Recent Developments and Challenges Implementing New and Improved Stress Intensity Factor (*K*) Solutions in NASGRO® for Damage Tolerance Analyses

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Abstract: Fatigue crack growth analysis software has been available to damage tolerance analysts for many years in either commercial products or via proprietary in-house codes. The NASGRO software has been publicly available since the mid-80s (known as NASA/FLAGRO up to 1999) and since 2000 has been sustained and further developed by a collaborative effort between Southwest Research Institute® (SwRI®), the NASA Johnson Space Center (JSC), and the members of the NASGRO Industrial Consortium. Since the stress intensity factor (*K*) is the foundation of fracture mechanics and damage tolerance analysis of aircraft structures, a significant focus of development efforts in the past fifteen years has been geared towards enhancing legacy *K* solutions and developing new and efficient numerical *K* solutions that can handle the complicated stress gradients computed by today's analysts using detailed finite element models of fatigue critical locations. This paper provides an overview of *K* solutions that have been recently implemented or improved for the analysis of geometries such as two unequal through cracks at a hole and two unequal corner cracks at a hole, as well as state-of-the-art weight function models capable of computing *K* in the presence of univariant and/or bivariant stress gradients and complicated residual stress distributions. Some historical background is provided to review how common *K* solutions have evolved over the years, including selective examples from the literature and from new research. Challenges and progress in rectifying discrepancies between older legacy solutions and newer models are reviewed as well as approaches and challenges for verification and validation of *K* solutions. Finally, a summary of current challenges and future research and development needs is presented. A key theme throughout the presentation of this paper will be how members of the aerospace industry have collaborated with software developers to develop a practical analysis tool that is used world-wide to support new design as well as the ongoing sustainment and airworthiness of commercial and military aircraft.

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  - Federal Aviation Administration (FAA)

Introduction

- Stress Intensity Factor ("*K*") is the foundation of fracture mechanics analysis for aircraft structures
  - Describes first-order effect of stress magnitude/distribution at a crack
  - Accounts for the geometry of both structure/component and crack
- Calculation of *K* is often the most important step in DTAs
- This presentation provides an overview of the current state of the art in *K* solution methods for practical aerospace DTA applications with an emphasis on new developments in the NASGRO software
- *Disclaimer: Not an exhaustive review*

Outline

- Background and Motivation
- Informal Historical Perspective
- Challenges and Resolution
- Verification and Validation
- Current and Future Challenges

Background & Motivation

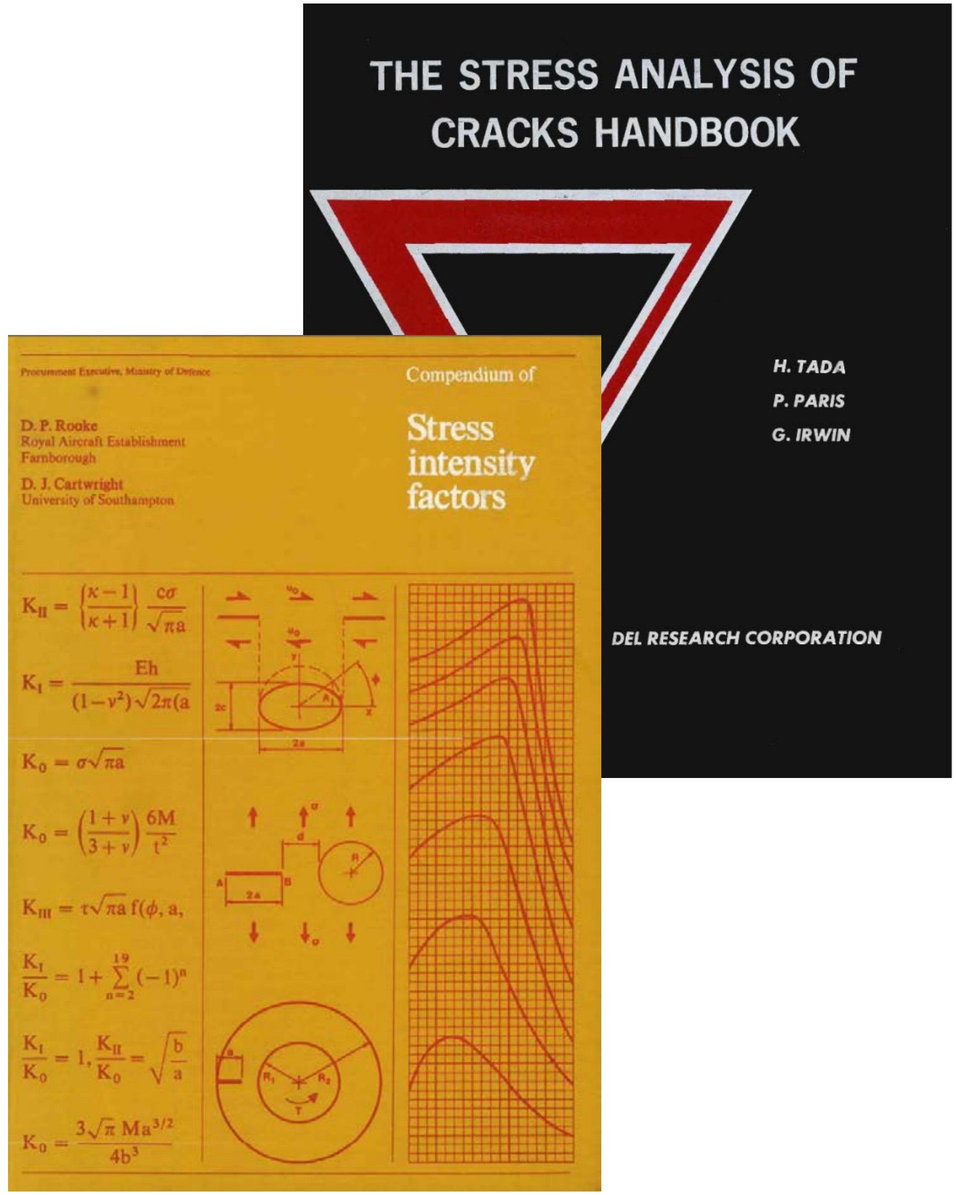
- NASA/FLAGRO was first developed by NASA-JSC in the mid-80s and contained about 30 *K*-solutions
- By the late-90s, NASGRO contained about 40 *K*-solutions
- Since 2000, the collaboration of SwRI®, NASA-JSC, and the NASGRO Consortium has more than doubled the number of *K*-solutions available in NASGRO (84)
- New and improved *K*-solutions are always ranked as high priority items by Consortium members for future development tasks

Informal Historical Perspective

- Handbooks
- Closed-Form Equations from FE Results
- Recent FE Methods
- Compounding Methods
- Weight Function Methods

Early Handbooks

- Tada, Paris, Irwin (1973)
  - Later editions 1985, 2000
- Rooke and Cartwright (1976)
- Murakami (1987)
- Valuable collections of many analytical and numerical solutions (many in graphical form)
- Some configurations of limited practical value
- Many solutions not readily usable for engineering purposes



Closed-Form Equations from Finite Element Results

I. S. RAJU and J. C. NEWMAN, Jr.

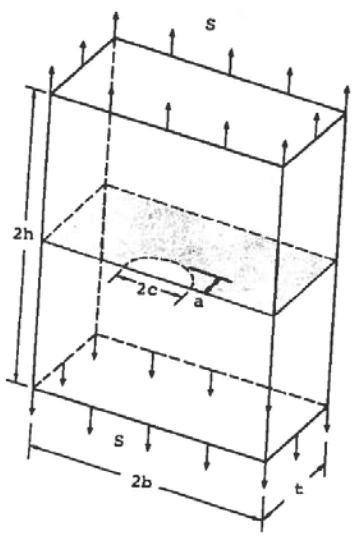
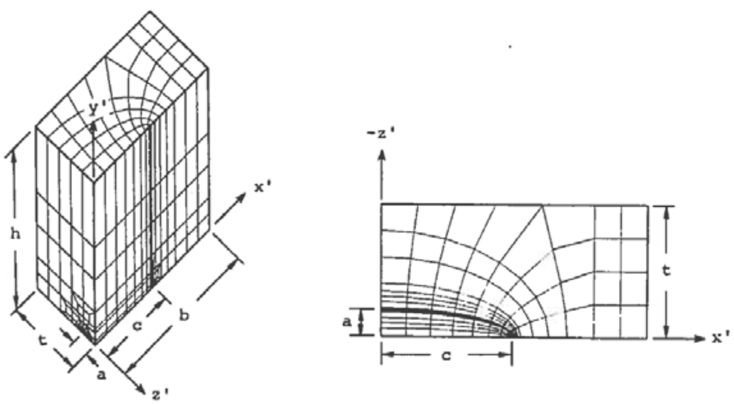


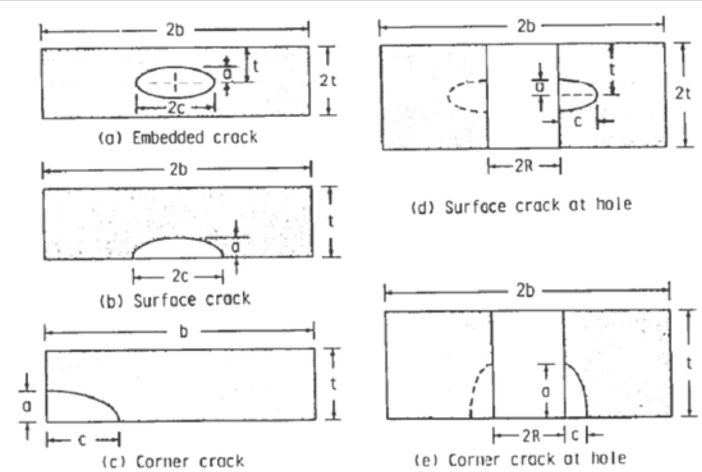
Fig. 1. Surface crack in a plate subjected to tension.

- Raju and Newman (1979ff)
- Finite element models with 6900 DOF
- "Correction factors" for various geometry considerations
- Incorporated in very early versions of NASA/FLAGRO (NASGRO)



(a) Specimen model. (b) Element pattern on y'= 0 plane.

Fig. 4. Finite-element idealization for a semi-elliptical surface crack.



