Broadband Microwave Wireless Power Transfer for Weak-Signal and Multipath Environments

Richard J. Barton  
Avionic Systems Division  
NASA Johnson Space Center  
Houston, TX USA  
richard.j.barton@nasa.gov

Abstract—In this paper, we study the potential benefits of using relatively broadband wireless power transmission WPT strategies in both weak-signal and multipath environments where traditional narrowband strategies can be very inefficient. The paper is primarily a theoretical and analytical treatment of the problem that attempts to derive results that are widely applicable to many different WPT applications, including space solar power SSP.

Keywords—Wireless Power Transfer

I. INTRODUCTION

One of the most persistently intriguing applications of space solar power (SSP) is that of supplying terrestrial power through the use of wireless power transfer (WPT) from satellites to ground stations equipped with large rectenna receiver arrays [1-3]. For the large-scale, high-power systems envisioned for this application, it is reasonable to assume that the power density at the receiver rectenna will be relatively high, and that the propagation characteristics from transmitter to receiver will be free of any significant multipath effects. Under these assumptions, the rectifiers in the rectenna elements can be operated in a high-efficiency range well above the rectifier turn-on threshold, the transmitter can be tuned to match a fairly stable channel frequency response, and the traditional approach of transmitting narrowband sinusoidal signals has no serious drawbacks. In more challenging applications, such as systems that transmit relatively low-power signals from space or systems incorporating relatively small rectenna arrays placed on tall buildings in an urban environment, it is much more likely that received power densities may be low or multipath propagation from surrounding buildings may cause significant fading at any particular fixed frequency. In such situations, as we show in this paper, there are significant potential benefits associated with transmitting much more broadband signals.

Similarly, as argued in [2], a reasonable path for the eventual development and deployment of large scale space-based power generation should include the development and testing of small-scale and medium-scale WPT systems. Numerous such small-scale and medium-scale applications have been studied [4-6]. Since many such systems would suffer from either weak-signal or multipath fading conditions, the broadband transmission strategies discussed in this paper could be very applicable.

This remainder of this paper is organized as follows. In Section II we derive the signal model used for the rectifier output signal and the processing gain associated with the corresponding harvested signal energy. In Section III we describe three different power transmission strategies that are compared in this paper. In Section IV, we describe the Monte Carlo simulation procedure adopted in the paper for numerical performance evaluation of different power transmission strategies and present the results of that simulation procedure applied to the three chosen transmission strategies. Finally, the conclusions to be drawn from the results presented in the paper are summarized in Section V.

II. TECHNICAL BACKGROUND

The basic model we adopt for studying the efficiency of microwave power transmission strategies is the complex baseband signal model for finite-energy, real-valued bandpass signals with center frequency $f_0$ Hz and two-sided bandwidth $B \ll f_0$ Hz. That is, if an arbitrary received signal is represented by $g(t)$ with Fourier transform $G(f)$, then we assume that $G(f) = 0$ for all positive frequencies $f$ outside of the interval $[f_0 - B/2, f_0 + B/2]$. In this case, the received signal $g(t)$ can always be represented as

$$g(t) = \text{Re}[e^{2\pi if_0 t} g_0(t)] = x(t) \cos(2\pi f_0 t) - y(t) \sin(2\pi f_0 t),$$

where

$$g_0(t) = x(t) + iy(t)$$

represents the so-called baseband- (or low-pass) equivalent signal for $g(t)$. It follows that

$$G_0(f) = \begin{cases} 2G(f + f_0), & |f| \leq B/2, \\ 0, & |f| > B/2. \end{cases}$$

The output from a perfect full-wave rectifier with $g(t)$ as the input, i.e., the model for the signal input to the final energy harvesting circuit, is given by
\[ |g(t)| = \sqrt{g^2(t)} \]
\[ = \sqrt{x^2(t)\cos^2(2\pi f_0 t) + y^2(t)\sin^2(2\pi f_0 t)} \]
\[ = \frac{1}{\sqrt{2}}x^2(t)[1 + \cos(4\pi f_0 t)] + \frac{1}{\sqrt{2}}y^2(t)[1 - \cos(4\pi f_0 t)] \]
\[ = -x(t)y(t)\sin(4\pi f_0 t) \]
\[ = \frac{1}{\sqrt{2}}[x^2(t) - y^2(t)]\cos(4\pi f_0 t) \]
\[ = \frac{1}{\sqrt{2}}|g_{le}(t)|^2 + \frac{1}{\sqrt{2}}h(t), \]

where
\[ h(t) = [x^2(t) - y^2(t)]\cos(4\pi f_0 t) - x(t)y(t)\sin(4\pi f_0 t). \]

