous studies (Blackwell 1954; Presley and Christensen 1997a) suggest that the following condition must also be met in order to achieve accuracy better than 1%:

\[ \frac{L}{r} > \left( \frac{\kappa}{0.0632r^2} \right)^{1/2}, \quad (12) \]

where \( L \) is one half of the probe length and \( r \) is the radius. Combining this with Eq. (3), we obtain:

\[ \tau < 0.0632 \cdot \frac{L^2}{\kappa} \quad (13) \]

In making measurements on granular materials such as dry soil and sands, there can be significant contact resistance between the probe and the material. The general practice among researchers is that use of a large diameter probe reduces this problem (e.g., Kömle et al., 2010). There have been limited cases of laboratory experiments in which researchers examined how the diameter of the probe affects needle probe measurements on granular materials. We are aware of
two such studies (Inaba, 1986; Campell et al., 2007). Each of these studies used multiple needle probes of different diameters in measuring one set of granular samples. Then, they compared them with thermal conductivity values obtained with the steady state measurement method (Beardsmore and Cull, 2001). The results obtained by Inaba (1986) suggest that use of probe diameter greater than the average grain diameter yields reliable measurements. Those by Campbell et al. (2007) suggest that even if the probe diameter is 2 orders of magnitude greater, due to contact resistance, it may significantly underestimate thermal conductivity. Both of these studies assumed that the steady state method yielded more accurate measurements than the needle probe does. Readers should be aware that the steady state method is indeed considered very reliable but only for measuring solid materials. For steady-state measurement on particulate materials, a sample must be packed in a metal container, and the texture of the material can be easily altered in the sample preparation process. Therefore, the thermal conductivity obtained for such a sample may not be the same as the one measured *in situ*.

**Technical Challenges in Use of the Conventional Needle Probe Method on Lunar and Planetary Regolith and Simulant in Near-Vacuum Conditions**

In near-vacuum conditions, heat transport through the matrix of planetary regolith (and its simulant) takes place primarily by radiation between the interfacing surfaces of the individual grains. Heat conduction by way of grain-to-grain contacts is minimal. Heat conduction through the gas that fills the pore spaces contributes less and less to the overall transport budget in decreasing air pressure. When researchers perform needle probe experiments on regolith/simulant samples in near-vacuum, they are actually measuring the combined effect of these three types of heat transport. Such ‘bulk’ or ‘effective’ thermal conductivities of regolith/simulant are 2 orders of magnitude less in near-vacuum conditions than in the earth’s atmosphere (e.g., Presley and Craddock, 2006).

The conventional needle probe technique (DeVries and Peck, 1958; Von Herzen and Maxwell, 1959) requires that the heating takes place long enough to satisfy the condition \( \tau > 14 \) at the minimum (Eq. 10). The less conductive the sample is, the longer time it takes to satisfy this condition for a given probe diameter. The upper limit of the heating time is set by Inequality 13, which is inversely proportional to the thermal diffusivity of the sample. For a needle probe of 2.4-mm diameter and 10-cm length, a heating time greater than ~20 seconds and less than ~300 seconds satisfies the condition for use of the 1-D approximation on terrestrial soils (Fig. 3A). The same probe would require a heating time of 3 hours or longer for measurements on the surface of the Moon. A probe of 2.5-cm diameter and 30-cm length (equivalent in size with the probe designed for the *InSight* mission, Spohn et al., 2012) would require a minimum of ~270 hours of heating for measurements on the Moon and that exceeds the upper limit for the 1-D approximation for the dimension of the probe (Fig. 3B).

For previous laboratory experiments conducted on lunar regolith (or simulant) samples in vacuum chambers on earth, researchers used very thin heat sources/sensors and did not require long heating time. For example, Cremer (1971) used a 32-AWG (0.2032-mm diameter) nichrome wire as both the heat source and the resistance thermometer. Fountain and West (1970) and Presley and Christensen (1997b) used a very thin (< 0.1-mm diameter) platinum wire
as the heat source. It was placed 2 mm away from a thermocouple in parallel, and together they were considered a needle probe.