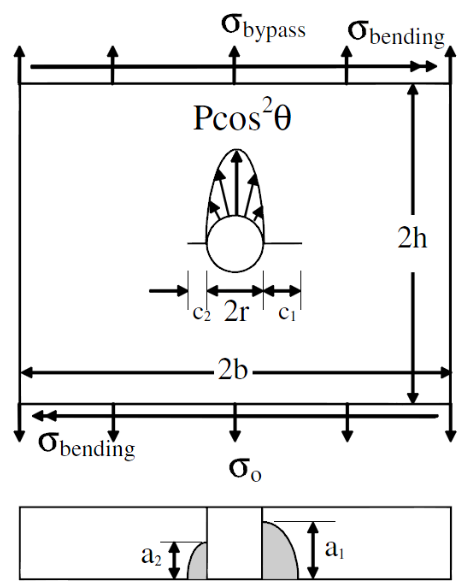
(a) Embedded crack
(b) Surface crack
(c) Corner crack
(d) Surface crack at hole
(e) Corner crack at hole

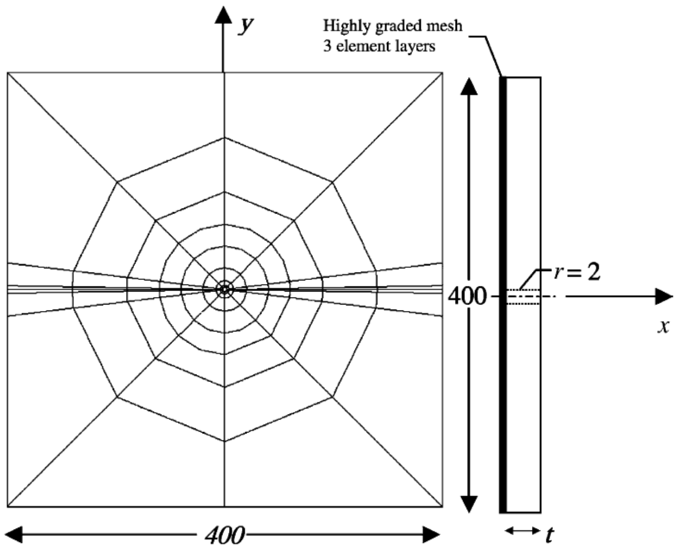
FIG. 2—*Embedded-, surface-, and corner-crack configurations (all cracks have elliptical fronts).*

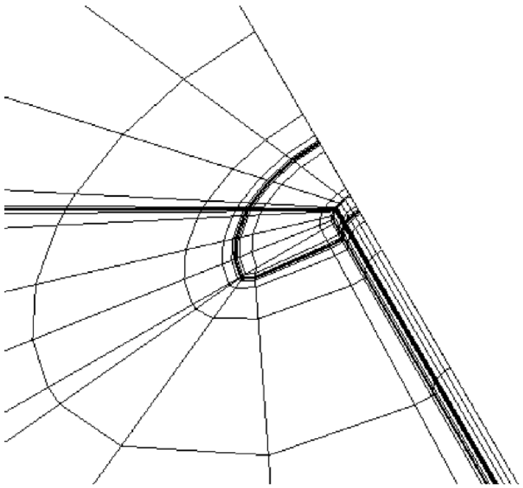
Recent Finite Element Methods

- **Fawaz and Andersson**
- ***p*-version FE method**
- **Very large solution matrices**
  - Unequal corner cracks at hole:
    - 7150 combinations of *R/t*, *a/t*, *a/c*
    - Over 5M *K* solutions

S.A. Fawaz, B. Andersson / Engineering Fracture Mechanics 71 (2004) 1235–1254







Use of New FE Methods

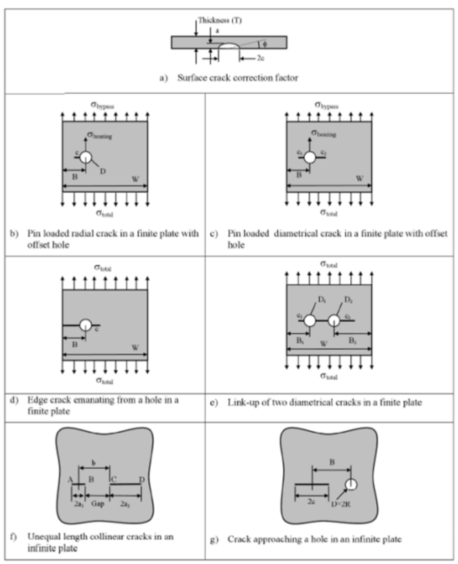
- **Automation and advanced computer power makes it possible to generate millions and millions of solutions**
- **How best to employ these new results?**
  - Calculate what you need, when you need it?
  - Evaluate/update/extend legacy solutions?
  - Develop “simple” equations?
  - Use directly as large interpolation tables?
- **Challenges:**
  - Computation time still too long for real-time use in design
  - Very large tables → large computer memory requirements
  - How to address other finite geometry effects? (e.g., offsets, plate width)
  - How to verify that all solutions are correct?

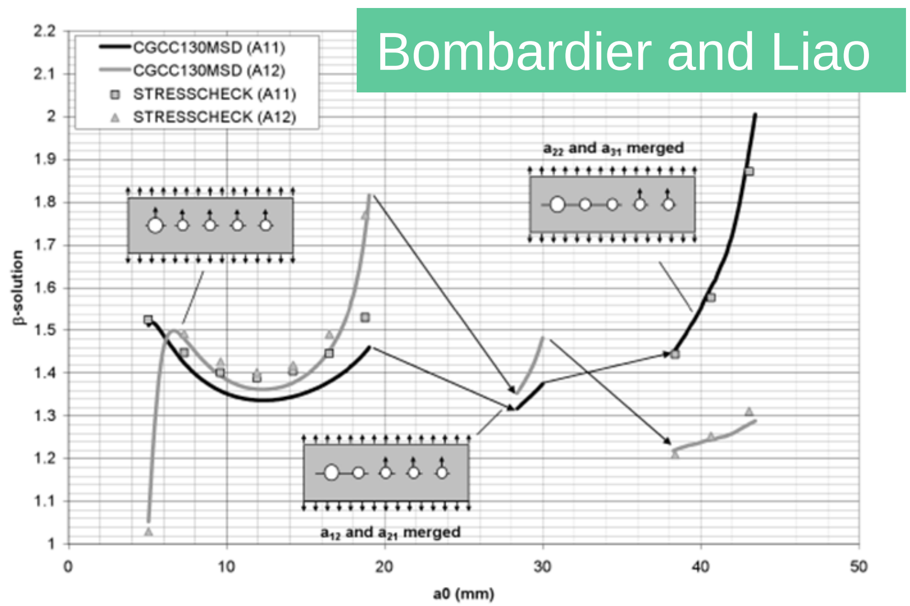
Compounding and Superposition Methods

- Compounding method originally published by Cartwright and Rooke (1974)

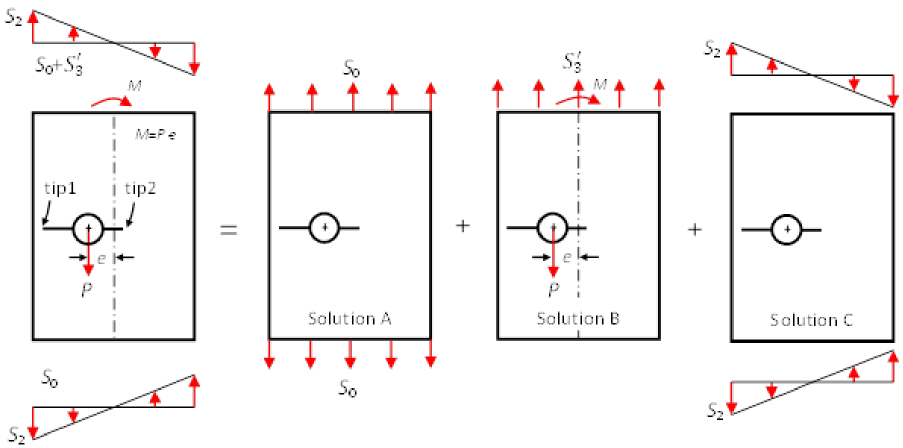
$$K = K_0 + [\sum_{n=1}^{N} (Kn - K_0)] + Ke$$

- Linear combinations of different loading and boundary effects
- Method is general but approximate
- Can be used to build up very complex solutions

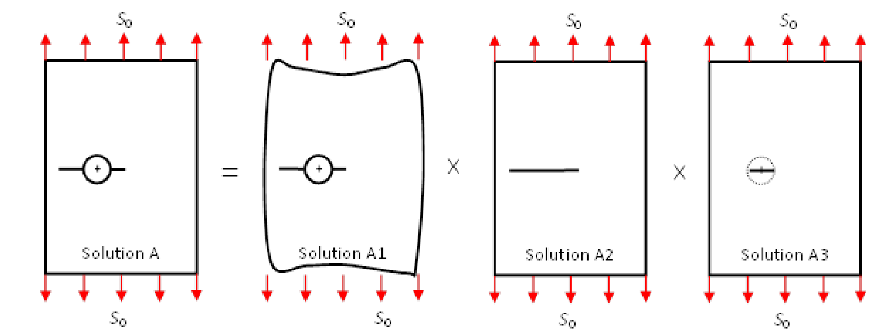


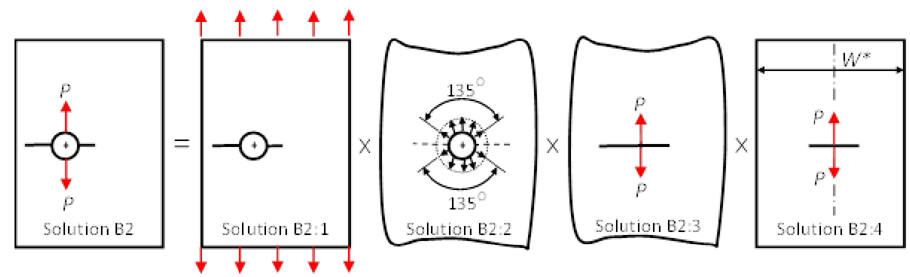


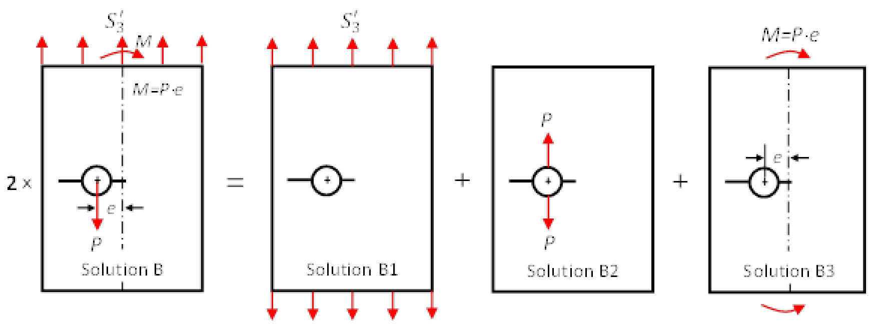
Compounding and Superposition Methods:
New TC23 Solution
Unequal Through Cracks at Hole

