1. Objective:
This study works to formulate convenient solutions to the problem of a thermoelectric couple operating under a time varying condition. Transient operation of a thermoelectric device will become increasingly common as thermoelectric technology permits new applications such as automotive and aerospace energy harvesting. In an effort to generalize the thermoelectric solution, Green’s functions are employed. This allows arbitrary time varying boundary conditions to be applied to the system without reformulation. The solution demonstrates that in thermoelectric applications of a transient nature Thermal Diffusivity Factor, Inductance Factor, and leg length ratio must be taken into account.

2. What are Green’s Functions?
- The Green’s function is an integral kernal which allows for simple expression of the solution for the problem of interest [1].
- Method applies to the linear operator \( L \) and the adjoint operator \( L^* \)

3. The Transient Couple:
- New applications of thermoelectrics demand transient operation.
- A number of numerical efforts have successfully characterized transient couples [2,3] but an analytic approach provides powerful design guidelines.
- Unlike the steady state couple, the transient couple depends additionally on material density and specific heat, which must be captured in Thermal Diffusivity Factor and Inductance Factor.

4. Thermal Diffusivity Factor:
- An On/Off cycle is studied, using a unit dimensionless heat flux on the hot shoe and fixed temperature on the cold shoe.
- Solution introduces a new dimensionless parameter, Thermal Diffusivity Factor \( \Gamma_\text{A,B} \).

5. The A/C Couple:
- The behavior of a couple under a sinusoidal heat flux is interesting for applications such as energy harvesting on a pulse detonation engine.
- Solution introduces the Inductance Factor \( \beta \) a dimensionless parameter with strong effect on amplitude and phase angle between thermal and electrical response.
- \( \beta \) and frequency are not found to alter periodic steady average values.

6. Conclusion:
The analytic solution of a transient couple leads to the introduction of a Thermal Diffusivity Factor \( \Gamma \) and an Inductance Factor \( \beta \). The behavior of couples as a function of these parameters has been investigated and for the case of \( \Gamma \) an optimal design point exists. This optimal \( \Gamma \) leads to the design guideline for the selection of optimal leg length ratio.

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References:

Nomenclature:
- \( \tau \) – Space Coordinate
- \( t \) – Time
- \( I \) – Current
- \( V \) – Voltage
- \( R \) – Resistance
- \( \beta \) – Thermal Conductivity
- \( \kappa \) – Thermal Diffusivity
- \( \rho \) – Density
- \( k \) – Electrical Conductivity
- \( c_p \) – Specific Heat
- \( n \) – Svedberg Coefficient
- \( L \) – Leg Length
- \( A \) – Leg Area
- \( W \) – Width
- \( E \) – Electrical Energy
- \( S \) – Svedberg Parameters
- \( M \) – Mass
- \( \text{Material Properties} \)

Fig. 1. a. the desired inhomogeneous problem to be solved. b. the corresponding problem to be solved to obtain the Green’s function. c. The desired solution in terms of the inhomogeneous function.

Fig. 2. Sketch of a transient thermoelectric couple, See “Nomenclature” section for clarification. Subscripts: (A & B) are n- and p-type legs, (C & H) are cold and hot shoes, \( \{\} \) is leg to leg average.

Fig. 3. Study of three geometrically and electrically similar couples under On/Off heat flux cycles; parameter of study is the Thermal Diffusivity Factor \( \Gamma \).

Fig. 4. Cycle conversion efficiency as a function of Thermal Diffusivity Factor \( \Gamma \). For geometrically and electrically similar couples

Fig. 5. Hot shoe temperature due to sinusoidal heat flux input.

Fig. 6. a. Power amplitude dependent on Inductance Factor \( \beta \) for 0.1 Hz. b. Set of power curves for a range of Inductance Factors \( \beta \). c. Power amplitude dependent on frequency for \( \beta = 0.001 \). d. Set of power curves for a range of frequencies.