Now, suppose we pass the output from the rectifier through a perfect low-pass filter with bandwidth \( B \). Note that since the functions \( g_{le}(t), x(t), \) and \( y(t) \) all have bandwidth less than or equal to \( B \), the corresponding functions \( |g_{le}(t)|^2, x^2(t) - y^2(t) \), and \( x(t)y(t) \) all have bandwidth less than or equal to \( 2B \). Hence, both \( |g_{le}(t)|^2 \) and the envelope of \( h(t) \) are slowly varying with respect to the frequency \( 2f_0 \), and the function \( h(t) \) averages to zero over periods of time for which \( |g_{le}(t)|^2 \) is roughly constant. Intuitively, this implies that the low-pass filtered output from the rectifier takes the form
\[ r(t) \approx \frac{1}{\sqrt{2}}|g_{le}(t)|. \]

Since the output from the rectifier is always low-pass filtered (generally to a bandwidth much smaller than \( B \)) to drive the energy storage load, it follows that for purposes of analyzing the behavior of different power transmission strategies, the signal of interest is just \( |g_{le}(t)| \). Furthermore, assuming that there is no loss in the energy storage process, the amount of energy harvested by the circuit, which is the quantity of interest for measuring the power transmission performance, is given by
\[ E = \int_{-\infty}^{\infty} |g(t)|^2 dt = \frac{1}{2} \int_{-\infty}^{\infty} |g_{le}(t)|^2 dt \]
\[ = \frac{1}{2} \int_{-B/2}^{B/2} |G_{le}(f)|^2 df = 2 \int_{-B/2}^{B/2} |G(f + f_0)|^2 df. \]

For the remainder of this document, we will drop the “le” subscripts and simply assume we are always referring to baseband-equivalent signals.

Now suppose that we transmit an arbitrary baseband signal \( s(t) \) with two-sided bandwidth \( B \) and Fourier transform \( S(f) \) over a (baseband) channel with impulse response \( h(t) \) and associated frequency response \( H(f) \). Then the processing gain associated with transmitting power via the signal \( s(t) \), assuming a perfect full-wave rectifier and a perfect energy storage device at the receiver, is just the energy in the received signal divided by the energy in the transmitted signal, which is given by
\[ G(s) = \left( \frac{\int_{-\infty}^{\infty} |r(t)|^2 dt}{\int_{-\infty}^{\infty} |s(t)|^2 dt} \right)^2 = \int_{-\infty}^{\infty} \left| \frac{s}{\sqrt{E_s}} * h(t) \right|^2 dt = \int_{-B/2}^{B/2} \left| \frac{S(f)}{\sqrt{E_s}} * H(f) \right|^2 df, \]

where
\[ E_s = \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-B/2}^{B/2} |S(f)|^2 df. \]

Note that this is just a normalization that computes the total energy output from the channel assuming that the transmitted signal always has unit energy. The processing gain is the metric we will use for comparing performance of different power transmission strategies throughout this document.

Unfortunately, practical rectifiers are not perfect, and a principle source of inefficiency in the power transmission process is the threshold effect required at the input of the rectifier before the circuit will begin conducting current. That is, the relationship between the input voltage and output voltage for a practical rectifier can be modeled as
\[ |r(t)| = \begin{cases} 0, & |r(t)| \leq \varepsilon, \\ |r(t)| - \varepsilon, & |r(t)| > \varepsilon, \end{cases} \]

where \( r(t) \) represents the received signal at the input to the rectifier, \( \varepsilon \) represents the threshold voltage for the rectifier, and \( |r(t)|_e \) represents the equivalent effective signal for purposes of energy harvesting at the output of the rectifier. The corresponding processing gain is given by
\[ G_e(s) = \int_{-\infty}^{\infty} |r(t)|/\sqrt{E_s}^2 dt = \int_{-\infty}^{\infty} \left| \frac{s}{\sqrt{E_s}} * h(t) \right|^2 dt \]
\[ \leq G_b(s) = G(s). \]