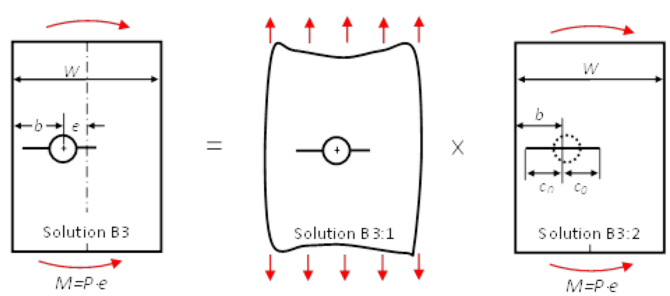


Y. Bombardier and M. Liao, SDM Conf., 2010.

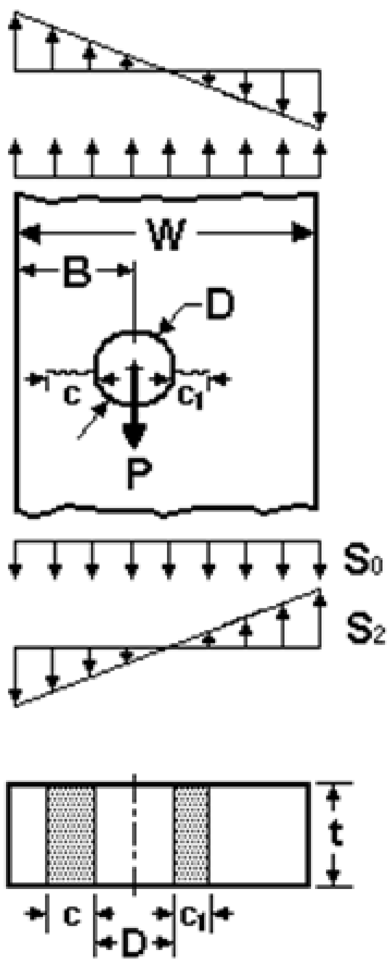


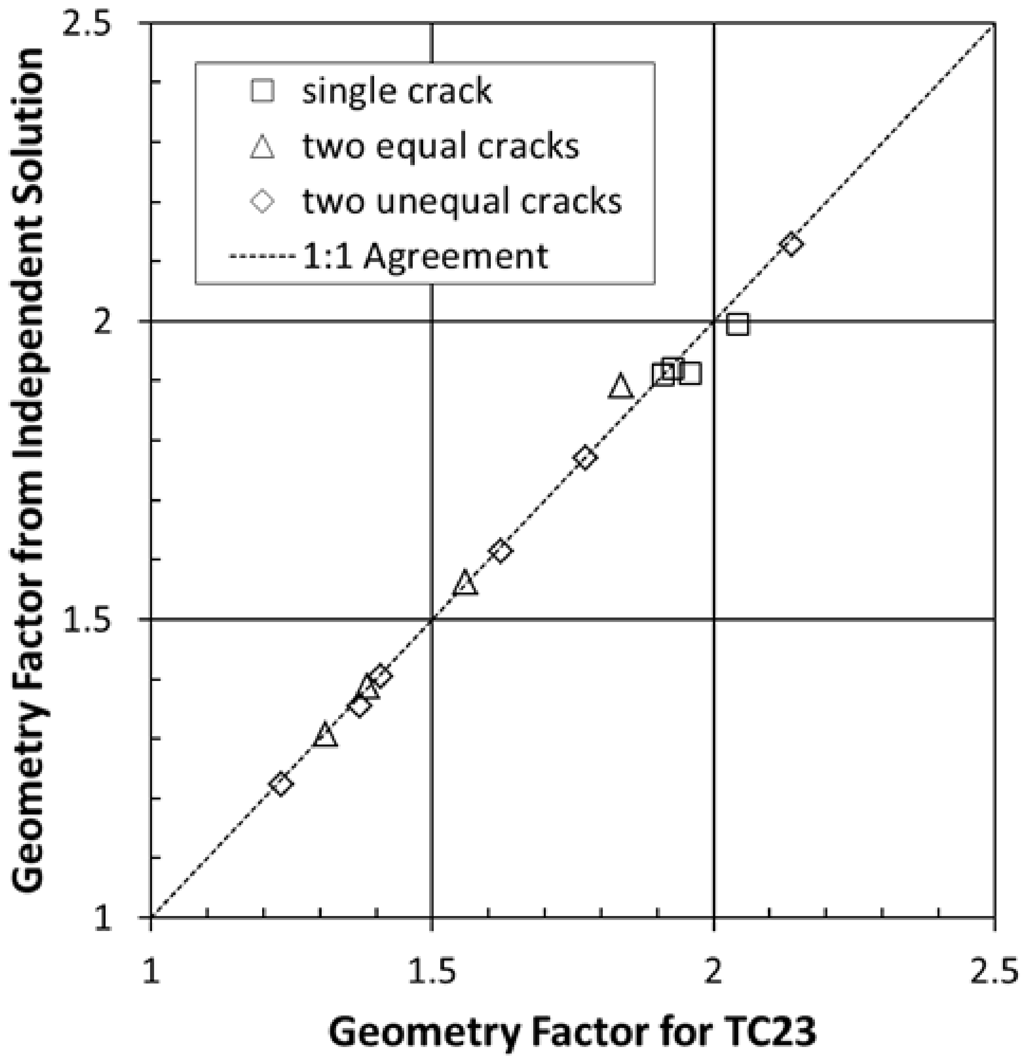




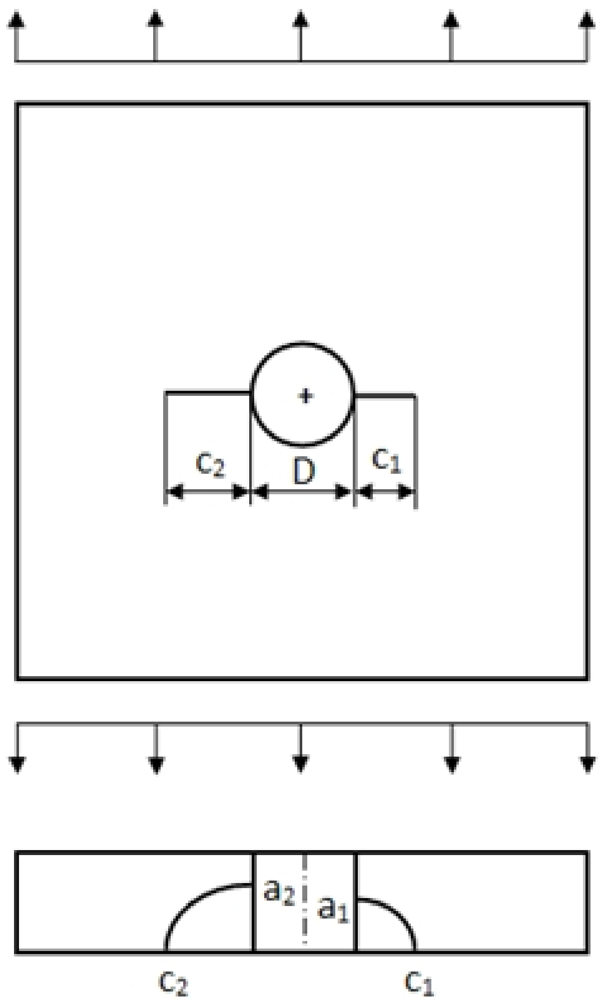


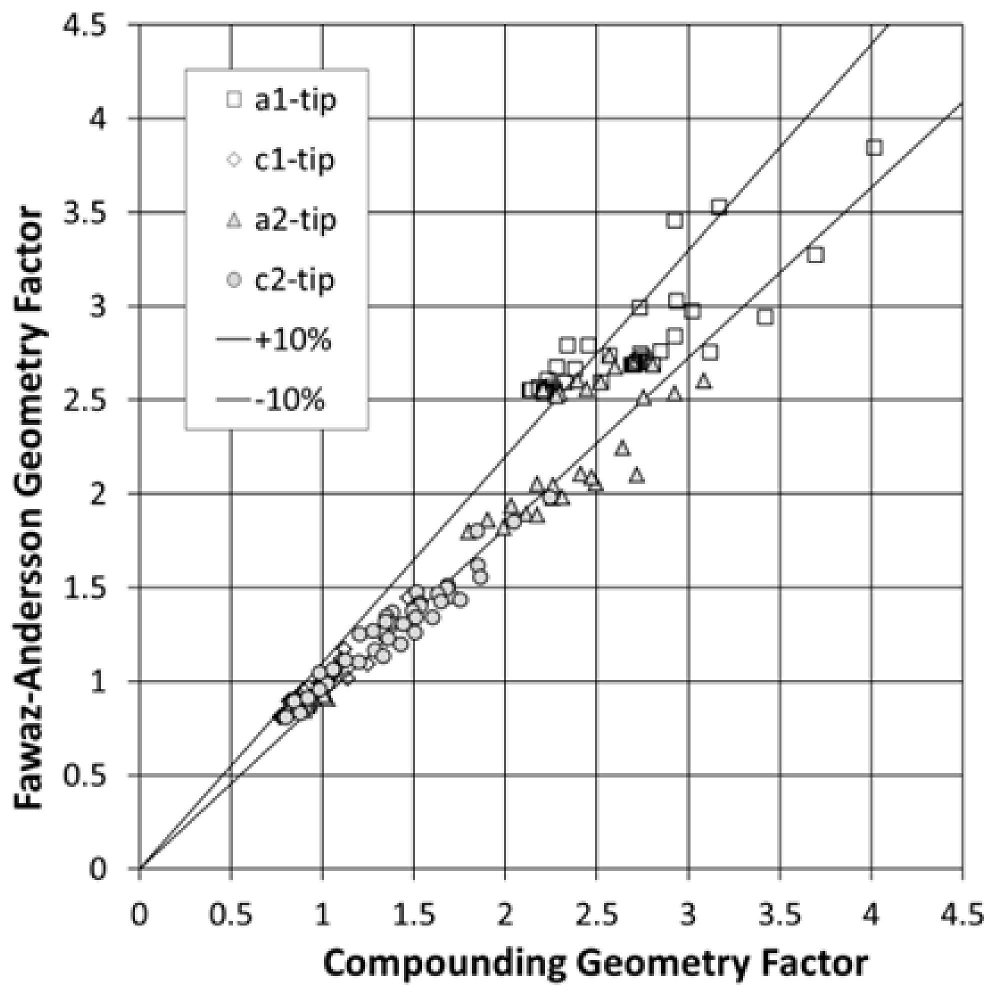
Compounding and Superposition Methods:
New TC23 Solution
Unequal Through Cracks at Hole