For \textit{in-situ} thermal conductivity measurement on a planetary/lunar surface, use of such a thin-wire ‘probe’ is not practical. Technologically, it would be very difficult (and expensive) to install such a thin wire to the required subsurface depths (\textit{e.g.}, 3 m for the Moon, Cohen et al., 2009) without breaking it. In addition, use of a very thin probe may not achieve good thermal contact with the individual grains of the regolith. Twenty to thirty percent of the lunar surface soil particles collected by the Apollo astronauts have diameters exceeding 0.2 mm (Carrier, 2001), and thus they may be too coarse to be measured accurately by the thin-wire probes.

The thermal conductivity instruments onboard the aforementioned space missions use large-diameter (1 to 2.5 cm) probes in order to insure good thermal contact with regolith. If they operate as conventional needle probes, they would require several tens of (or more than a hundred) hours of heating time. Even with their lengths (30 to 50 cm), they cannot avoid the axial heat loss affecting measurements for such long duration of heating. From space missions’ operational point of view, it is desirable to minimize the time required for \textit{in-situ} thermal conductivity, or any other \textit{in-situ} experiment. In case of the Apollo heat flow experiment, the 2.5-cm diameter probe was heated for 40 hours (Langseth \textit{et al.}, 1972), while it would have required 270 hours of heating. Therefore, determination of thermal conductivity required sophisticated computer simulations that aim to duplicate the experimental results (\textit{e.g.}, Grott \textit{et al.}, 2010). The heat flow instruments for the Rosetta and InSight missions share similar design problems with the Apollo heat flow probe, and thus they would also require computer simulations with a number of loosely constrained parameters in constraining the thermal conductivity of the target regolith.

\textbf{Needle Probe Experiments on JSC-1A Lunar Regolith Simulant}

The present study envisions a scenario in which a needle probe (or a line heat source) is utilized for \textit{in-situ} thermal conductivity measurement on the surface of the Moon or other extra-terrestrial body in near-vacuum conditions. While using a probe thick enough to ensure mechanical strength and good thermal contact with regolith particles, we try to minimize the time required for measurement. We also want a simple and robust data reduction scheme so that it does not require detailed computer simulation of the heating experiment in obtaining the thermal conductivity of the regolith.

In developing such an algorithm, we carried out a series of needle probe experiments on the JSC-1A lunar regolith stimulant (Schrader \textit{et al.}, 2010) in a vacuum chamber. A commercially available needle probe, \textit{TR-1} of Decagon Devices was used in most of our experiments. The probe uses a stainless steel tube of 2.4-mm outside diameter and 10-cm length for casing. It contains strands of \textit{Evanohm} Alloy heater wire and a thermistor placed at its center. \textit{RBC 4300} thermal epoxy is used as the tube filling. A plastic pail holding roughly 3-liter of JSC-1A was placed inside a vacuum chamber of 60-cm by 60-cm by 60-m. The simulant had been compacted by vibration prior to the experiments. The needle probe was inserted near the center of the pail. The needle probe was controlled by the \textit{KD2 Pro} data logger of the Decagon Devices.
JSC-1A lunar regolith simulant has a median particle size of 0.1-mm diameter, and ~99% of the particles have diameters less than 1 mm (Zeng et al., 2010). The diameter of the TR-1 probe (2.4 mm) is more than 20 times the median diameter of the simulant. Contact resistance between the probe and the grains should not be a major issue. In order to confirm this, we conducted a side-by-side experiment of TR-1 and another smaller-diameter (1.3 mm) probe, KS-1 of Decagon Devices under atmospheric pressure. The two probes yielded thermal conductivity values that are within 0.8% of each other at 0.27 W/m·K. This suggests that even the 1.3-mm diameter probe is large enough not to be affected by the grain size of the simulant. Otherwise, the thermal conductivity measured by KS-1 should have been significantly less than that by TR-1.