### III. Transmission Strategies

We consider three different transmission strategies for comparison: a baseline approach assuming no channel knowledge; a narrowband approach that is optimal in terms of maximizing the processing gain in the absence of a significant rectifier threshold effect; and a broadband approach that is optimal in terms of peak channel power output rather than total energy output, which should be nearly optimal for maximizing the processing gain in the presence of a significant threshold effect.
A. Baseline Approach

In the situation where nothing is known a priori about the frequency response of the channel between the transmitter and receiver in a power transmission scheme, a reasonable approach is to transmit a flat power spectrum across the available bandwidth of \( B \) Hz. Hence, as a point of reference, we assume that the Fourier transform of the transmitted signal takes the form

\[
S_{\text{ref}}(f) = \begin{cases} 
1, & |f| \leq B/2, \\
0, & |f| > B/2.
\end{cases}
\]

The processing gain associated with this reference signal transmitted across an arbitrary channel with impulse response \( h(t) \) and frequency response \( H(f) \) is given by

\[
G_0(s_{\text{ref}}) = \frac{1}{B} \int_{-\infty}^{\infty} |h(t)|^2 dt = \frac{1}{B} \int_{-\infty}^{\infty} |H(f)|^2 df,
\]

assuming no rectifier threshold and

\[
G_0(s_{\text{ref}}) = \int_{-\infty}^{\infty} |h(t)|/\sqrt{B} dt,
\]

in general.

B. Narrowband Approach

If the impulse response \( h(t) \) and corresponding frequency response \( H(f) \) of the channel are known, then the processing gain can be maximized subject to an arbitrary time constraint on the transmitted signal by transmitting all of the energy in a narrow band around the frequency at which the channel frequency response is maximized. That is, if we assume that the transmitted signal must be essentially time limited to some interval \([0,T]\), then the processing gain will be approximately maximized by transmitting a signal of the form

\[
S_{\text{max}}(t) = \begin{cases} 
\exp(2\pi j f_{\text{max}} t), & 0 \leq t \leq T, \\
0, & \text{otherwise},
\end{cases}
\]

where

\[
f_{\text{max}} = \arg \max_{|f|/B \leq 1/2} |H(f)|.
\]

The processing gain for this optimal, time-limited narrowband signal is given by

\[
G_0(s_{\text{max}}) = \frac{1}{T} \int_{-T/2}^{T/2} \left| \int_{-T/2}^{T/2} e^{2\pi j f_{\text{max}} x} h(t-x) dx \right|^2 dt
\]

\[
= \frac{1}{T} \int_{0}^{T} \int_{0}^{T} |H(f)| \text{sinc}(\pi T |f_{\text{max}} - f|) df dt
\]

assuming no rectifier threshold, where

\[
r_{\text{max}}(t) = \int_{0}^{T} e^{2\pi j f_{\text{max}} x} h(t-x) dx,
\]

and

\[
G_0(s_{\text{max}}) = \int_{-\infty}^{\infty} \left| h(t) \right|^2 dt / \int_{-\infty}^{\infty} \left| H(f) \right|^2 df,
\]

it is worth noting that most WPT systems transmit a sinusoidal signal at a single frequency. Hence, the narrowband approach discussed here is really the most common power beaming approach. Of course, in a free-space environment, any frequency (or any waveform at all for that matter) will produce exactly the same results, and most previous work on power beaming has assumed a free-space environment. In the sequel, we compute the behavior of both the best-case sinusoid illustrated in this section as well as the worst-case sinusoid, which corresponds to transmitting on the frequency with the minimum possible frequency response rather than the maximum. Clearly, if one adopts a narrowband approach without attempting to match the signal to the channel, the worst-case scenario is just as likely to prevail as the best-case scenario, and computing both brackets the behavior on the channel for narrowband signals in general.

C. Broadband Approach

If the impulse response \( h(t) \) and corresponding frequency response \( H(f) \) of the channel are known, then the processing gain associated with this reference signal transmitted across an arbitrary channel with impulse response \( h(t) \) and frequency response \( H(f) \) is given by

\[
G_0(s_{\text{ref}}) = \frac{1}{B} \int_{-\infty}^{\infty} |h(t)|^2 dt = \frac{1}{B} \int_{-\infty}^{\infty} |H(f)|^2 df,
\]

assuming no rectifier threshold and

\[
G_0(s_{\text{ref}}) = \int_{-\infty}^{\infty} |h(t)|/\sqrt{B} dt,
\]

in general.

