T-matrix modeling of linear depolarization by morphologically complex soot and soot-containing aerosols

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1. Introduction

Black carbon (BC) and BC-containing particles form a climatically important category of tropospheric aerosols exerting a direct radiative forcing of climate, inhibiting as well as facilitating cloud formation, and reducing the albedo of ice and snow surfaces, thereby affecting the overall energy budget of the terrestrial climate system [1–11]. They can also limit atmospheric visibility and have a highly negative effect on human health [12]. Owing to the notoriously complex morphology of such particles [13–30], theoretical modeling of their scattering and absorption properties is a highly nontrivial task and has often been based on approximate approaches with poorly defined accuracy and range of applicability. However, the growing need for much improved knowledge of BC and BC-containing aerosols and their climatic, ecological, and visibility effects imposes strict limitations on quantitative uncertainties in particle scattering and absorption properties entering optical characterization and remote sensing applications as well as atmospheric radiation budget computations [31–42].

Electron microscope images of soot particles show that they are emitted as complex chain structures (e.g., Ref. [16]), which tend to collapse upon humidification and/or as the particles age (e.g., Refs. [17, 22, 24, 43]), thereby changing their optical properties. Recently Ueda et al. [28] classified soot-containing particles into five major types: soot aggregates, dome-shaped particles, particles having satellite structures, single nearly spheroidal particles, and clusters of spheroidal units. R.K. Chakrabarty et al. (private communication) have conducted a literature survey and identified three major biomass burning aerosol habits: fractal soot aggregates,
individual and semi-externally mixed tar balls [25], and carbonaceous super-aggregates [30] (we use the definitions of external, semi-external, and internal mixtures of particles according to Ref. [44]). One also encounters situations when soot clusters are semi-externally or internally mixed with other aerosol types [15, 27].

Some of the prevailing morphologies of BC and BC-containing particles are illustrated in Fig. 1. One might think that the extreme complexity of these morphologies can limit one’s ability to compute the scattering and absorption properties of realistic BC and BC-containing aerosol models with requisite accuracy. Yet the rapid growth of computer power coupled with the availability of efficient numerically-exact computer solvers of the Maxwell equations [45–48] has led to a recent explosion in the number of papers based on first-principle computations. The (superposition) T-matrix method (TMM) [46, 49–55] and the discrete dipole approximation (DDA) [56, 57] (or the closely related volume integral equation formulation [58, 59]) appear to be the most frequently used techniques, as exemplified by Refs. [60–87] and [87–95], respectively. However, other approaches such as the finite difference time domain method (FDTDM), the pseudo-spectral time domain method, and the finite element—boundary integral technique are also gaining popularity [96–104].

It is commonly believed that the DDA and FDTDM provide greater flexibility in modeling complex particle morphologies. However, the currently available TMM codes (e.g., Refs. [46, 51, 105–107]) enable one to easily model the morphologies shown in Fig. 1a–d while providing much greater computational efficiency. Furthermore, it is expected that the superposition TMM code described in Ref. [105] will soon be extended to handle the internal mixing scenarios exemplified by Fig. 1e. Perhaps the only morphological detail that may defy the superposition TMM codes is partial overlap of otherwise quasi-spherical soot monomers resulting in so-called sintered soot aggregates. However, the results of Ref. [87] suggest that the effect of sintering on the scattering and absorption properties of soot aggregates is likely to be insignificant.

In our previous publications [44, 64–66], we had used the superposition TMM code described in Ref. [108] to analyze the potential effects of aggregation and heterogeneity on light scattering and absorption by complex tropospheric soot and soot-containing aerosols. The computations described below are based on the new Fortran 90 version of the code which makes use of dynamic memory allocation and vector arithmetic features that enable efficient execution on both parallel and serial machines [105, 109]. The primary objective of this paper is to analyze in detail the dependence on the morphology of BC aerosols of an important optical characteristic called the linear depolarization ratio (LDR). This characteristic is often measured in laboratory experiments [110, 111] and, in the case of the exact backscattering direction, in lidar remote-sensing research [112–115], but appears to be studied inadequately on the basis of first-principle numerical computations. Our paper is motivated by the recent publication [98] in which the FDTDM was used to compute the LDR for smoke clusters of up to four monomers in order to analyze implications of depolarization lidar observations from the Cloud—Aerosol Lidar and