We also obtained the density and the specific heat of the simulant at the room temperature and atmospheric pressure. The density (1940 kg/m³) was obtained from measurement of the volume and weight of the simulant in the container. The density value conforms to those previously reported on the same material (Zeng et al., 2010). The Decagon system also comes with a thermal diffusivity probe, SH-1. It uses the ‘dual needle’ technique which uses two parallel, closely-spaced (~5 mm) probes. One of the probes generates heat and the other detects the arrival of the heat that conducted through the sample (Beardsmore and Cull, 2001). Based on this technique, we obtained the thermal diffusivity of the JSC-1A to be 1.8 x 10⁻⁷ m²/s in room temperature and pressure. By solving Eq. (4) for c, using $K = 0.27$ W/m·K and $\rho = 1940$ kg/m³, $\kappa = 1.8 \times 10^{-7}$ m²/s, specific heat of JSC-1A is obtained as 773 J/kg.

The needle probe experiments were conducted in chamber pressures ranging from 0.13 Pa (1 mTorr) to 101 kPa (760 Torr). At each heating experimental run, chamber pressure was held constant. At low pressures (<6.7 kPa or 50 Torr), the probe was heated for 5 minutes, and at higher pressures, the probe was heated for 2.5 minutes. The data obtained in the high pressure range show linear relationship between the temperature and the natural logarithm of the time after 40 seconds into the heating cycle (Fig. 1B), while the low-pressure datasets do not (Fig. 4). The difference here is that while the condition $\tau > 14$ was met for the data obtained in the high pressure range, it was not for those in the low pressure range (Fig. 3A).

The Data Reduction Method for Fast Needle Probe Measurement in Near Vacuum

To minimize the time of measurement, we develop a reduction algorithm that utilizes data for $\tau < 3$. In the past, such an approach was considered impractical, primarily because it requires evaluation of the function $G (\alpha, \tau)$ (Eq. 5), rather its approximation, the natural logarithm of the time (Eq. 8). Evaluating $G$ is far more computationally intensive, and it requires prior knowledge of the heat capacities of the sample and the probe (Eq. 2). When the needle probe technique was proposed in the 1950s, it was a major computational challenge to evaluate $G$, but not for present-day computers. Regarding the requirement of the prior knowledge of the heat capacity parameters, for the present data set obtained on the JSC-1A simulant, we have the density (1940 kg/m³) and the specific heat (773 J/kg·K) measured at room temperature and pressure. Therefore, we first discuss how the thermal conductivity can be obtained when the heat capacity of the sample is known. Later, we will discuss cases without prior knowledge of the heat capacity.
In evaluating $G$ (Eq. 5), the volumetric heat capacity of the sample (density multiplied with specific heat), the thermal conductivity of the sample, the volumetric heat capacity of the probe, and the radius of the probe must be independently known. The radius of the probe can be measured accurately ($r = 1.2$ mm). We also have a rough estimate of the volumetric heat capacity of the probe ($2.23 \times 10^6$ J/m$^3$·K), based on knowledge of the materials used. We assume that the volumetric heat capacity of the sample ($\rho c = 1500 \times 10^6$ J/m$^3$·K) in vacuum is the same as that at 1 atm., because heat capacity of the air in the sample’s pore spaces should be negligible in this case. At 1 atm., the volume of the air subjected to heating by the probe is very small, and in vacuum there is no air. Knowledge of these parameters allows determination of $\alpha = 1.35$.

That leaves the thermal conductivity of the sample as the only unknown. It can be constrained by the iterative algorithm below:

1. Assume a thermal conductivity value, $K_{est}$.
2. For $K_{est}$, evaluate $G(\alpha, r)$ for the time range of the experiment.
3. Obtain a thermal conductivity value ($K_{slope}$) from the slope of $T$ vs. $G$.
4. If $K_{slope} = K_{est}$, the iteration stops. If not, use $K_{slope}$ as the new $E_{est}$ and start over.

This algorithm converges to a $K$ value within a reasonable tolerance ($\pm 0.0002$ W/m•K) in several iterations, because $G$ changes monotonically with $\tau$ (Fig. 2).

Figure 5A shows how thermal conductivity values change through the iteration from an initially assumed value of 0.01 W/m·K to the final values for the samples pressured at 6.67 Pa and 66.7 Pa, the same datasets shown in Fig. 4. Figure 5B shows the $T$ vs. $G$ plots for the same set. The very early part of the data ($\tau < 0.5$) were excluded from consideration, because they deviate from the linear trends established for the later parts of these data. We believe that the deviation of the early part of the data from Eq. 1 is due to the fact that the thermal properties of the probe itself are affecting the measurement. While the measurement theory assumes that the probe is a perfect conductor, in reality it is not. It takes a finite amount of time to fully heat the probe in the beginning of the experiment, and that affects these early temperature measurements.