Compounding and Superposition Methods:
Unequal Corner Cracks at Hole





Weight Function Methods

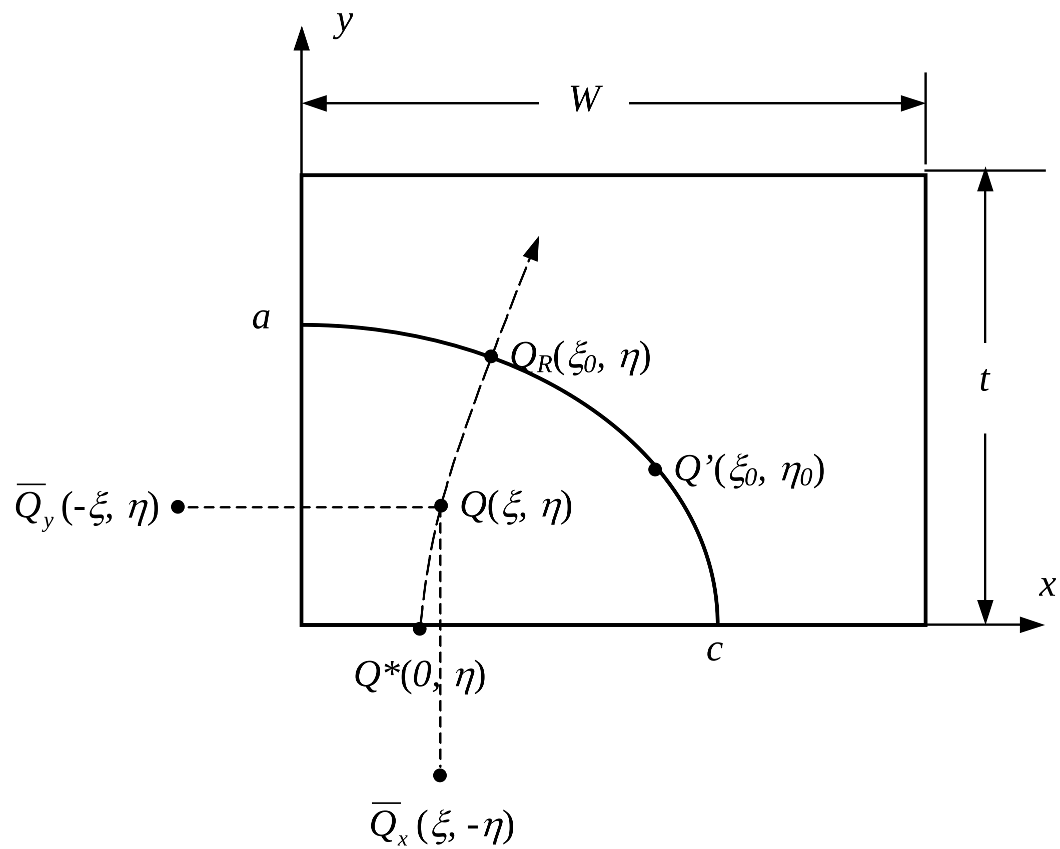
- Calculate *K* for an arbitrary stress gradient on the crack plane in the corresponding <u>uncracked</u> body

$$K = \int W(x)\sigma(x)dx$$

- Most WF formulations are for one-dimensional cracks in univariant stress fields
- Glinka has published widely-used WF formulations for part-through cracks in univariant stress fields

Weight Function Methods:
New Bivariant WF Formulation

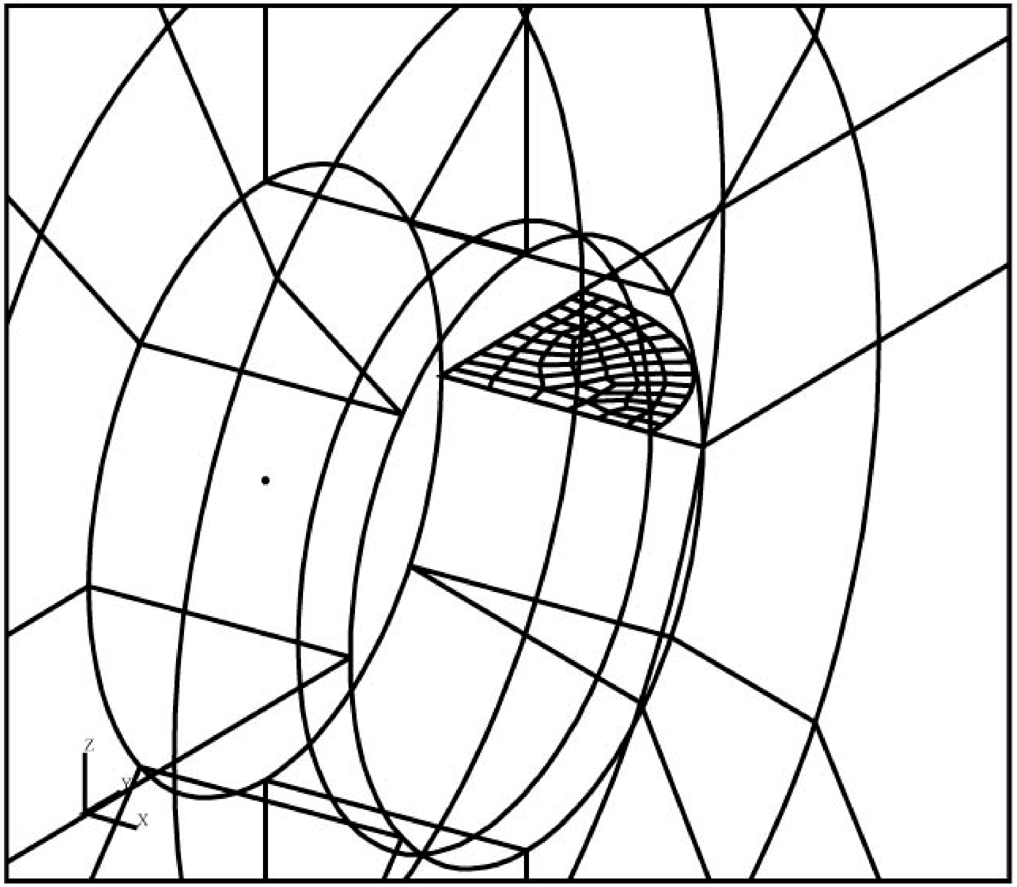
- New point WF formulation including free boundary and finite geometry effects
- Stresses can vary arbitrarily in all directions on the crack plane
- Improved accuracy and efficiency over previous methods
- Lee et al, *FFEMS* 31 (11) 2008



$$K^{a,c} = \int_0^a \int_0^{c\sqrt{1-\frac{y^2}{a^2}}} \sigma(x,y) \frac{\sqrt{R^2 - r^2}}{\pi \ell^2_{QQ^{a,c}} \sqrt{\pi R}} \left(1 + \frac{\ell^2_{QQ^{a,c}}}{\ell^2_{\bar{Q}_x Q^{a,c}}} + \frac{\ell^2_{QQ^{a,c}}}{\ell^2_{\bar{Q}_y Q^{a,c}}}\right) \left[1 + \Pi_1^{a,c}\sqrt{1-\frac{r}{R}} + \Pi_2^{a,c}\left(1-\frac{y}{y'}\right) + \Pi_3^{a,c}\left(1-\frac{x}{x'}\right)\right] dxdy$$

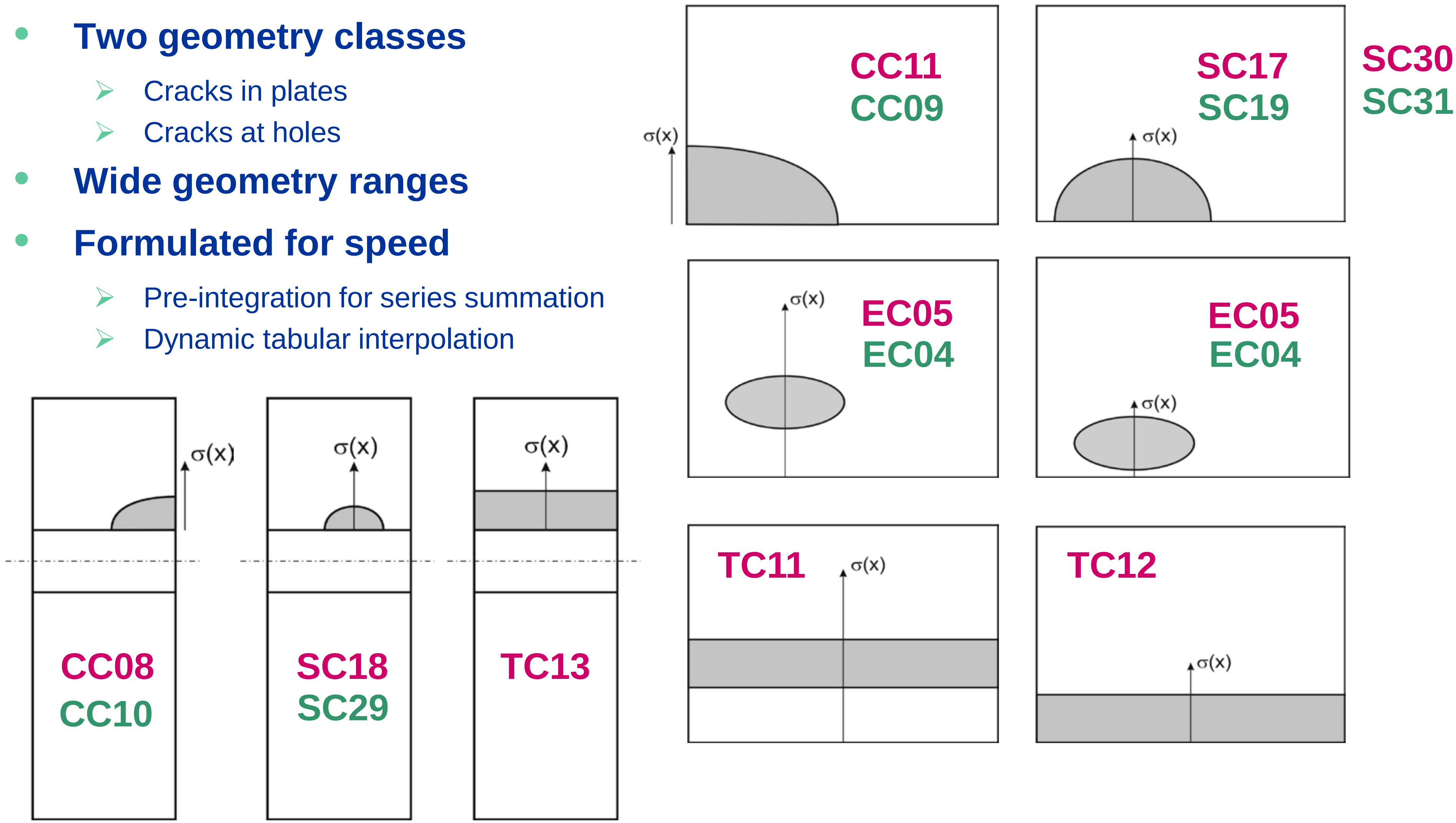
Weight Function Methods:
Numerical Generation of Reference Solutions

- These univariant and bivariant WF SIF formulations require large number of accurate reference solutions over wide geometry ranges
- Uniform tension, linear gradient loadings on crack face
- Hybrid FADD3D BE-FE software used to generate these solutions
  - Highly accurate
  - Limited meshing requirements



