Computational Fluid Dynamics Uncertainty Analysis applied to Heat Transfer over a Flat Plate

Curtis Groves
Ph.D. Candidate, University of Central Florida, Orlando, FL
Fluids Engineer, NASA Kennedy Space Center, FL

Marcel Ilie, Ph.D.
Former Assistant Professor, University of Central Florida, Orlando, FL

Paul Schallhorn, Ph.D.
Environments and Launch Approval Branch Chief, NASA Kennedy Space Center, FL
Problem

• Computational Fluid Dynamics (CFD) is being used without proper validation
• Experimental Data is expensive
  – Shrinking Budgets
• Pairing experimental data, uncertainty analysis, and analytical predictions provides a comprehensive approach to verification and is the current state of the art. (ASME V&V 20-2009)
• A method is sought to conservatively envelop the exact solution using CFD only
  – Example Heat Transfer over a flat plate is presented
Comparison error:

\[ E = S - D \]

Validation uncertainty,

\[ u_{\text{val}} = k \left( \sqrt{u_{\text{num}}^2 + u_{\text{input}}^2 + u_D^2} \right) \]

\[ \delta_{\text{model}} \in \left[ E - u_{\text{val}}, E + u_{\text{val}} \right] \]
Without Experimental Data??

Proposed Methodology: **Conservative estimate to envelop true value**

If there is no experimental data, D=0, δ_D=0, and u_D=0.

\[ E = S - D = S \]

\[ \delta_s = S - T \]

\[ E = S - D = T + \delta_s - (T + \delta_D) = \delta_s - \delta_D = \delta_s \]

\[ u_{val} = k \left( \sqrt{u_{num}^2 + u_{input}^2 + u_D^2} \right) \]

\[ u_{val} = k \left( \sqrt{u_{num}^2 + u_{input}^2} \right) \]

Report the simulated result, S as

\[ S^{+}_{val} u_{val} \]
Without Experimental Data -continued

- Report $S \pm u_s$
- $k$ – value (Use Student-$t$ Distribution)
- Treat all input variables as ‘random’ and run separate CFD case
- Treat as an oscillatory convergence parameter

$$U_{Oscillatory} = \frac{1}{2} (S_U - S_L)$$

<table>
<thead>
<tr>
<th>Number of Cases</th>
<th>Degrees of Freedom</th>
<th>Confidence 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>6.314</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2.92</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2.353</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>2.132</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>2.015</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>1.943</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>1.895</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>1.86</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>1.833</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>1.812</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>1.796</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>1.782</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
<td>1.771</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
<td>1.761</td>
</tr>
<tr>
<td>16</td>
<td>15</td>
<td>1.753</td>
</tr>
<tr>
<td>17</td>
<td>16</td>
<td>1.746</td>
</tr>
<tr>
<td>18</td>
<td>17</td>
<td>1.74</td>
</tr>
<tr>
<td>19</td>
<td>18</td>
<td>1.734</td>
</tr>
<tr>
<td>20</td>
<td>19</td>
<td>1.729</td>
</tr>
<tr>
<td>21</td>
<td>20</td>
<td>1.725</td>
</tr>
<tr>
<td>22</td>
<td>21</td>
<td>1.721</td>
</tr>
<tr>
<td>23</td>
<td>22</td>
<td>1.717</td>
</tr>
<tr>
<td>24</td>
<td>23</td>
<td>1.714</td>
</tr>
<tr>
<td>25</td>
<td>24</td>
<td>1.711</td>
</tr>
<tr>
<td>26</td>
<td>25</td>
<td>1.708</td>
</tr>
<tr>
<td>27</td>
<td>26</td>
<td>1.706</td>
</tr>
<tr>
<td>28</td>
<td>27</td>
<td>1.703</td>
</tr>
<tr>
<td>29</td>
<td>28</td>
<td>1.701</td>
</tr>
<tr>
<td>30</td>
<td>29</td>
<td>1.699</td>
</tr>
<tr>
<td>31</td>
<td>30</td>
<td>1.697</td>
</tr>
<tr>
<td>41</td>
<td>40</td>
<td>1.684</td>
</tr>
<tr>
<td>51</td>
<td>50</td>
<td>1.676</td>
</tr>
<tr>
<td>61</td>
<td>60</td>
<td>1.671</td>
</tr>
<tr>
<td>81</td>
<td>80</td>
<td>1.664</td>
</tr>
<tr>
<td>101</td>
<td>100</td>
<td>1.66</td>
</tr>
<tr>
<td>121</td>
<td>120</td>
<td>1.658</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$\infty$</td>
<td>1.645</td>
</tr>
</tbody>
</table>
Cornell University posted a Fluent Example Problem

\[ h = c \left( \frac{\rho V L}{\mu} \right)^{4/5} \frac{k}{L} \]

- \( U_\infty = 1 \text{ m/s} \)
- \( \mu = 6.667e-7 \text{ kg/(m·s)} \)
- \( k = 9.4505e-4 \text{ W/(m·K)} \)
- \( C_p = 1006.43 \text{ J/(kg·K)} \)
- \( T_p = 413 \text{ K} \)
- \( T_\infty = 353 \text{ K} \)
- \( P_\infty = 101325 \text{ Pa} \n \]
- \( \text{Re}_L = 1.5e6 \)
- \( \text{Pr} = 0.71 \)

