Bayesian Statistics and Uncertainty Quantification for Safety Boundary Analysis in Complex Systems

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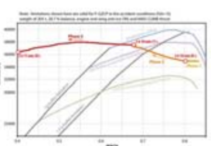
Abstract

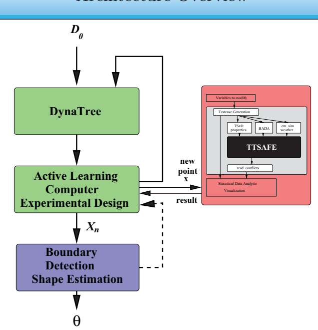
The analysis of a safety-critical system often requires detailed knowledge of safe regions and their high-dimensional non-linear boundaries. We present a statistical approach to iteratively detect and characterize the boundaries, which are provided as parameterized shape candidates. Using methods from uncertainty quantification and active learning, we incrementally construct a statistical model from only few simulation runs and obtain statistically sound estimates of the shape parameters for safety boundaries.

Introduction



- All spacecraft, aircraft, and other complex systems can only work safely within a given operational envelope (Figure shows the flight path (red) of the ill-fated flight AF447 as altitude over mach number; important boundaries are shown in gray colors)
- Multiple, non-linear boundaries in a high-dimensional parameter space and slow/expensive simulation runs limit the use of current analysis techniques like single-variable and linear techniques.
- We use statistical emulation and hierarchical Bayesian modeling to quantify the uncertainites in models and make reliable predictions of complex phenomena like number, location, and shapes of boundaries.

Architecture Overview

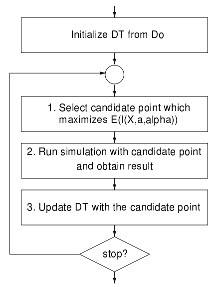


- We use DynaTrees: dynamic regression trees and sequential tree model for online applications [Taddy,Gramacy,Polson 2011]
  - Recursive partition of input space
  - Particle learning for posterior simulation

$$p([T,S]_t|[x,y]^t) = \int p([T,S]_t|[T,S]_{t-1})dP([T,S]_{t-1}|[x,y]^t)$$
$$\propto \int p([T,S]_t|[T,S]_{t-1},[x,y]_t)\int p([x,y]_t|[T,S]_{t-1})dP([T,S]_{t-1}|[x,y]^{t-1})$$

  solved with resampling and propagation
- High efficiency through tree-based partitioning in higher dimensions
- Particle mechanism suitable for active learning and experimental design

Active Learning Architecture



- General goal: candidate points should be near boundaries
- Maximum entropy $Y = -\sum_{c \in c_1,\ldots,c_n} p_c \log p_c$ is too greedy
- Active Learning McKay (ALM): select maximum variance
- Active Learning Cohn (ALC): maximize reduction in predictive variance
- Expected Improvement (EI): maximize posterior expectation of improvement statistic

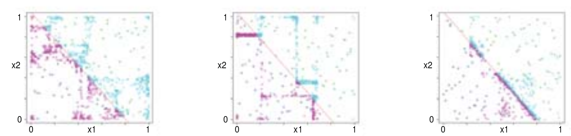
Limitation: ALM, ALC, EI do not take boundaries into account.

Our Extension: Boundary-EI

- Focus on x with $0.5-\epsilon \leq \hat{y}(x) \leq 0.5+\epsilon$ for $0 < \epsilon$
- Improvement (Jones 1998, Ranjan 2008): $I(x) = \epsilon^2(x) - \min\{(y(x)-0.5)^2, \epsilon^2(x)\}$
- Expectation of $I(x)$: ($\alpha > 0$, $\epsilon(x) = \alpha s(x)$, std deviation $s(x)$, $y(x) \sim N(\hat{y}(x), s^2(x))$)

$$\begin{aligned} E[I(x)] = & -\int_{0.5-\alpha s(x)}^{0.5+\alpha s(x)} (y-\hat{y}(x))^2 \phi\left(\frac{y-\hat{y}(x)}{\sigma(x)}\right) dy \\ & +2(\hat{y}-0.5)\sigma^2(x)\left[\phi\left(\frac{0.5-\hat{y}(x)}{\sigma(x)}+\alpha\right) - \phi\left(\frac{0.5-\hat{y}(x)}{\sigma(x)}-\alpha\right)\right] \\ & +(\alpha^2\sigma^2(x) - (\hat{y}(x)-0.5)^2)\left[\Phi\left(\frac{0.5-\hat{y}(x)}{\sigma(x)}+\alpha\right) - \Phi\left(\frac{0.5-\hat{y}(x)}{\sigma(x)}-\alpha\right)\right] \end{aligned}$$

- Term 1 variability of response in ϵ neighborhood
  Term 2 farther away and in areas with high variance
  Term 3 is active close to estimated boundary



ALC EI Bounday-EI

Modeling Boundary Shapes

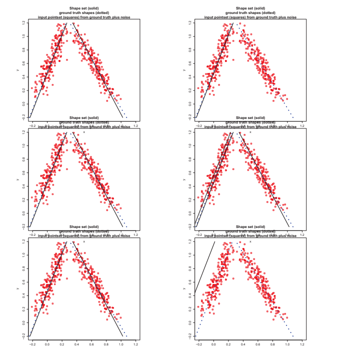
- Task: estimate shapes of boundaries given points X_n near boundaries
- Boundary shapes can incorporporate physics and domain knowledge
- Shape dictionary can be provided by domain expert

Metrics for shape estimation

Completeness $\overline{D}^2_{X,\mathcal{S}} = \frac{\sum_{x \in X} d^2_{X,\mathcal{S}}(x)}{|X|}$

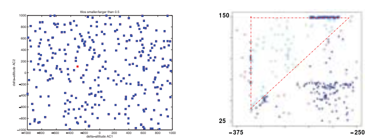
Minimality $\overline{D}^2_{\mathcal{S}} = \frac{\sum_{S_i \in \mathcal{S}} \sum_{s_i \in S_i} d^2_{S_i,\mathcal{S}_{-i}}(s_i)}{\sum_{S_i \in \mathcal{S}} |S_i|}$

Summary $\overline{D}^2_{\mathcal{S},X_n} = \frac{\sum_{a=1}^{l} \sum_{s \in S_a} d^2_{S_a,X_n}(s)}{\sum_{a=1}^{l} |S_a|}$



Experimental Result

Uncertainty in TTSAFE (Terminal Tactical Separation Assured Flight Environment) track data. We analyzed TTSAFE behavior with respect to bias in the measured Radar data.



Summary

- We developed a statistical framework to support analysis and uncertainty quantification of non-linear complex systems.
- We used Bayesian statistical methology in combination with active learning techniques for efficient detection and characterization safety regions and their boundaries.
- Case studies include NASA Intelligent Flight Control System (IFCS) and Terminal Tactical Separation Assured Flight Environment (TTSAFE) for Next Generation Air Traffic Control.
- Future work will focus on further uncertainty quantification study, optimization of the active learning for high dimensional spaces, and application of the framework to other domains.

References

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