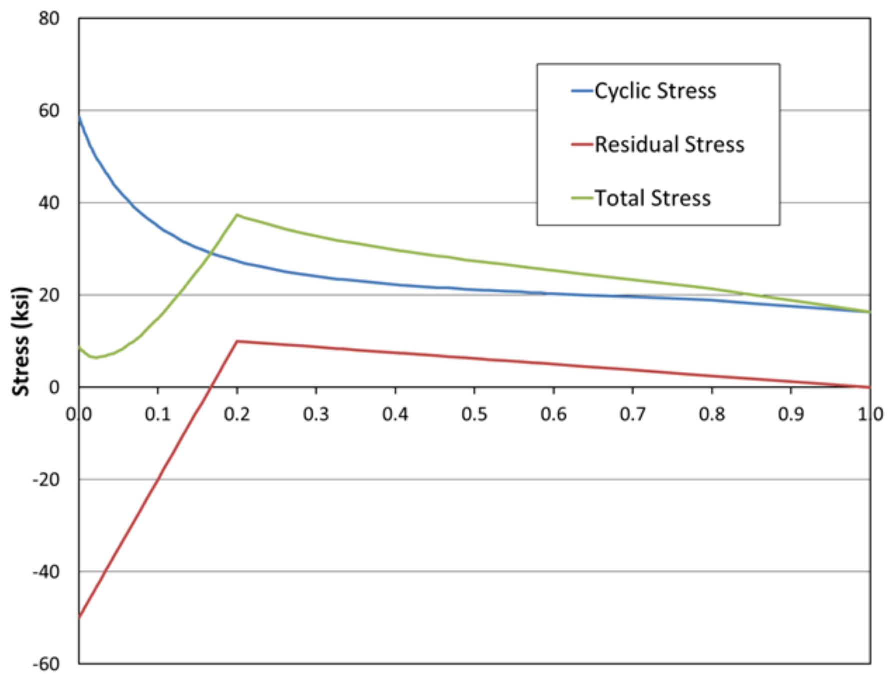
Weight Function Methods: New Family of Univariant and Bivariant WF Solutions

- **Two geometry classes**
  - Cracks in plates
  - Cracks at holes
- **Wide geometry ranges**
- **Formulated for speed**
  - Pre-integration for series summation
  - Dynamic tabular interpolation

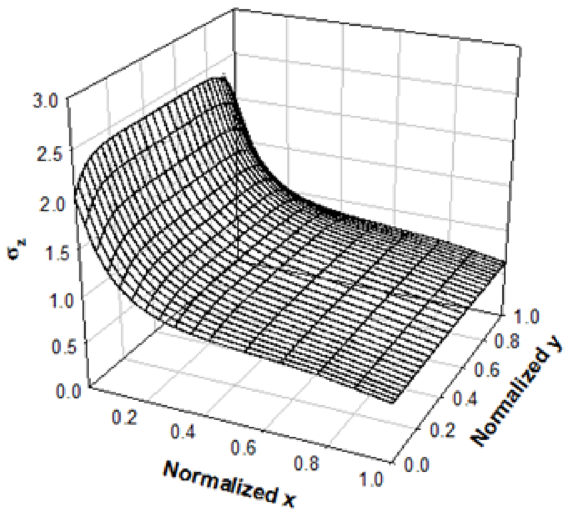


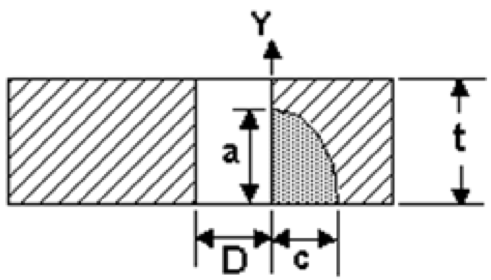
Weight Function Methods: New Family of Univariant and Bivariant WF Solutions

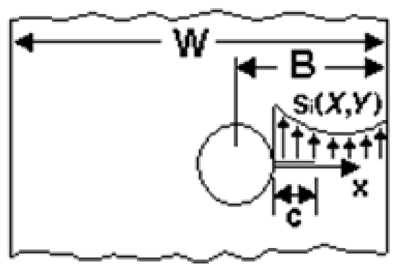
- **Capable of computing *K* for:**
  - Complicated nonlinear gradients
  - Residual stress gradients
  - Superposition of gradients having different length scales
  - Deep cracks:
    - Large a/t
    - Large a/c (tunneling)
  - Recent improvements for shallow surface cracks (small a/c)

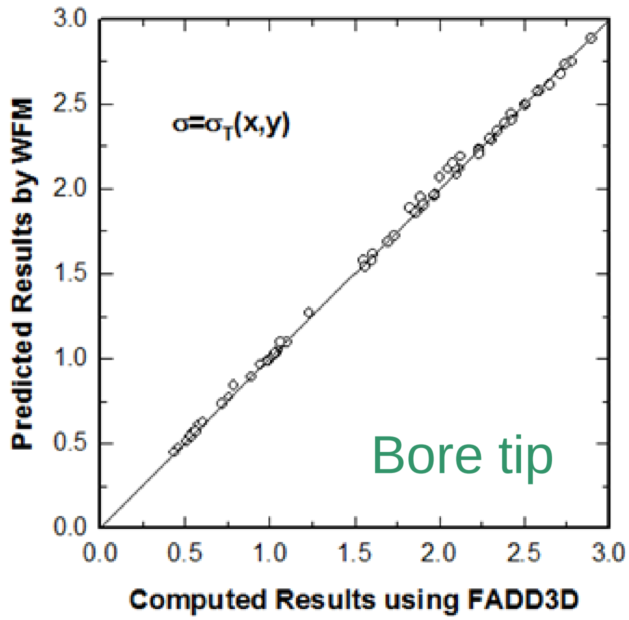


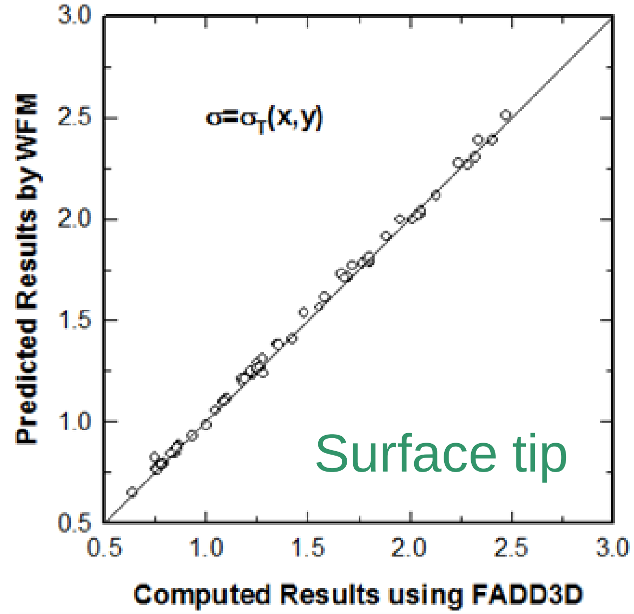
Weight Function Methods:
Verification of CC10
Bivariant Corner Crack at Hole





