1. Objective:
This study works to formulate convenient solutions to the problem of a thermoelectric couple operating under a time varying condition. Transient operation of a thermoelectric device will become increasingly common as thermoelectric technology permits new applications such as automotive and aerospace energy harvesting. In an effort to generalize the thermoelectric solution, Green’s functions are employed. This allows arbitrary time varying boundary conditions to be applied to the system without reformulation. The solution demonstrates that in thermoelectric applications of a transient nature, Thermal Diffusivity Factor, Inductance Factor, and leg length ratio must be taken into account.

3. The Transient Couple:
- New applications of thermoelectrics demand transient operation.
- A number of numerical efforts have successfully characterized transient couples [2,3] but an analytic approach provides powerful design guidelines.
- Unlike the steady state couple, the transient couple depends additionally on material density and specific heat, which must be captured in Thermal Diffusivity Factor and Inductance Factor.

4. Thermal Diffusivity Factor:
- An On/Off cycle is studied, using a unit dimensionless heat flux on the hot shoe and fixed temperature on the cold shoe.
- Solution introduces a new dimensionless parameter, Thermal Diffusivity Factor \( \Gamma \).

5. The A/C Couple:
- The behavior of a couple under a sinusoidal heat flux is interesting for applications such as energy harvesting on a pulse detonation engine
- Solution introduces the Inductance Factor \( \beta \) a dimensionless parameter with strong effect on amplitude and phase angle between thermal and electrical response.
- \( \beta \) and frequency are not found to alter periodic steady average values.

6. Conclusion:
The analytic solution of a transient couple leads to the introduction of a Thermal Diffusivity Factor \( \Gamma \) and an Inductance Factor \( \beta \). The behavior of couples as a function of these parameters has been investigated and for the case of \( \Gamma \) an optimal design point exists. This optimal \( \Gamma \) leads to the design guideline for the selection of optimal leg length ratio.

Optimal Length
\[
\frac{L_A}{L_B} = \sqrt{\frac{2 \alpha_B}{\alpha_A}} + 1
\]
\[
\alpha = 1 + \frac{\alpha_B}{\alpha_A}
\]

The authors would like to thank: Ben Kowalski, Tom Sabo, and Ray Babuder NASA Cooperative Agreement: NNX08AB43A

References:

Nomenclature:
- \( \alpha \) - Space Coordinate
- \( j \) - Current
- \( v \)- Voltage
- \( R \) - Resistance
- \( k \) - Thermal Conductivity
- \( a \) - Thermal Diffusivity
- \( \rho \) - Density
- \( c_p \) - Specific Heat
- \( \Gamma \) - Thermal Diffusivity
- \( L \) - Leg Length
- \( \Gamma_A \) - Svebeck Coefficient
- \( \Gamma \) - Temperature
- \( L \) - Leg Area
- \( \beta \) - Electrical Conductivity

Dependant Variables
- \( \beta \) - Inductance
- \( \alpha \) - Diffusivity
- \( \rho \) - Density
- \( c_p \) - Specific Heat
- \( L \) - Leg Length
- \( \Gamma \) - Svebeck Coefficient

Materials Properties
- Dependant Variables
- \( \beta \) - Inductance
- \( \alpha \) - Diffusivity
- \( \rho \) - Density
- \( c_p \) - Specific Heat
- \( L \) - Leg Length
- \( \Gamma \) - Svebeck Coefficient

The Green's function is an integral kernel which allows for simple expression of the solution for the problem of interest [1].
- Method applies to the linear operator \( L \) and the adjoint operator \( L^* \).

Fig. 1. a. the desired inhomogeneous problem to be solved. b. the corresponding problem to be solved to obtain the Green's function. c. The desired solution in terms of the inhomogeneous function.

Fig. 4. Cycle conversion efficiency as a function of Thermal Diffusivity Factor \( \Gamma \). For geometrically and electrically similar couples, cycle conversion efficiency is optimized for \( \Gamma = 1 \). For selected materials the leg length ratio can be optimally designed as follows:

Optimal Length
\[
\frac{L_A}{L_B} = \sqrt{\frac{2 \alpha_B}{\alpha_A}} + 1
\]
\[
\alpha = 1 + \frac{\alpha_B}{\alpha_A}
\]

Fig. 5. Hot shoe temperature due to sinusoidal heat flux input.

Fig. 6. a. Power amplitude dependent on Inductance Factor \( \beta \) for 0.1 Hz b. Set of power curves for a range of Inductance Factors \( \beta \). c. Power amplitude dependent on frequency for \( \beta = 0.001 \). d. Set of power curves for a range of frequencies.