Fig. 1. (a) Transmission electron microscope (TEM) image of a soot cluster [29], (b) Scanning electron microscope image of a collapsed (fractal dimension approaching 3) chamise smoke particle [24], (c) TEM images of nearly spheroidal soot particles [28], (d) TEM image of an aggregate formed by a soot cluster and an ammonium sulfate particle [15] and (e) TEM image of internally mixed soot aggregates. Arrows show solid phases inside aqueous droplets [27].
Infrared Pathfinder Satellite Observation (CALIPSO) satellite [115]. We extend the analysis of Ref. [98] by considering a more comprehensive and representative set of soot-particle models and using what we believe to be more relevant refractive indices.

2. Basic definitions and notation

The key single-scattering characteristics of randomly oriented particles forming a macroscopically isotropic and mirror-symmetric turbid medium are the ensemble-averaged scattering and extinction cross sections per particle and the elements of the normalized scattering matrix [46]. In the standard $(I, Q, U, V)$ representation of polarization, the normalized $4 \times 4$ Stokes scattering matrix has the well-known block-diagonal structure [46]:

$$
\mathbf{F}(\theta) = \begin{bmatrix}
    a_1(\theta) & b_1(\theta) & 0 & 0 \\
    b_1(\theta) & a_2(\theta) & 0 & 0 \\
    0 & 0 & a_3(\theta) & b_3(\theta) \\
    0 & 0 & -b_2(\theta) & a_4(\theta)
\end{bmatrix},
$$

(1)

where $0 \leq \theta \leq 180^\circ$ is the scattering angle. The $(1,1)$ element of the scattering matrix, $a_1(\theta)$, is traditionally called the phase function, satisfies the normalization condition

$$
\frac{1}{\pi} \int_0^\pi d\theta \sin \theta a_1(\theta) = 1
$$

(2)

and describes the angular distribution of the scattered intensity in the case of unpolarized incident light.

If the incident light is polarized linearly in the scattering plane then $Q^{\text{inc}} = I^{\text{inc}}$ and $U^{\text{inc}} = V^{\text{inc}} = 0$. It is easily seen that the corresponding angular distributions of the co-polarized and cross-polarized scattered intensities are given by

$$
\frac{1}{2} [P^{\text{co}} + Q^{\text{co}}] \propto \frac{1}{2} [a_1(\theta) + 2b_1(\theta) + a_2(\theta)],
$$

(3)

$$
\frac{1}{2} [P^{\text{cross}} - Q^{\text{cross}}] \propto \frac{1}{2} [a_1(\theta) - a_2(\theta)].
$$

(4)

Therefore, the LDR can be defined according to

$$
\Delta(t) = \frac{a_1(\theta) - a_2(\theta)}{a_1(\theta) + 2b_1(\theta) + a_2(\theta)}.
$$

(5)

For spherically symmetric scatterers, the LDR vanishes owing to the identity $a_1(\theta) = a_2(\theta)$. Otherwise it may carry useful implicit information on particle morphological properties.

At exactly the backscattering direction, $b_1(180^\circ) = 0$, and so

$$
\Delta(180^\circ) = \frac{a_1(180^\circ) - a_2(180^\circ)}{a_1(180^\circ) + a_2(180^\circ)} = \delta_l,
$$

(6)

where $\delta_l$ is the conventional lidar linear depolarization ratio [46, 112–115].

3. Numerical results and discussion

Using the (superposition) TMM, we calculated the LDR for several BC and BC-containing aerosol morphologies shown in Fig. 2. The fractal aggregate model satisfies the following statistical scaling law [13, 14]:

$$
N = k_0 \frac{R_g}{a},
$$

(7)

where $R_g$, called the radius of gyration, is a measure of the overall aggregate radius; the fractal prefactor $k_0$ and the fractal dimension $D_i$ define the compactness of the aggregate, higher values corresponding to more compact clusters; $a$ is the mean monomer radius; and $N$ is the number of monomers in the fractal. Fig. 2b and c represents a model of the post-collapse stage in the BC cluster evolution [17, 22, 24, 43], while Fig. 2e and f are intended to model BC particles shown in Fig. 1c. The practical relevance of the morphology shown in Fig. 2d is less certain at this point, but its inclusion in the analysis is justified by the relative ease of modeling and by potentially significant remote-sensing and radiation-budget implications [82].