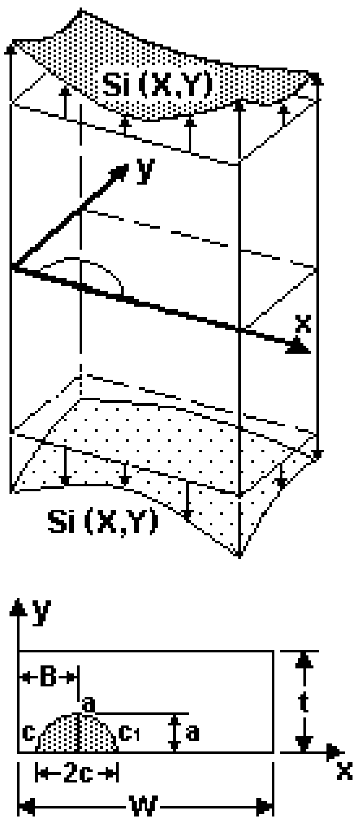


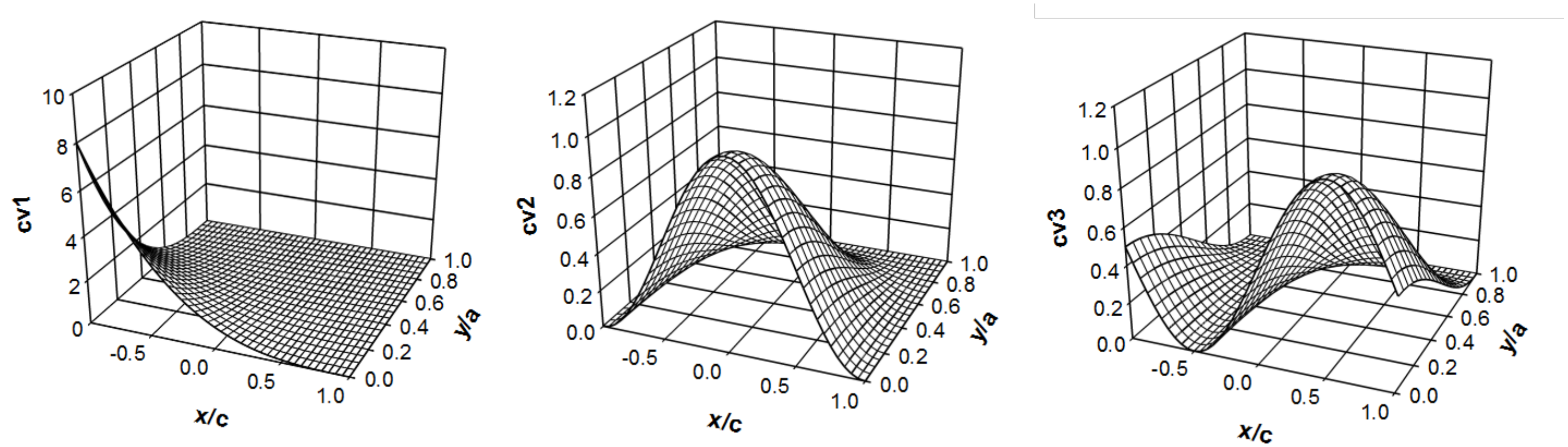


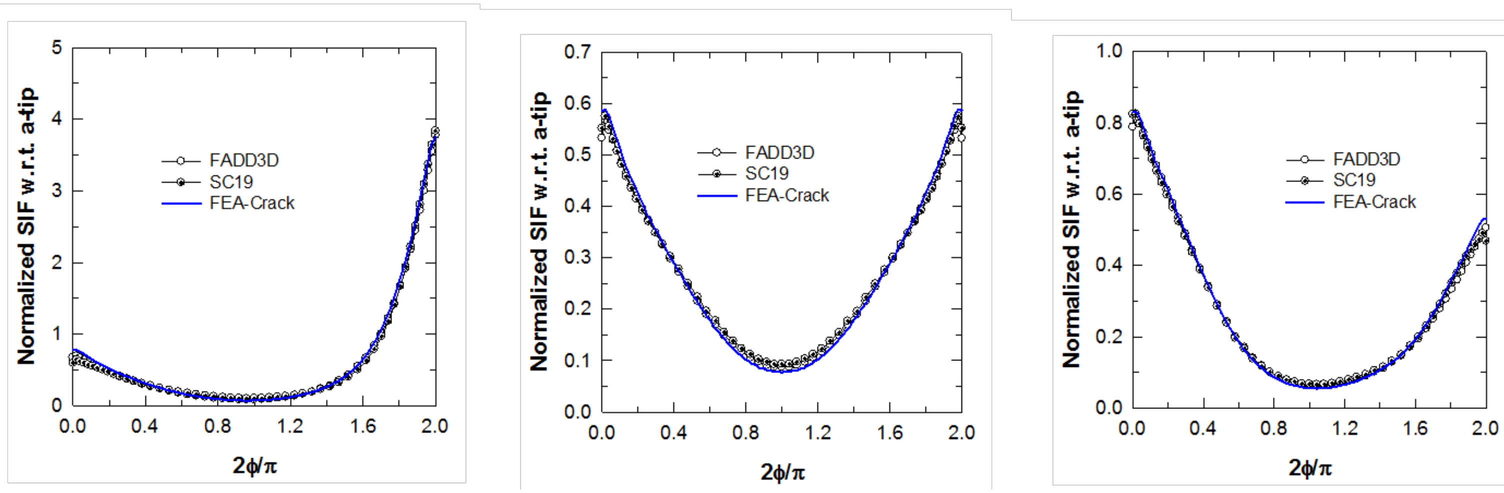


96 different geometry combinations

Weight Function Methods:
Verification of SC19
Bivariant Surface Crack

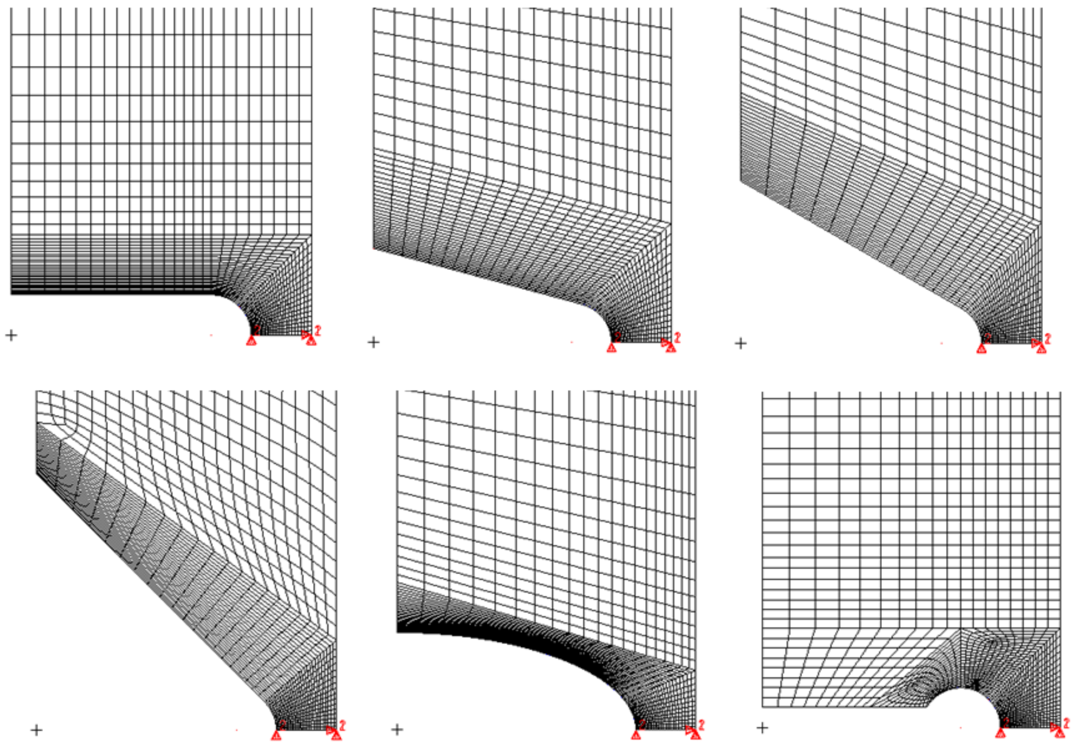


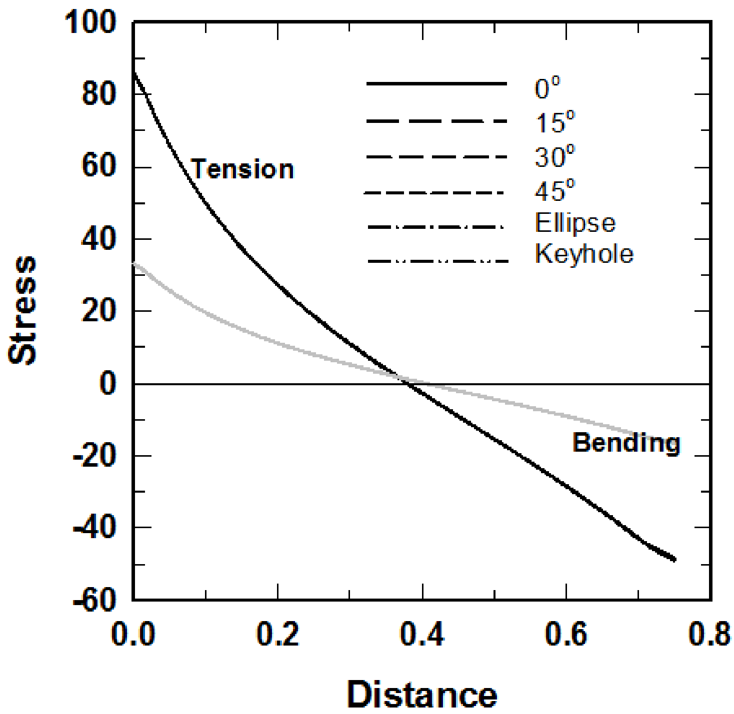




Weight Function Methods:
Derivation of New Solutions for Cracks at Arbitrary Edge Notches

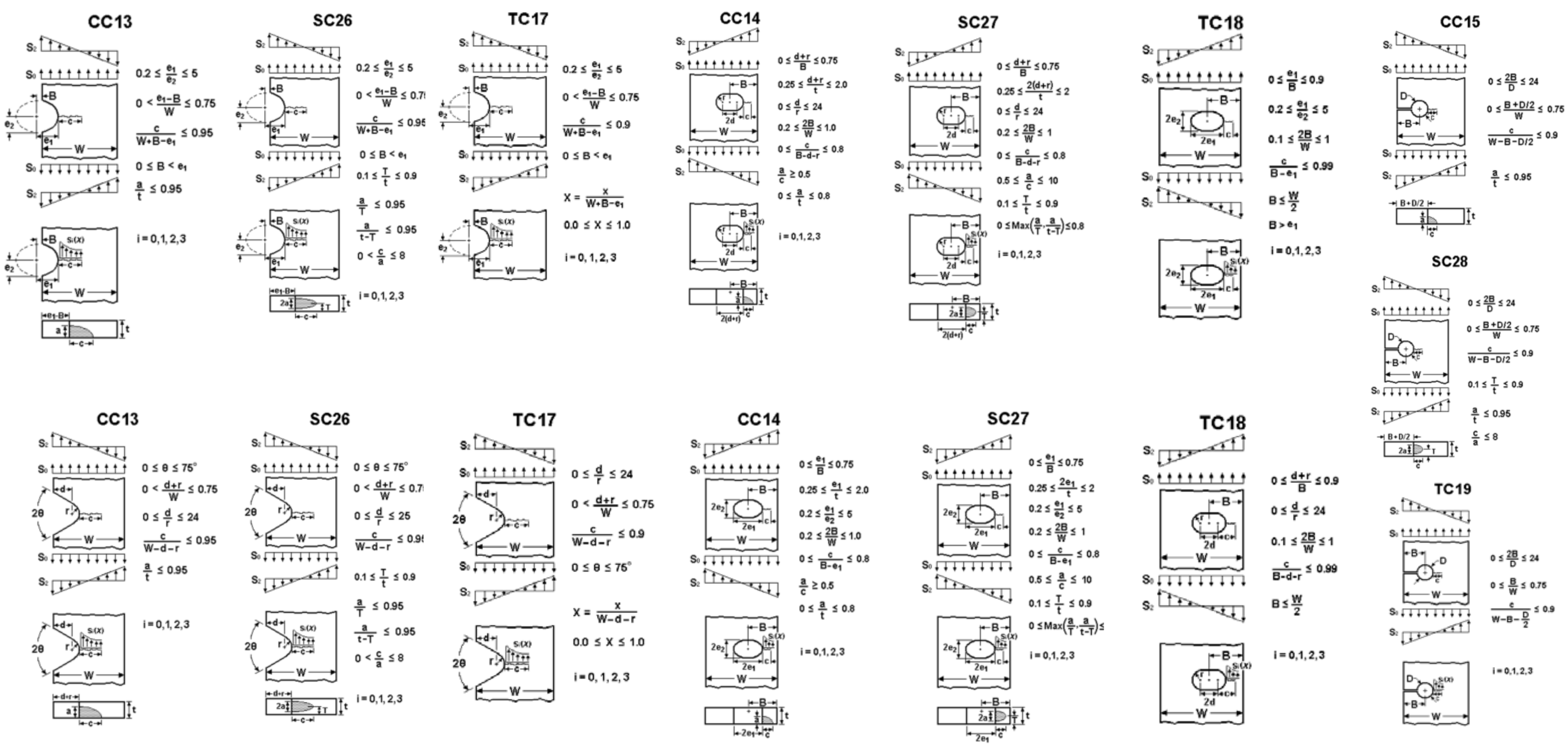
- Stress field ahead of arbitrary notches is a function of the notch root radius and the total notch depth





Weight Function Methods:
Family of New Solutions for Cracks at Arbitrary Notches/Slots/Holes

- Corner/Surface/Through crack at elliptical or angled edge notch
- Surface/Corner/Through crack at embedded slot or elliptical hole
- Surface/Corner/Through crack at round hole with broken ligament

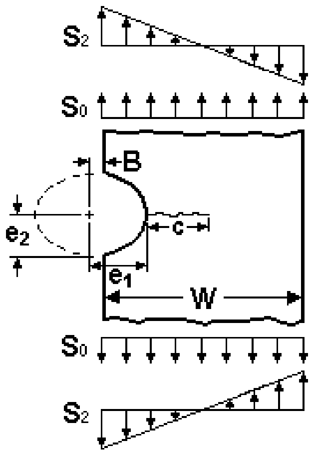


Weight Function Methods:

Family of New Solutions for Cracks at Arbitrary Notches/Slots/Holes

Note geometry details

CC13



$0.2 \le \frac{e_1}{e_2} \le 5$

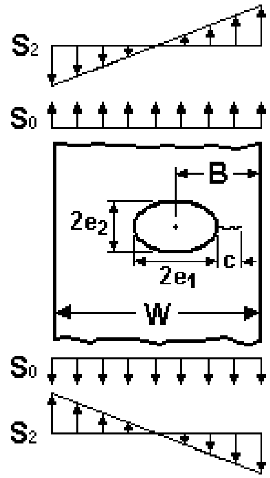
$0 < \frac{e_1 - B}{W} \le 0.75$

$\frac{c}{W + B - e_1} \le 0.95$

$0 \le B < e_1$

$\frac{a}{t} \le 0.95$

CC14



$0 \le \frac{e_1}{B} \le 0.75$

$0.25 \le \frac{e_1}{t} \le 2.0$

$0.2 \le \frac{e_1}{e_2} \le 5$

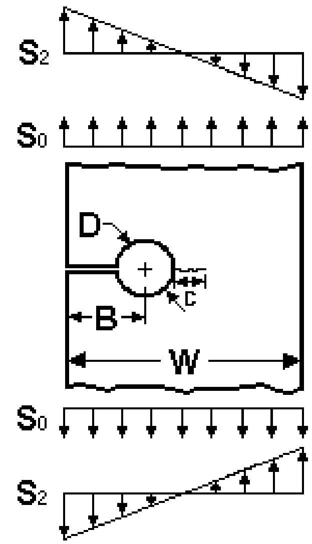
$0.2 \le \frac{2B}{W} \le 1.0$

$0 \le \frac{c}{B - e_1} \le 0.8$

$\frac{a}{c} \ge 0.5$

$0 \le \frac{a}{t} \le 0.8$

CC15



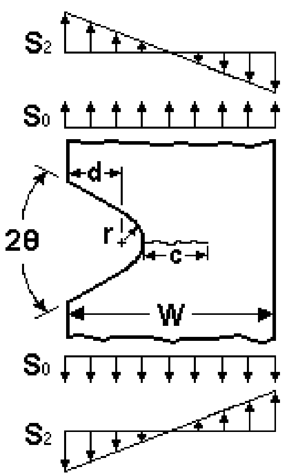
$0 \le \frac{2B}{D} \le 24$

$0 \le \frac{B + D/2}{W} \le 0.75$

$\frac{c}{W - B - D/2} \le 0.9$

$\frac{a}{t} \le 0.95$

CC13



$0 \le \theta \le 75^\circ$

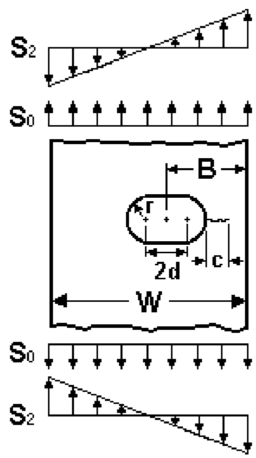
$0 < \frac{d + r}{W} \le 0.75$

$0 \le \frac{d}{r} \le 24$

$\frac{c}{W - d - r} \le 0.95$

$\frac{a}{t} \le 0.95$

CC14



$0 \le \frac{d + r}{B} \le 0.75$

$0.25 \le \frac{d + r}{t} \le 2.0$

$0 \le \frac{d}{r} \le 24$

$0.2 \le \frac{2B}{W} \le 1.0$

$0 \le \frac{c}{B - d - r} \le 0.8$

$\frac{a}{c} \ge 0.5$

$0 \le \frac{a}{t} \le 0.8$

Challenges and Resolution

- ***K*** **solutions have been accumulating for 30+ years**
- **Multiple solutions are now available for the same geometry**
  - They don't all have the same geometry scope
  - They don't all have the same loading capabilities
  - They don't all give the same answers!
  - They are not easily reconciled!
  - Which is the most accurate?
- **Other factors make it even more difficult to compare and evaluate different solutions**
  - For example, how to treat *K* solutions at the free surface for a part-through crack?
- **The path forward: intelligent combination of experience and methods**

Software Sustainment (Example)

- **Multiple *K*-solutions cases were available in NASGRO for corner-crack-at-hole geometries**
  - CC02, CC04, CC07 (legacy models)
  - CC08, CC10 (newer WF models)
- **These different crack cases all had slightly different capabilities, but they sometimes gave inconsistent results, leading to confusion**
- **It was not clear which solution was the most accurate**
- **Should we attempt to adjust the existing solutions, or should we attempt to develop a new (replacement) solution?**

Review of Existing Solutions

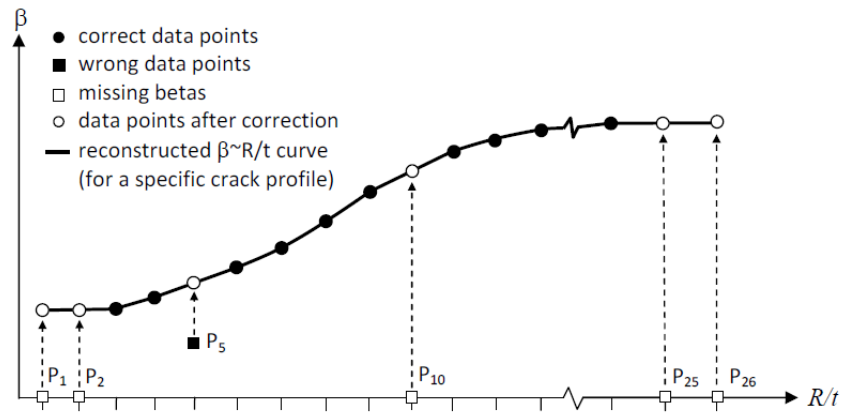
- Detailed review of the existing solutions (CC02/04/07/08/10) confirmed inconsistency of results
- No easy way to reconcile these inconsistencies
  - Some solutions use multiple correction factors or equation fits on top of the original Newman-Raju FE results
  - Some solutions use completely different matrices of results that are themselves fundamentally inconsistent with each other
- The original Newman-Raju results, while remarkably accurate in many cases, are based on ~1980 technology (meshes with 7900 DOF) and have limited geometry ranges

Review of New Fawaz-Andersson Solutions

- **The new Fawaz-Andersson solutions (2004) appeared to be reliable and superior**
  - Employed much larger and more sophisticated FE models
  - Agrees with Newman-Raju results in many cases
  - Covers a much wider overall geometry range
  - Includes pin loading results in addition to tension and out-of-plane bending
  - The raw F-A database had a few obvious problems that needed to be fixed (challenging due to large size of database)
  - F-A data were also available for consistent extension to solution for two unequal corner cracks at hole

Reconstruction of F-A Database

- **Detected interrogation**
- **Identified and repaired anomalie**
  - Missing values
  - Incorrect values
- **Limits expanded for a/t ⇨ 0**

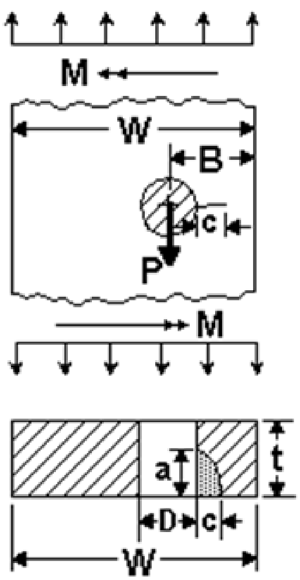


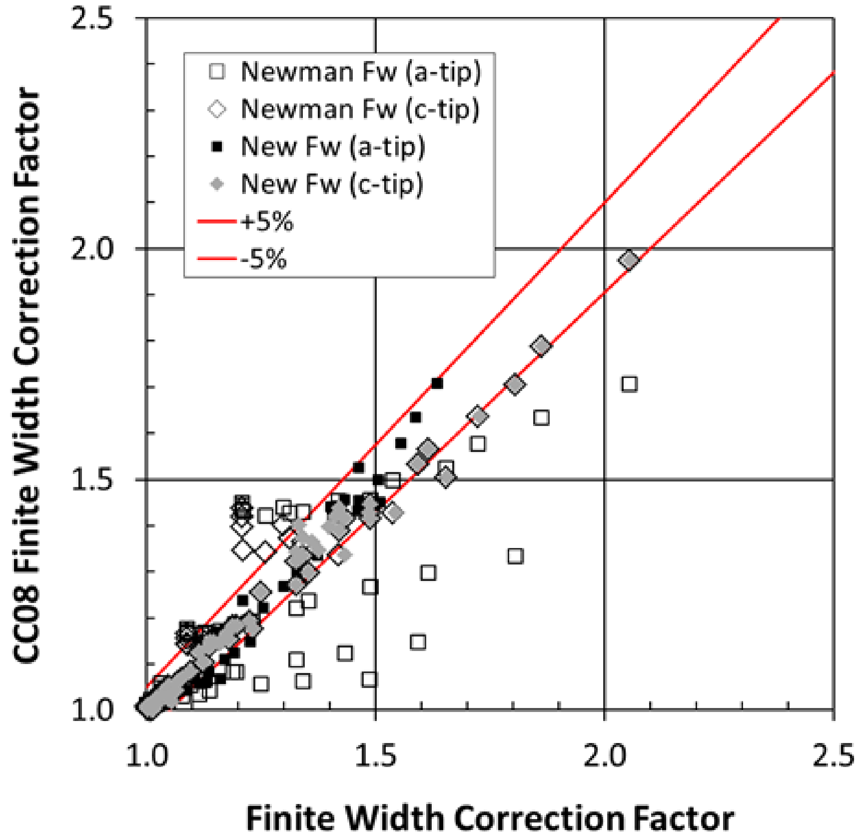
- **Overall size of database reduced by more than 20X:**
  - Removed unnecessary data
  - Reduced domain size but maintained overall limits
  - Binary file storage
- **Now easily useable without burdensome file size**

New Single-Corner-Crack-at-Hole Solution (CC16)

- **Start with (repaired) Fawaz-Andersson database**
  - $0.1 \le a/c \le 10$, $0 \le a/t \le 0.99$, and $0.1 \le R/t$
  - Remote tension, remote out-of-plane bending, pin loading
- **Use existing CC08 WF solution to guide refinements**
  - New finite width correction factor
  - Hole offset correction factor
  - $a/t = 0$ solution

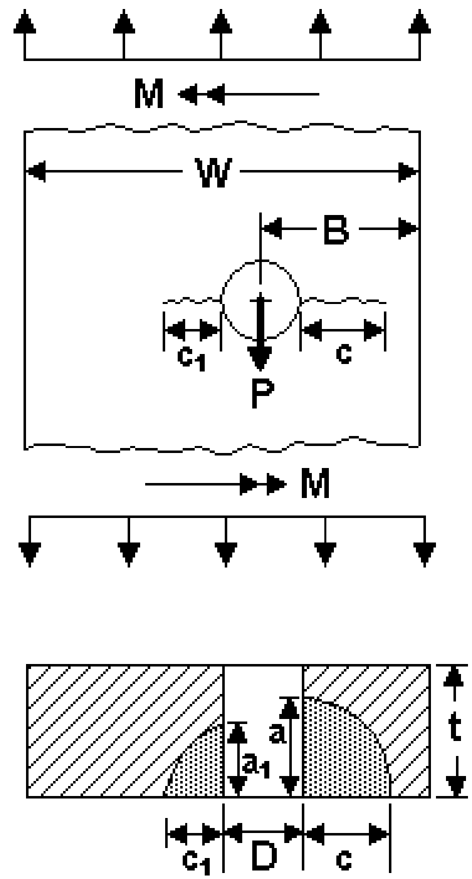
    (from Kt considerations)
- **Generate additional FE solutions for verification**