It is worth noting that most WPT systems transmit a sinusoidal signal at a single frequency. Hence, the narrowband approach discussed here is really the most common power beaming approach. Of course, in a free-space environment, any frequency (or any waveform at all for that matter) will produce exactly the same results, and most previous work on power beaming has assumed a free-space environment. In the sequel, we compute the behavior of both the best-case sinusoid illustrated in this section as well as the worst-case sinusoid, which corresponds to transmitting on the frequency with the minimum possible frequency response rather than the maximum. Clearly, if one adopts a narrowband approach without attempting to match the signal to the channel, the worst-case scenario is just as likely to prevail as the best-case scenario, and computing both brackets the behavior on the channel for narrowband signals in general.

If the impulse response \( h(t) \) and corresponding frequency response \( H(f) \) of the channel are known, then the peak output power from the channel can be maximized subject to an arbitrary energy constraint, by transmitting the time-reverse of the channel impulse response. That is (assuming for simplicity that the impulse response is essentially time limited to the interval \([0,T]\)), transmitting a signal of the form

\[
S_{\text{R}}(t) = \begin{cases} 
h(T-t), & 0 \leq t \leq T, \\
0, & \text{otherwise},
\end{cases}
\]

maximizes the peak output power of the channel over all transmitted signals with energy

\[
E_t = \int_{-\infty}^{\infty} \left| h(t) \right|^2 dt,
\]

with the maximum peak power output occurring at time \( t = T \). For practical purposes, this implies that \( S_{\text{R}}(t) \) also approximately maximizes the processing gain of the channel in the presence of a substantial threshold effect. The processing gain for the peak-power optimal broadband signal is given by

\[
G_0(s_{\text{R}}) = \int_{-\infty}^{\infty} \left| r_{\text{R}}(t) \right|^2 dt / \int_{-\infty}^{\infty} \left| h(t) \right|^2 dt
\]

\[
= \int_{-\infty}^{\infty} \left| h(T-x) \right| \left| h(t-x) \right|^2 dx dt
\]

\[
= \int_{-\infty}^{\infty} \left| H(f) \right|^2 dx,
\]
assuming no rectifier threshold, where
\[ r_{\text{B}}(t) = \int_{-\infty}^{\infty} h(T-x)h(t-x)dx , \]
and
\[ G_{\text{c}}(s_{\text{B}}) = \int_{-\infty}^{\infty} f_{\text{c}}(t_{\text{c}})/\sqrt{E_{\text{B}}} dt_{\text{c}} , \]
in general.

IV. SIMULATION RESULTS

To give some idea of how the various transmission strategies work over a broad range of channels parameterized by the amount of reverberation in the environment, we have completed a simulation study of performance using a standard random Gaussian channel model. The channel model used is the so-called wide-sense-stationary, uncorrelated scattering (WSSUS) model \([7]\) that is frequently adopted for statistical analysis of small-scale fading on Rayleigh fading channels, coupled with a simple square-law path-loss model. While this is certainly not a completely generic channel model, it does capture most of the essential channel characteristics that impact the efficiency of microwave power transmission.

A. Power Delay Profile

The power delay profile \(S(\tau)\) for a WSSUS channel is the inverse Fourier transform of the spaced-frequency correlation function \(\varphi(f)\) for the channel and represents the distribution of the energy in the channel impulse response over delay time \(\tau\). The channel models used to generate performance results in this document assumed a causal power delay profile parameterized by a power-decay parameter \(\alpha\) and a (totally arbitrary) maximum delay spread of \(T\) given by

\[ S(\tau) = \begin{cases} \frac{\alpha}{1-(1+\alpha T) e^{-\alpha T} \left(e^{-\alpha(T-\tau)}-1\right)}, & 0 \leq \tau \leq T, \\ 0, & \text{otherwise}. \end{cases} \]