We first calculated the LDRs for (i) low-density fractal aggregates with a fractal dimension $D_l=1.82$ and prefactor $k_0=1.19$, composed of 125 identical BC spherules with a radius $r=20$ nm; (ii) 125 BC spherules with a radius $r=20$ nm densely compacted inside a circumscribing sphere with a radius $R_g=150$ nm; (iii) 125 BC spherules with a radius $r=20$ nm randomly covering a larger non-absorbing sulfate particle with a radius of $R_g=150$ nm; (iv) an equiprobable shape mixture of externally mixed BC spheroids with axis ratios 1/2, 1/1.8, 1/1.6, 1/1.4, 1/1.2, 1.2, 1.4, 1.6, 1.8, 2; and (v) an equiprobable shape mixture of externally mixed BC circular cylinders with diameter-to-length ratios 1/2, 1/1.8, 1/1.6, 1/1.4, 1/1.2, 1.2, 1.4, 1.6, 1.8, 2. The wavelength is fixed at 550 nm, the refractive index of the soot particles is $m=1.75+0.435$, and the equal-volume-sphere radius $R_{50}$ of the BC material is $100$ nm for all five particle morphologies. The latter value represents the effective radius of BC aerosols.

Fig. 2. Simulated soot and soot-containing aerosol habits: (a) fluffy fractal BC aggregates, (b) compact nearly-spherical BC aggregates, (c) compact nearly-spheroidal BC aggregates, (d) host aerosol particles randomly coated with multiple soot spherules, (e) BC spheroids and (f) BC cylinders.
adopted in the NASA Goddard Institute for Space Studies general circulation model (A.A. Lacis, personal communication). The results for habits (i)–(iii) in Fig. 3a and c are ensemble-averaged over 10 random realizations of each morphological type in order to provide a more realistic modeling of natural particle mixtures (cf. Refs. [76, 116]) and ensure smooth depolarization curves.

The numerical results for the five nonspherical habits (i)–(v) are shown in Fig. 3a. A remarkable feature of the depolarization curve for the low-density BC fractals is a sharp maximum at $\Theta \approx 90^\circ$. To explain this feature, it is instructive to recall the form of the normalized Stokes scattering matrix for randomly oriented Rayleigh particles with an anisotropic polarizability, including those with shape-induced anisotropy [117–119]:

$$F_k(\Theta) = \frac{3}{3y+1} \begin{bmatrix}
 y + \cos^2 \Theta & -\sin^2 \Theta & 0 & 0 \\
 -\sin^2 \Theta & 1 + \cos^2 \Theta & 0 & 0 \\
 0 & 0 & 2\cos \Theta & 0 \\
 0 & 0 & 0 & 3\cos \Theta 
\end{bmatrix}.$$  

(8)
Depending on the particle shape, the parameter \( y \) can vary from 1 to 13, the former value corresponding to spherically symmetric particles. Eqs. (5) and (8) yield the following formula for the Rayleigh LDR:

\[
\Delta_R(\Theta) = \frac{y-1}{y+1+2\cos^2 \Theta}
\]  

(9)

It is clear that \( \Delta_R(\Theta) \) vanishes for spherically symmetric scatterers, but a degree of shape anisotropy causes a symmetric maximum centered at \( \Theta \approx 90^\circ \).

This conclusion is illustrated in Fig. 4 by depicting \( \Delta_R(\Theta) \) for several values of \( y \). The Rayleigh LDR behaves like a delta function for \( y=1.002 \), but the height of the peak relative to the background decreases and its width increases substantially by the time \( y \) reaches the value 2.

We can thus conclude that linear depolarization produced by the low-density BC fractals behaves as that of randomly oriented Rayleigh scatterers with a weakly anisotropic polarizability. This result is not surprising. In the single-scattering approximation, wherein each monomer is assumed to be excited only by the incident field (see Chapter 7 of Ref. [120]), the LDR of a BC cluster composed of spherical monomers would be exactly zero. This implies that a non-zero LDR must result from electromagnetic interactions between the monomers [110]. However, the strongly absorbing nature of the BC monomers acts to suppress multi-monomer interactions, which makes it reasonable to assume that for a fluffy aggregate the non-zero LDR is caused primarily by double-scattering interactions between pairs of adjacent monomers, the size parameter of such monomer doublets still being in the Rayleigh domain. It is thus the shape-induced anisotropic polarizability of these Rayleigh doublets that causes the sharp side-scattering peak in the angular profile of the LDR for fluffy BC fractals.