<https://confluence.cornell.edu/display/SIMULATION/FLUENT+-+Forced+Convection+over+a+Flat+Plate>
Example Heat Transfer over Flat Plate

- Heat Transfer Correlations, Traditional
- The uncertainty analysis will follow the methodology laid out by Coleman and Steele (Experimentation and Uncertainty Analysis for Engineers, 2nd ed, J. Wiley and Sons, 1999). This methodology is in line with the ISO Guide to the Expression of Uncertainty in Measurement (1993).

\[
U = \left( \sum_{i=1}^{J} \left( \frac{\partial r}{\partial X_i} \right)^2 B_i^2 \right) + 2 \sum_{i=1}^{J} \sum_{k=i+1}^{J} \left( \frac{\partial r}{\partial X_i} \right) \left( \frac{\partial r}{\partial X_k} \right) [B_i B_k]_{\text{correlated}} + \sum_{i=1}^{J} \left( \frac{\partial r}{\partial X_i} \right)^2 P_i^2 \right)^{1/2}
\]

<table>
<thead>
<tr>
<th>Bias</th>
<th>Correlated</th>
<th>Random</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ h = c \left( \frac{\rho VL}{\mu} \right)^{4/5} \frac{k}{L} ]</td>
<td>[ \frac{dh}{dV} = \frac{4ck\rho}{5\mu (\frac{LV\rho}{\mu})^{4/5}} ]</td>
<td>[ \frac{dh}{d\mu} = -\frac{4CVk\rho}{5\mu^2 (\frac{LV\rho}{\mu})^{4/5}} ]</td>
</tr>
<tr>
<td>[ \frac{dh}{d\rho} = \frac{4ckV}{5\mu (\frac{LV\rho}{\mu})^{4/5}} ]</td>
<td>[ \frac{dh}{dL} = \frac{4CVk\rho}{5L\mu (\frac{LV\rho}{\mu})^{4/5}} - \frac{Ck (\frac{LV\rho}{\mu})^{4/5}}{L^2} ]</td>
<td>[ \frac{dh}{dc} = \frac{k}{L} \left( \frac{\rho VL}{\mu} \right)^{4/5} ]</td>
</tr>
<tr>
<td>[ \frac{dh}{dk} = \frac{c}{L} \left( \frac{LV\rho}{\mu} \right)^{4/5} ]</td>
<td>[ \frac{dh}{dc} = \frac{k}{L} \left( \frac{\rho VL}{\mu} \right)^{4/5} ]</td>
<td></td>
</tr>
</tbody>
</table>
• Heat Transfer Correlation Uncertainty

\[ U_h = \left( \left( \frac{\partial h}{\partial \nu} \right)^2 B_v^2 \right) + \left( \left( \frac{\partial h}{\partial \rho} \right)^2 B_\rho^2 \right) + \left( \left( \frac{\partial h}{\partial k} \right)^2 B_k^2 \right) + \left( \left( \frac{\partial h}{\partial \mu} \right)^2 B_\mu^2 \right) + \left( \left( \frac{\partial h}{\partial L} \right)^2 B_L^2 \right) + \left( \left( \frac{\partial h}{\partial c} \right)^2 P_c^2 \right) + 2 \left( \frac{\partial h}{\partial \rho} \right) \left( \frac{\partial h}{\partial k} \right) B_\rho B_k + 2 \left( \frac{\partial h}{\partial \rho} \right) \left( \frac{\partial h}{\partial \mu} \right) B_\rho B_\mu + \right. \\
\left. 2 \left( \frac{\partial h}{\partial k} \right) \left( \frac{\partial h}{\partial \mu} \right) B_k B_\mu \right)^{1/2} \]
Example Heat Transfer over Flat Plate

- Plug in Partial Derivatives

\[ U_h = \left( \left( \frac{4ck\rho}{5\mu (L/V, \rho)} \right)^{2} B_v^2 \right) + \left( \left( \frac{4ckV}{5\mu (L/V, \rho)} \right)^{1/5} B_\rho^2 \right) + \left( \frac{c}{L} \left( \frac{L/V, \rho}{\mu} \right)^{4/5} B_k^2 \right) + \]

\[ \left( \frac{\partial h}{\partial \mu} \right)^{2} B_\mu^2 \right) + \left( \left( \frac{4ckV}{5\mu (L/V, \rho)} \right)^{1/5} \right) B_L^2 \right) + \left( \frac{k}{L} \left( \frac{L/V, \rho}{\mu} \right)^{4/5} \right) P_C^2 \right) + \]

\[ 2 \left( \frac{4ckV}{5\mu (L/V, \rho)} \right) \left( \frac{c}{L} \left( \frac{L/V, \rho}{\mu} \right)^{4/5} \right) B_\rho B_k + 2 \left( \frac{4ckV}{5\mu (L/V, \rho)} \right) \left( - \frac{4ckV}{5\mu^2 (L/V, \rho)^{1/5}} \right) B_\rho B_\mu + \]

\[ 2 \left( \frac{c}{L} \left( \frac{L/V, \rho}{\mu} \right)^{4/5} \right) \left( - \frac{4ckV}{5\mu^2 (L/V, \rho)^{1/5}} \right) B_k B_\mu \left( \frac{1/2}{1/2} \right) \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity, V</td>
<td>3%</td>
</tr>
<tr>
<td>Density, rho</td>
<td>3%</td>
</tr>
<tr>
<td>Thermal Conductivity, k</td>
<td>3%</td>
</tr>
<tr>
<td>Viscosity, ( \mu )</td>
<td>3%</td>
</tr>
</tbody>
</table>
• Numerically Evaluating (Traditional):