We have tried both the new and the conventional reduction schemes on all of our needle probe experimental data. In the new scheme, thermal conductivity is obtained from the slope of a $T$ vs. $G$ plot (Fig. 5B). In the conventional scheme, thermal conductivity is obtained from the slope a $T$ vs. the log of $t$ (Fig. 1B). Suitability of either technique may be measured by the linearity of these plots. In other words, if any of these plots are curved, the experiment deviates from the mathematical model which the reduction scheme uses. In Figure 6, linear correlation coefficients for both groups of plots are shown as a function of the chamber pressure. It is evident that at low pressures ($< 1,000$ Pa), the $T$ vs. $G$ plots consistently shows superior linearity, and the new reduction scheme is preferred. In the lowest range of pressures ($< 100$ Pa), the standard reduction scheme overestimated thermal conductivity by 100% to 150%. At 1,000 Pa, thermal conductivity of the simulant is $\sim 0.06$ W/m·K (Fig. 7) and the 5 minutes of heating time roughly corresponds to $\tau = 8.3$. At higher pressures ($> 1,000$ Pa), both plots show high linear correlation coefficients ($> 0.999$), and thermal conductivity values obtained by the two schemes match within 3%. 
The Data Reduction Method When Volumetric Heat Capacity of the Sample is Unknown

The needle probe data reduction scheme proposed here requires knowledge of some parameters that are not required by the conventional scheme. They are the radius of the probe, the volumetric heat capacity (density multiplied with specific heat) of the probe, and the volumetric heat capacity of the regolith/simulant. The radius of the probe can be easily measured. The volumetric heat capacity of the probe can be determined by pre-flight laboratory tests. The volumetric heat capacity of the regolith must be determined from other data obtained from the landing site. In the future, when a lunar/planetary lander makes in-situ thermal conductivity measurement of the regolith, we cannot count on its also having a capability for measuring regolith heat capacity as well. However, there may be ways for estimating the heat capacity based on other available observations. For example, it is reasonable to expect that such a landing mission will be preceded by an orbital remote sensing mission which yields information on the surface geology of the targeted site. It is also reasonable to expect that the lander is equipped with cameras and other instruments that can characterize minerals and texture of the regolith of the landing site. The present discussion focuses on how the volumetric heat capacity of the regolith may be estimated and how the uncertainty of such estimates may affect the determination of thermal conductivity.

At a fixed temperature (e.g., 300 K), dry terrestrial sands made of non-clay, rock-forming minerals have similar specific heat values (Bowers and Hanks, 1962; Table 1). In near-vacuum conditions on the lunar and planetary surfaces, temperature is the primary controlling factor of the specific heat of the regolith material (Robie et al., 1970; Cremers, 1975). Temperature of the regolith can be measured accurately by the thermal conductivity probe. Given that various sands have similar specific heat values, a researcher may be able to estimate the in-situ specific heat of the regolith of the landing site within a few tens of percent by relying on temperature measurements and other geological observations.

Bulk density of regolith is much less sensitive to temperature, given that the thermal expansion rates of common rock-forming minerals are in the order of $10^{-5}$. Previous studies (Fountain and West 1970; Horai, 1981) have shown that more densely packed lunar regolith materials tend to yield higher thermal conductivity values, and vice versa. It may be possible, at least for the Moon, to obtain an empirical relationship between these two parameters.

By taking advantage of the relationship between temperature and specific heat and that between density and thermal conductivity, a more generalized iterative algorithm may be constructed for thermal conductivity determination:

1. Assume a thermal conductivity value, $K_{est}$.
2. Obtain the volumetric heat capacity of the regolith as $\rho(K_{est}) \cdot c(T)$, and use it to evaluate $G(\alpha, \tilde{\varepsilon})$.
3. Obtain a thermal conductivity value ($K_{slope}$) from the slope of $T$ vs. $G$.
4. If $K_{slope} = K_{est}$, the iteration stops. If not, use $K_{slope}$ as the new $E_{est}$ and start over.