Technically, for a WSSUS model associated with a Rayleigh fading channel, the impulse response of the channel \(h(\tau)\) is a zero-mean, complex Gaussian random process with variance characterized by

\[ E\left[|h(\tau)|^2\right] = \frac{1}{\delta} \int_{-\infty}^{1/\delta} S(\tau) d\tau , \]
for any value of \(\delta > 0\). Hence, for small values of \(\delta\), we have

\[ E\left[|h(\tau)|^2\right] \approx E\left[\frac{1}{\delta} \int_{-1/\delta}^{1/\delta} S(\tau) d\tau \right] = \frac{1}{\delta} \int_{-1/\delta}^{1/\delta} \delta S(\tau) d\tau . \]

For large values of the bandwidth \(B\) with \(\delta = 1/B\), this relationship becomes

\[ E\left[|h(t)|^2\right] \approx E\left[\int_{-1/\delta}^{1/\delta} S(\tau) d\tau \right] = B^2 \int_{-1/\delta}^{1/\delta} S(\tau) d\tau , \]
which can be used to generate a sequence of independent, zero-mean, complex Gaussian random variables \(h_k = h(kB)\), for \(k = 0, 1, \ldots, TB\), that represent samples of the impulse response of a baseband WSSUS channel smoothed to have an approximate bandwidth of \(B\) Hz (more or less).

The channels considered in this study have power delay profiles corresponding to \(B = 80\) MHz, \(T = 40\) \(\mu\)sec, and \(\alpha = \{10^2, 10^3, 10^5, 10^6\}\). The channels corresponding to \(\alpha = 10^5\) have a very long impulse response (equivalently, a great deal of environmental multipath and a delay spread much longer than the reciprocal of the bandwidth), and channels corresponding to \(\alpha = 10^6\) have only a single sample of non-zero impulse response (equivalently, a channel with only a single resolvable path and a delay spread shorter than the reciprocal of the bandwidth).

B. Examples with no threshold effect

As a baseline for comparison of different strategies, we start by considering the behavior of the three transmission schemes discussed above assuming no rectifier threshold effect. In particular, we compute the channel outputs from a sample of 100 channel realizations for each of four different transmitted signals:

1. The reference bandlimited impulse signal. In this case, the output is just the impulse response of the bandlimited random baseband channel.
2. The time-reverse of the impulse response of the channel. In this case, the output is a very “peaky” signal that is considerably delayed with respect to the other three outputs.
3. The best-case sinusoidal signal for the random realization of the channel.
4. The worst-case sinusoidal signal for the random realization of the channel.

Throughout the remainder of this document, the term “Reference” refers to the behavior of the channel assuming the reference bandlimited impulse signal was transmitted and no rectifier threshold effect is present. In contrast, the term “Impulse” also refers to the transmission of the reference bandlimited impulse signal but with the effect of any rectifier threshold taken into consideration. The three other signals are referred to as “TR” for transmission of the time-reverse of the impulse response, “MaxFreq” for transmission of the best-case sinusoid, and “MinFreq” for transmission of the worst-case sinusoid.

The normalized amount of energy that can theoretically be harvested on these channels (i.e., the processing gain) as a function of transmission range, relative to an arbitrary initial range (this can be thought of as 1 meter, for convenience), is illustrated in Figures 1-5. In these figures, the Reference and Impulse curves are exactly the same (as they should be for a
threshold of zero), and all of the curves decrease at a rate of 20 dB per decade of relative range increase, corresponding to the assumed free-space path-loss exponent of 2.

Figure 1. Processing gain for $\alpha = 10^5$ with no threshold effect.

Figure 2. Processing gain for $\alpha = 10^6$ with no threshold effect.

Figure 3. Processing gain for $\alpha = 10^7$ with no threshold effect.

Figure 4. Processing gain for $\alpha = 10^8$ with no threshold effect.

Figure 5. Processing gain for $\alpha = 10^9$ with no threshold effect.

There are several points of interest to be observed in these figures:

1. Because there is assumed to be no rectifier threshold effect, the curves corresponding the Reference and Impulse signals are exactly the same.

2. As predicted, for the channel with the no multipath ($\alpha = 10^0$), the energy that can theoretically be harvested over the channel is the same for any unit-energy transmitted signal.