To substantiate this conclusion, we plot in Fig. 3b the LDRs for two-sphere BC clusters with touching components and in random orientation. Both spheres have the same radius (20 or 25 nm) and the same refractive index (1.75+0.435 or 1.95+0.79). All three LDR curves are almost perfectly symmetric with respect to the 90° scattering angle and are described by anisotropy parameters \( y \) between \( \sim 1.01 \) and \( \sim 1.03 \) (cf. Fig. 4). Furthermore, the angular width of the green-curve maximum at the \( \Delta=10\% \) level in Fig. 3b is virtually the same as that of the dashed curve in Fig. 4. The green curve from Fig. 3b is reproduced as the dashed green curve in Fig. 3a. Although the wings of the two green curves in Fig. 3a diverge owing to the residual effects of multi-monomer interactions in the \( N=125 \) soot fractal, the angular profiles of the corresponding side-scattering maxima are hardly distinguishable down to the \( \Delta=5\% \) level.

The multi-monomer interactions can be expected to be much stronger in the case of a densely packed BC aggregate owing to the fact that almost every monomer is now surrounded by many closely positioned monomers rather than being quasi-isolated. As a consequence, the resulting LDR must be affected by the overall shape of the cluster. It is seen indeed from Fig. 3a that packing the same number of BC monomers into a compact nearly spherical cluster reduces the magnitude of the depolarization maximum by almost two orders of magnitude. The maximum LDR value for habit (iii) is even smaller, owing to the scattering dominance of the large spherical host causing near-zero linear depolarization.

The depolarization curves for the spheroid and cylinder shape mixtures in Fig. 3a are rather close to each other and show the highest \( \delta_i \) values. They also reveal pronounced maxima at side-scattering angles which can be explained by the size parameter of the particles still being in the vicinity of the Rayleigh domain.

Fig. 3c parallels Fig. 3a, but the cumulative equal-volume-sphere radius \( R_{eq} \) of the soot material is now 150 nm; the number of monomers in models (i) and (ii) is 422; the radius of the circumscribing sphere in model (ii) is 225 nm; and the radius of the sulfate host in model (iii) is 260 nm. The other fractal parameters of model (i) remain unchanged.

One can see that now the depolarization curves for the spheroid and cylinder mixtures lack the pronounced quasi-Rayleigh maximum at side-scattering angles and reveal significantly increased backscattering values. The likely explanation of this behavior is that the 50% increase in the particle size moves their size parameter farther from the Rayleigh domain. The curve for the fluffy fractal aggregate is hardly different from that in Fig. 3a, which is consistent with the above explanation of its quasi-Rayleigh origin. Finally, models (ii) and (iii) produce very low LDR values, which, again, is an expected consequence of strengthened electromagnetic interactions for the nearly spherical model (ii) and increased scattering dominance of the spherical sulfate host in model (iii).

Fig. 5a compares the LDRs for randomly oriented fractal soot aggregates at a wavelength of 550 nm. The fractal parameters \( D_i \) and \( k_0 \) are fixed at 1.82 and 1.19, respectively, thereby representing the mean values.
Fig. 5. (a) LDR (in %) vs. scattering angle \( \theta \) at a wavelength of 550 nm for different low-density fractal models. (b) As in panel (a), but for groups of \( N \) soot monomers uniformly distributed throughout a spherical volume having a radius of \( R_c = 250 \) nm. (c) LDRs for a sulfate host randomly coated with \( N \) soot spherules. The wavelength is 550 nm in the left-hand and middle panels and 628 nm in the right-hand panel. The refractive index of the host sulfate particle is 1.44. (d) LDR increase with increasing \( r \) and \( R_s \). The wavelength is fixed at 628 nm, and the refractive indices are 1.44 for the sulfate host and 1.75 + 0.435 for the BC spherules.
obtained by Sorensen and Roberts [121] in their diffusion-limited cluster aggregation simulations. The refractive index $m$ is equal to 1.75 $+$ $i$0.435 in the left-hand panel and to 1.95 $+$ $i$0.79 in the middle and right-hand panels. The BC monomer radius $r$ is equal to 20 nm in the left-hand and middle panels and increases to 25 nm in the right-hand panel. The number of monomers in the fluffy fractal clusters varies from 5 to 500.