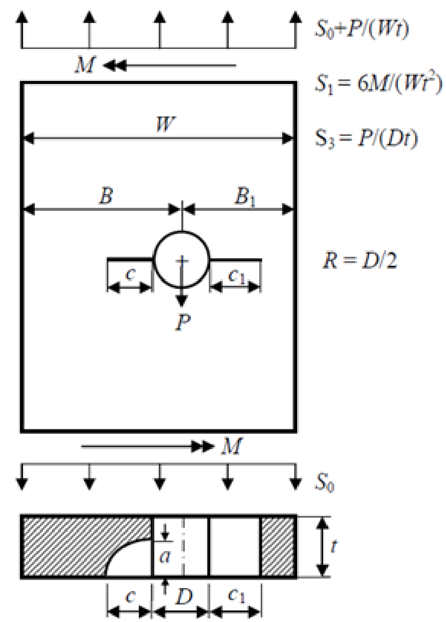
New Unequal-Corner-Crack-at-Hole Solution (CC17)

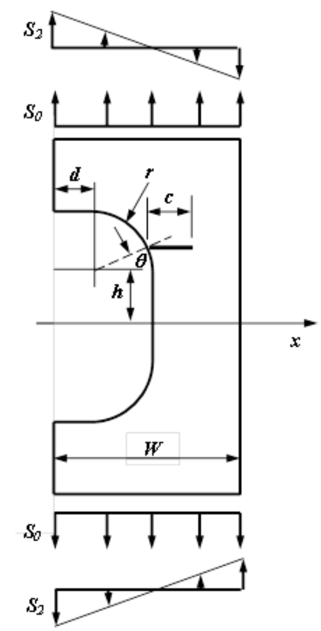
- **Start with (repaired) Fawaz-Andersson database**
  - $0.2 \leq a/c \leq 5$, $0 \leq a/t \leq 0.95$, and $0.125 \leq R/t \leq 10$
  - Remote tension, remote out-of-plane bending, pin loading
- **Derive new "equivalent hole" method for finite geometry effects**
  - Account for effects of crack on the other side of the hole
  - Then use new CC16 correction factors
  - Validated with additional FE solutions
- **Entirely consistent with CC16**
- **Verified with extensive additional numerical analyses**



Related Ongoing Development Activities

- **Improved finite-width correction factors for pin loading**
- **Hybrid through crack (TC) and corner crack (CC) at a hole model**
- **Through crack at rectangular edge notch with rounded corners, univariant WF**





$$0 \le \frac{d}{r} \le 10$$

$$0.05 \le \frac{d+r}{W} \le 0.5$$

$$0 \le \frac{h}{r} \le 5$$

Model Verification & Validation

- ***Verification:*** Process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model

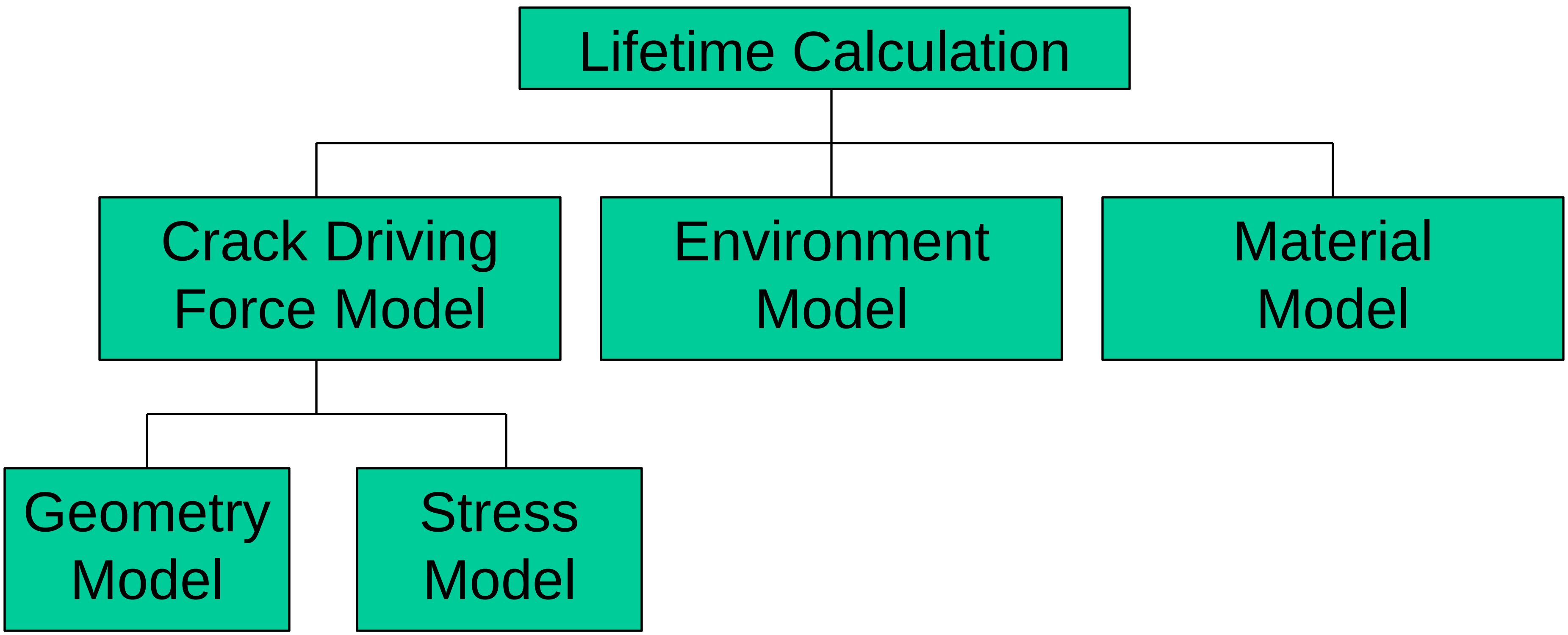
  Math issue: "Solving the equations right"

- ***Validation:*** Process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model

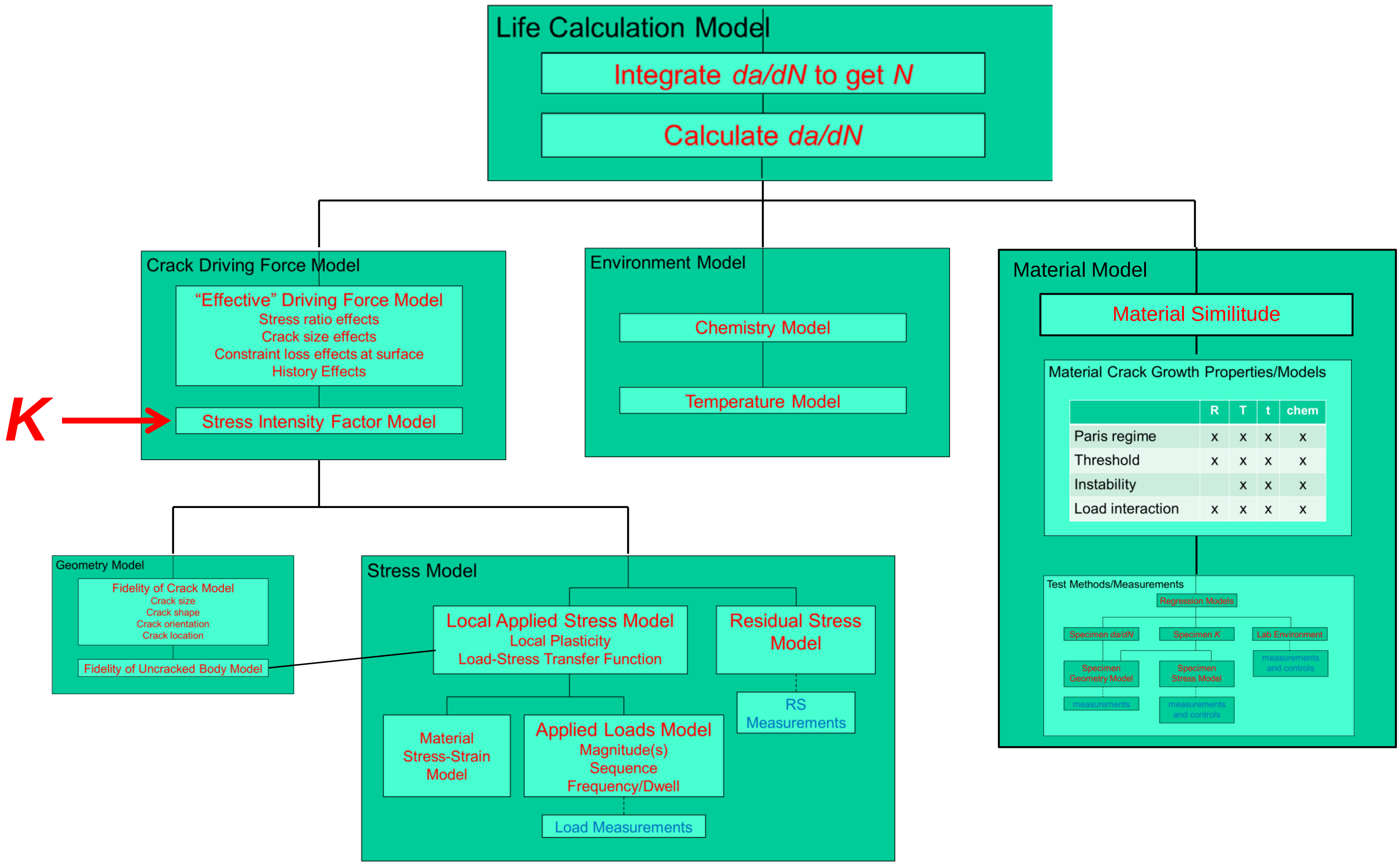
  Physics issue: "Solving the right equations"

Hierarchical Approach to V&V

- Following the paradigm of ASME V&V 10-2006, V&V should be performed step-by-step in a hierarchical, building-block approach



Detailed Draft Hierarchy for V&V of FCG Lifetime Analysis



Some Current and Future Challenges

- **Crack configurations with many degrees of freedom**
  - Cracks at countersunk holes
  - Cracks at lugs
  - Multiple-site damage, including crack interaction and link-up
  - Continuing damage
  - Cracks in stiffened structures
- **Irregular crack shapes (not straight or part-elliptical)**
- **Contact stresses at fasteners**
- **Constraint loss for crack tips near surfaces**
- **Structural load redistribution**
- **Speed issues: faster computers vs. bigger problems**

Concluding Remarks

- *K* solutions have been available to support engineering analysis for fracture control for 40+ years
  - Many legacy solutions have been used for 30+ years
- Recent resurgence of interest and activity in developing new and improved *K* solutions
  - Faster computers, improved numerical methods, new formulations
- New *K* solutions are more widely available today (and easier to use) in sophisticated engineering software
- Continued collaborations between the research community and industry are needed to ensure that this technology growth continues and addresses the significant number of remaining needs and challenges

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