Defining the actual formulae for $c(T)$ and $\rho(K)$ is outside the scope of the present study. However, it is obvious that any such formula would involve some uncertainty. If thermal conductivity is very sensitive to errors associated with evaluation of $\rho c$, this algorithm may not be practically implemented. In Table 2, we show cases where $\rho c$ values are under- and over-
estimated by 20% for chamber pressure at 6.67 Pa and 66.7 Pa. Thermal conductivity values are 7 to 8% lower for the overestimated heat capacity and 8 to 11% higher for the underestimated heat capacity. In either case, the error in thermal conductivity determination is less than that of the heat capacity.

One way to improve accuracy would be to use materials of relatively low heat capacity in constructing the probe. A low heat capacity of the probe translates to a high $\alpha$ value (Eq. 2). The $G$ function becomes less sensitive to $\alpha$ with increasing $\alpha$ values (Fig. 2). That makes the thermal conductivity determination also less sensitive to the uncertainty in the estimates of the heat capacity of the regolith ($\rho_c$). In the examples given in Table 2, the overestimation of $\rho_c$ resulted in less error in thermal conductivity determination than the underestimation did. This is because the overestimation also resulted in a higher $\alpha$.

Table 1: Specific heat measurements obtained for some terrestrial sand samples at 29° C to 37° C (after Bowers and Hanks, 1962).

<table>
<thead>
<tr>
<th>Material</th>
<th>Specific Heat (J/kg·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derby loamy sand</td>
<td>880</td>
</tr>
<tr>
<td>Cass loam</td>
<td>880</td>
</tr>
<tr>
<td>Sarpy fine sandy loam</td>
<td>920</td>
</tr>
<tr>
<td>Coarse quartz sand</td>
<td>790</td>
</tr>
</tbody>
</table>

Table 2: Variation in obtained thermal conductivity due to uncertainty in the volumetric heat capacity of the sample.

<table>
<thead>
<tr>
<th>Air pressure (Pa)</th>
<th>$\rho_c$ ($10^6$ J/m³·K)</th>
<th>1.2</th>
<th>1.5</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.67</td>
<td>Thermal Conductivity (W/m·K)</td>
<td>0.0062</td>
<td>0.0056</td>
<td>0.0052</td>
</tr>
<tr>
<td>66.7</td>
<td>Thermal Conductivity (W/m·K)</td>
<td>0.0100</td>
<td>0.0093</td>
<td>0.0085</td>
</tr>
</tbody>
</table>

Conclusions

For in-situ thermal conductivity measurement on an extra-terrestrial body in near-vacuum condition, a large diameter (1 to 2.5 cm) probe is necessary in ensuring mechanical strength and good thermal contact with regolith. The conventional data reduction method would require a heating experiment lasting several tens of hours ($\tau > 10$) in carrying out one set of measurements using such a probe. Long-duration experiments would also result in significant heat loss in the axial direction of the probe, and that would negatively affect accuracy of the measurement. The new data reduction algorithm proposed here allows use of data for $\tau = 0.5$ to 1 and would reduce the time of experiment by an order of magnitude. As shown in the series of experiments conducted on JSC-1A lunar simulant in a vacuum chamber, using the new data reduction algorithm, a heating duration of 5 minute is enough for a commercial 2.4-mm diameter probe in a chamber pressure less than 1 Pa, while the same probe using the conventional data reduction would reach its low-pressure limit at ~1000 Pa (Fig. 7), unless heating time is extended.
This new data reduction scheme requires prior/independent knowledge of volumetric heat capacity of the regolith ($\rho c$), while the conventional scheme does not. That is a disadvantage. However, it should be possible to make a reasonable estimate of $\rho c$ based on observations from an orbiting spacecraft and other instruments on the lander. In addition, constructing the probe with materials of lower heat capacity would make the thermal conductivity determination less sensitive to the uncertainty in the $\rho c$ estimation. Overall, the improvement proposed here would enable researchers to use relatively simple, standardized hardware and a straightforward data reduction method for in-situ thermal conductivity measurements on lunar/planetary landing missions in which time management on the surface is critical.