Upon realizing that it is difficult in practice to achieve measurement accuracies for $\Delta I(\Theta)$ significantly better than a percent, one has to conclude from Fig. 5a that the sensitivity of the LDR to parameters of fluffy BC clusters is rather weak. Perhaps the only detectable manifestation of multi-monomer interactions in the fluffy BC fractals is the widening of the side-scattering peak with increasing $N$. Also noticeable are larger $\delta_i$ values for the soot clusters with $m = 1.95 + i0.79$.

Fig. 5b is intended to further test our hypothesis of the quasi-Rayleigh origin of the side-scattering peak in LDR curves for low-density clusters of small BC monomers. The idea is to keep the radius of the circumscribing sphere $R_p$ fixed while increasing the number of uniformly distributed monomers from 5 to 500, thereby varying the expected packing-density effect on $\Delta I(\Theta)$. The wavelength is fixed at 550 nm.

One can see that if $N=5$ then the average distance between the monomers is so large that the electromagnetic interaction between the monomers is weak and cannot cause LDRs approaching 100%. $N=10$ maximizes the double-scattering contribution and causes LDR values reaching 90%. Further increase in the number of BC monomers increases the multiple-scattering contribution and eventually results in a densely packed cluster having a nearly spherical shape and LDRs smaller than 1%.

Fig. 3d compares the LDRs for a single randomly oriented prolate BC spheroid with an aspect ratio of 2 and $R_{ve} = 150$ nm with those for randomly oriented near-spheroidal clusters built of a number of 20-nm radius BC monomers (see Fig. 2c). In each case the soot monomers are packed into a prolate spheroidal volume with the same aspect ratio 2 and the same equal-volume-sphere radius $R_{ve} = 150$ nm using the cluster-generation procedure described in Ref. [122]. The BC refractive index is taken to be 1.75 $+$ $i$0.435. It is seen that with growing $N$, the cluster LDR approaches that of the homogeneous BC spheroid. In particular, the height of the side-scattering maximum decreases while its width increases, and the backscattering LDR significantly increases. These traits are consistent with the growing strength of electromagnetic interactions between multiple densely packed monomers and the resulting dependence of the LDR on the overall cluster nonsphericity.

Fig. 5c and d depicts the LDRs for the aerosol model shown in Fig. 2d, i.e., a larger sulfate particle coated with $N$ soot spheres. One can see that it takes BC spherule radii in excess of 30 nm to generate LDR values significantly exceeding 1%.

4. Concluding remarks

The results of our extensive TMM computations show that the measurement of the LDR over the entire range of scattering angles can be a powerful means of morphological characterization of BC and BC-containing particles. In particular, the detection and profiling of the quasi-Rayleigh maximum at side-scattering angles can help evaluate the compactness state of BC clusters and trace the evolution of low-density fluffy fractals into densely packed aggregates. Of course, more research is needed to analyze factors that can potentially influence the uniqueness and accuracy of optical particle characterization based on measurements of the LDR angular profile. In particular, it is unclear whether a priori information on particle chemistry is required and whether a degree of monomer polydispersity can affect data interpretation.

Backscattering lidar measurements can also carry useful information. For example, on the basis of high spectral resolution lidar measurements, Burton et al. [123] have concluded that $\delta_i$ at 532 nm for fresh smoke is typically very low (less than 2–5%) and also typically lower than for more aged smoke. This result is consistent with the magnitude and variability of previous lidar observations of smoke by Sassen [112]. The CALIPSO data also showed volume (including molecular and particulate) lidar LDRs $\sim 2\%$ for South-African smoke [124]. These lidar data are generally consistent with low $\delta_i$ values produced by fluffy fractals in Figs. 3a, c, and 5a as well as by densely packed near-spheroidal aggregates in Fig. 3d. To reproduce larger lidar LDRs typical of the aged smoke (3–8%), one would need to use alternative particle models such as shape mixtures of BC spheroids and cylinders and/or large host aerosols covered with relatively large BC spherules. Needless to say, more theoretical and experimental work is still required to draw more definitive conclusions.

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