Statistical Mechanics of Turbulent Dynamos

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Incompressible magnetohydrodynamic (MHD) turbulence and magnetic dynamos, which occur in magnetofluids with large fluid and magnetic Reynolds numbers, will be discussed. When Reynolds numbers are large and energy decays slowly, the distribution of energy with respect to length scale becomes quasi-stationary and MHD turbulence can be described statistically. In the limit of infinite Reynolds numbers, viscosity and resistivity become zero and if these values are used in the MHD equations ab initio, a model system called ideal MHD turbulence results. This model system is typically confined in simple geometries with some form of homogeneous boundary conditions, allowing for velocity and magnetic field to be represented by orthogonal function expansions. One advantage to this is that the coefficients of the expansions form a set of nonlinearly interacting variables whose behavior can be described by equilibrium statistical mechanics, i.e., by a canonical ensemble theory based on the global invariants (energy, cross helicity and magnetic helicity) of ideal MHD turbulence. Another advantage is that truncated expansions provide a finite dynamical system whose time evolution can be numerically simulated to test the predictions of the associated statistical mechanics. If ensemble predictions are the same as time averages, then the system is said to be ergodic; if not, the system is nonergodic. Although it had been implicitly assumed in the early days of ideal MHD statistical theory development that these finite dynamical systems were ergodic, numerical simulations provided sufficient evidence that they were, in fact, nonergodic. Specifically, while canonical ensemble theory predicted that expansion coefficients would be (i) zero-mean random variables with (ii) energy that decreased with length scale, it was found that although (ii) was correct, (i) was not and the expected ergodicity was broken. The exact cause of this broken ergodicity was explained, after much investigation, by greatly extending the statistical theory of ideal MHD turbulence. The mathematical details of broken ergodicity, in fact, give a quantitative explanation of how coherent structure, dynamic alignment and force-free states appear in turbulent magnetofluids. The relevance of these ideal results to real MHD turbulence occurs because broken ergodicity is most manifest in the ideal case at the largest length scales and it is in these largest scales that a real magnetofluid has the least dissipation, i.e., most closely approaches the behavior of an ideal magnetofluid. Furthermore, the effects grow stronger when cross and magnetic helicities grow large with respect to energy, and this is exactly what occurs with time in a real magnetofluid, where it is called selective decay. The relevance of these results found in ideal MHD turbulence theory to the real world is that they provide at least a qualitative explanation of why confined turbulent magnetofluids, such as the liquid iron that fills the Earth's outer core, produce stationary, large-scale magnetic fields, i.e., the geomagnetic field. These results should also apply to other planets as well as to plasma confinement devices on Earth and in space, and the effects should be manifest if Reynolds numbers are high enough and there is enough time for stationarity to occur, at least approximately. In the presentation, details will be given for both theoretical and numerical results, and references will be provided.