Acknowledgment

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REFERENCES


FIGURE CAPTIONS

Figure 1  A: Temperature (°C) versus time (sec) for the needle probe experiments on JSC-1A conducted at two different pressures (6.3 kPa and 12.5 kPa). Thermal conductivity obtained from each experiment is also shown in W/m•K. Heater power was 5.30 W/m and 5.26 W/m, respectively.  B: Temperature (°C) versus the natural logarithm of time (sec) for the same set of experimental data with only the measurements after t = 20 sec were used.

Figure 2  A graph of G vs τ (1 to 10) for α = 1, 2, and 3.

Figure 3  A: The heating times (sec) equivalent for τ = 0.5, 3, 14, and 0.0632*L²/κ for a for a cylindrical probe with 2.4-mm diameter and 10-cm length, obtained for a range of thermal diffusivity values, 1 x 10⁻⁹ to 1 x 10⁻⁶ m²/s. B: The heating times (sec) equivalent for τ = 0.5, 3, 14, and 0.0632*L²/κ for a cylindrical probe with 25-mm diameter and 30-cm length, a size equivalent for the heat flow probe designed for the InSight mission (Spohn et al., 2012), obtained for a range of thermal diffusivity values, 1 x 10⁻⁹ to 1 x 10⁻⁶ m²/s.

Figure 4  Temperature (°C) versus the natural logarithm of time (sec) for the needle probe experiments on JSC-1A conducted at two different pressures (6.67 Pa and 66.7 Pa). Only the measurements made after t = 20 sec were used. We used low heater power (0.4 W/m and 0.3 W/m, respectively) in order to avoid overheating of the sensor.

Figure 5  A: a graph showing how Kₑₛₑₜ, starting from 0.01, converges to its final value for the case of chamber pressures 6.67 Pa and 66.7 Pa.  B: Temperature versus G plots for the same experiments shown in Fig. 4 at chamber pressures 6.67 Pa and 66.7 Pa.

Figure 6  For the heating experiments conducted in a range of pressure from 0.067 Pa to 100 kPa, linear correlation coefficients for temperature versus G plots (diamonds) are shown as function of the chamber pressures of the corresponding experiments. Linear correlation coefficients for temperature versus the natural log of time (circles) are also shown. It can be seen that, in pressures less than 1000 Pa, the correlation coefficients for the temperature versus G plots are consistently higher than those for the temperature versus the log of time plots, while in higher pressure, they are about the same.

Figure 7  Thermal conductivity values obtained for the JSC-1A stimulant plotted against the chamber pressures in which the individual needle probe experiments were conducted. The diamonds represent the thermal conductivity values obtained by the use of the new data reduction method. The circles represent the values obtained by the use of the conventional data reduction method. For pressures less than 1,000 Pa, the conventional reduction scheme was not used because of the low linearity of the temperature versus the log of time plots (Fig. 6).
Fig. 1A

A collection of all the figures in page size.
Fig. 1B

- $0.15 \text{ W/mK}$
- $6.3 \text{ kPa}$
- $0.18 \text{ W/mK}$
- $12.5 \text{ kPa}$
Fig. 2
Thermal Diffusivity ($m^2/s$)

Heating Time (sec)

$\tau = \frac{0.0028 L^2}{K}$

Fig. 3A

Fig. 3A

terrestrial soils

regolith in lunar vacuum
Thermal Diffusivity (m²/s)

Heating Time (sec)

\[ \tau = 3 \]
\[ \tau = 14 \]
\[ \tau = 0.0628 \frac{L^2}{k} \]

Regolith in lunar vacuum

Terrestrial soils

Fig. 3B
Fig. 4

Temperature (deg. C) vs. Ln(time)

- 66.7 Pa
- 6.67 Pa
$G(\beta_6, \gamma_8)$

Temperature Rise (K)

6.67 Pa, 0.0052 W/mK

66.7 Pa, 0.0085 W/mK

Fig. 5B
Linear Correlation Coefficient

Chamber Pressure (Pa)

Fig. 6
Fig. 7