**Heat Transfer Coefficient**

**Uncertainty in Heat Transfer Coefficient**
**Proposed Methodology using CFD Only**

<table>
<thead>
<tr>
<th>CFD Uncertainty Cases</th>
<th>Number of Cases</th>
<th>Degrees of Freedom</th>
<th>Confidence 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse Grid</td>
<td>2</td>
<td>1</td>
<td>6.314</td>
</tr>
<tr>
<td>Medium Grid</td>
<td>3</td>
<td>2</td>
<td>2.92</td>
</tr>
<tr>
<td>Fine Grid</td>
<td>4</td>
<td>3</td>
<td>2.353</td>
</tr>
<tr>
<td>Velocity Low</td>
<td>5</td>
<td>4</td>
<td>2.132</td>
</tr>
<tr>
<td>Velocity High</td>
<td>6</td>
<td>5</td>
<td>2.015</td>
</tr>
<tr>
<td>Density Low</td>
<td>7</td>
<td>6</td>
<td>1.943</td>
</tr>
<tr>
<td>Density High</td>
<td>8</td>
<td>7</td>
<td>1.895</td>
</tr>
<tr>
<td>Thermal Conductivity High</td>
<td>9</td>
<td>8</td>
<td>1.86</td>
</tr>
<tr>
<td>Thermal Conductivity Low</td>
<td>10</td>
<td>9</td>
<td>1.833</td>
</tr>
<tr>
<td>Viscosity Low</td>
<td>11</td>
<td>10</td>
<td>1.812</td>
</tr>
<tr>
<td>Viscosity High</td>
<td>12</td>
<td>11</td>
<td>1.730</td>
</tr>
<tr>
<td>SA Turbulence Model</td>
<td>13</td>
<td>12</td>
<td>1.782</td>
</tr>
<tr>
<td>kWSST Turbulence Model</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ U_h = \left( \left( \frac{\partial h}{\partial \ell} \right)^2 B_v^2 \right) + \left( \frac{\partial h}{\partial \rho} \right)^2 B_\rho^2 + \left( \frac{\partial h}{\partial k} \right)^2 B_k^2 + \left( \frac{\partial h}{\partial \mu} \right)^2 B_\mu^2 + \left( \frac{\partial h}{\partial c} \right)^2 P_c^2 + 2 \left( \frac{\partial h}{\partial \rho} \right) \left( \frac{\partial h}{\partial k} \right) B_\rho B_k + 2 \left( \frac{\partial h}{\partial \rho} \right) \left( \frac{\partial h}{\partial \mu} \right) B_\rho B_\mu + 2 \left( \frac{\partial h}{\partial k} \right) \left( \frac{\partial h}{\partial \mu} \right) B_k B_\mu \right)^{1/2} \]

\[ u_{val} = 1.782 \times \left| \frac{1}{2} (S_U - S_L) \right| \]
Results for proposed methodology

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average Difference in htc (W/m²K)</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulence</td>
<td>0.693102673</td>
<td>1</td>
</tr>
<tr>
<td>Grid</td>
<td>0.130514851</td>
<td>2</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.117431782</td>
<td>3</td>
</tr>
<tr>
<td>Density</td>
<td>0.117431683</td>
<td>4</td>
</tr>
<tr>
<td>k (thermal conductivity)</td>
<td>0.069466139</td>
<td>5</td>
</tr>
<tr>
<td>Viscosity</td>
<td>0.021837228</td>
<td>6</td>
</tr>
</tbody>
</table>
Comparison of Methods

• Traditional vs. Proposed

Heat Transfer Coefficient

Uncertainty in Heat Transfer Coefficient

CFD Uncert High

CFD Uncert Low

Uncertainty in Proposed Method

Uncertainty in Traditional Method
Comparison of Methods

• Traditional vs. Proposed Average Heat Transfer Coefficient over Flat Plate

  – Traditional
    \[ h_{\text{avg}} = 2.66 \pm 0.74 \text{ [W/m}^2\text{K]} \]

  – Proposed CFD,
    \[ h_{\text{avg}} = 2.73 \pm 1.39 \text{ [W/m}^2\text{K]} \]
Conclusion

• Proposed Method Envelops the True value and uses only CFD Data to Estimate the Uncertainty for Heat Transfer over a Flat Plate
  – No Testing

• Proposed methodology can be used to conservatively estimate the uncertainty in CFD models