3. Also as predicted, the amount of energy that can theoretically be harvested from the channel is always maximized by transmitting the MaxFreq signal and minimized by transmitting the MinFreq signal; however, the penalty associated with transmitting the Reference/Impulse signal or the TR signal relative to the MaxFreq signal remains relatively small over the range of channels studied while the penalty associated with transmitting the MinFreq signal can become extremely large.

4. Finally, notice that the difference between the energy harvested from the MaxFreq and Reference/Impulse
signals is maximized on the channel with the greatest delay spread \( \alpha = 10^5 \), which is exactly what one would expect. On the other hand, the difference between the energy harvested from the MaxFreq and MinFreq signals is not maximized on the channel with the greatest delay spread, which is somewhat counter-intuitive. In fact, this results from the interaction between the relatively short reference transmission time \( T \) and the transient behavior on the channel. If the reference transmission time were increased sufficiently, the transient behavior on all of the channels would become negligible, and the difference in energy harvested between the MaxFreq and MinFreq signals would also be maximized on the channel with the greatest delay spread.

By far the most important conclusion to be drawn from the above is summarized by rephrasing observation number 3: \textit{With no channel knowledge, transmission of a narrowband signal at an arbitrary frequency can be a very bad idea when there is no significant rectifier effect, but transmitting a simple broadband impulse under those conditions always performs relatively well.}

C. Examples with a threshold effect

As discussed previously, any real rectifier will only begin conducting current when the input voltage exceeds some activation threshold. If the input signal to the rectifier is large relative to the threshold, then the effect of the threshold on the amount of energy that can be harvested at the output of the rectifier will be negligible. However, as the input signal voltage decreases, the amount of energy harvested drops at an ever increasing rate relative to the decrease in input voltage.

To illustrate the effect of a rectifier threshold on energy harvesting performance, we reran the same set of simulations as above assuming that the rectifier threshold was fixed at a value of \( 9 \times 10^{-3} \) magnitude units (proportional to volts). This value was chosen as it is approximately 80 dB (i.e., a reduction in magnitude by a factor of 10,000) below the maximum peak magnitude for any of the channel output signals seen in the simulations. There is no special significance to the chosen value for the rectifier threshold. It was picked simply because it is insignificant relative to the peak magnitude of any of the channel output signals at the initial reference range (as might be expected in practice) but is still large enough to have a significant impact on energy harvested over the span of transmission ranges considered in the simulations.

The amount of energy that can theoretically be harvested on these channels with the rectifier threshold taken into account is illustrated in Figures 6-10. Once again, the figures illustrate the energy harvested as a function of relative range, but in these figures, only the curve corresponding to the Reference signal, which ignores the rectifier threshold, decreases at a rate of 20 dB per decade of relative range increase. All of the others decrease at a greater rate, which increases with relative distance.
The points of interest to be observed in these figures are as follows:

1. For channels with no significant multipath ($\alpha = 10^0$), the rectifier threshold has almost no effect on either of the two broadband signals (Impulse and TR), which are identical in this case, but an enormous effect on a narrowband signal at any frequency.

2. As the level of multipath on the channel increases, the TR signal, continues to show very little loss in harvested energy over the entire 30-dB span of relative range. The Impulse signal, shows more degradation but still significantly outperforms all narrowband signals at long ranges.

3. On channels with significant multipath, the narrowband signal matched to the channel response continues to outperform the broadband signals at shorter ranges, but decays much more quickly as the range increases.

V. CONCLUSION

In summary, the following conclusions can be drawn regarding microwave WPT when rectifier thresholds, which always come into play at some point, are taken into account:

1. With no channel knowledge, transmitting a narrowband signal at an arbitrary frequency can be a devastatingly bad idea, and even in the best case, will not outperform a simple broadband impulse at long range.

2. For channels with no significant multipath, transmitting a narrowband signal is always a bad idea, and transmitting a simple broadband impulse comes close to being the optimal strategy at any range.

3. When channel knowledge is available, the optimal strategy over short range is a narrowband signal at the frequency matched to the maximum channel frequency response, but as the range increase, transmitting the time-reverse of the channel impulse response comes close to being optimal and significantly outperforms the best narrowband signal at long ranges. For this case, it should be noted that when both transmitter and receiver are equipped with large antenna arrays, the transmitting-element-to-receiving-element channel impulse response may vary significantly across the array, which makes matching the transmitted signal to the channel response very problematic if not impossible.

REFERENCES


