Bayesian Statistics and Uncertainty Quantification for Safety Boundary Analysis in Complex Systems

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Abstract

The analysis of a safety-critical system often requires detailed knowledge of safe regions and their high-dimensional non-linear boundaries. We present a statistical approach to iteratively detect and characterize the boundaries, which are provided as parametrized shape candidates. Using methods from uncertainty quantification and active learning, we incrementally construct a statistical model from only few simulation runs and obtain statistically sound estimates of the shape parameters for safety boundaries.

Introduction

• All spacecraft, aircraft, and other complex systems can only work safely within a given operational envelope (Figure shows the flight path (red) of the ill-fated flight AF447 as altitude over much number important boundaries are shown in gray colors)
• Multiple, non-linear boundaries in a high-dimensional parameter space and costly/expensive simulation runs limit the use of current analysis techniques like single-variable and linear techniques
• We use statistical emulation and hierarchical Bayesian modeling to quantify the uncertainties in models and make reliable predictions of complex phenomena like number, location, and shapes of boundaries.

Bayesian Approach

• We use DynaTrees: dynamic regression trees and sequential tree model for online applications [Taddy,Gramacy,Polson 2011]
  – Recursive partition of input space
  – Particle-learning for posterior simulation
  
\[ p(T,S)[x,y] = \frac{p(x,y)[T,S_x=x,T,S_y=y]p(T,S_x=x,S_y=y)}{p(x,y)[T,S_x=x,T,S_y=y]} \]

Active Learning Architecture

1. Select candidate points with maximum Q (Acquisition)
2. For each candidate point and obtain result
3. Update Q with the new point

• General goal: candidate points should be near boundaries
• Minimum entropy \( Y = -\sum_{i=1}^{N} p(y_i) \log p(y_i) \) is too greedy
• Active Learning McKay (ALM): select maximum variance
• Active Learning Cohn (ALC): maximize reduction in predictive variance
• Expected Improvement (EI): maximize posterior expectation of improvement statistic

Limitation: ALM, ALC, EI do not take boundaries into account.

Our Extension: Boundary-EI

• Focus on \( x \) with \( 0.5 - \epsilon \leq y(x) \leq 0.5 + \epsilon \) for \( \epsilon > 0 \)
• Improvement (Jones 1998, Ranjan 2008): \( I(x) = c(x) - \min(y(x), 0.5)^2 + c(x) \)
• Expectation of \( I(x) \) \( \alpha > 0, c(x) = \sigma(x), \) and deviation \( s(x), y(x) = N(y(x), c(x)) \)

\[
E[I(x)] = -\int (y - y(x))^2 q_y \left( y \mid x \right) dy
+ \int y(y - y(x))^2 \left[ 0.5 \log |\sigma^2(y(x))| + \sigma^2(y(x)) \right. \\
\left. + \phi \left( \frac{y - y(x)}{\sigma(y(x))} \right) \phi \left( \frac{{-y + y(x)}}{\sigma(y(x))} \right) \right] dx
+ \alpha \int \sigma^2(y(x)) (y(x) - 0.5)^2 \left[ \frac{\sigma^2(y(x))}{\sigma^2(y(x))} + \alpha \right. \\
\left. - \Phi \left( \frac{y(x) - 0.5}{\sigma(y(x))} \right) \Phi \left( \frac{-y(x) + 0.5}{\sigma(y(x))} \right) \right] dx
\]

• Term 1 variability of response in \( y \) neighborhood
• Term 2 further away and in areas with high variance
• Term 3 is active close to estimated boundary

Modeling Boundary Shapes

• Task: estimate shapes of boundaries given points \( X_n \) near boundaries
• Boundary shapes can incorporate physics and domain knowledge
• Shape dictionary can be provided by domain expert

Metrics for shape estimation

Completeness \( P_{Na} = \sum_{i=0}^{N} p_1 \left( y_i \mid x, S \right) \)

Minimality \( P_{Na} = \sum_{i=0}^{N} p_1 \left( y_i \mid x, S \right) \)

Summary \( P_{Na} = \sum_{i=0}^{N} p_1 \left( y_i \mid x, S \right) \)

Experimental Result

Uncertainty in TTSafe (Terminal Tactical Separation Assured Flight Environment) track data. We analyzed TTSafe behavior with respect to bias in the measured Radar data.

Summary

• We developed a statistical framework to support analysis and uncertainty quantification of non-linear complex systems.
• We used Bayesian statistical methodology in combination with active learning techniques for efficient detection and characterization safety regions and their boundaries.
• Case studies include NASA Intruder Flight Control System (IFCS) and Terminal Separation Assured Flight Environment (TTSafe) for Next Generation Air Traffic Control.
• Future work will focus on further uncertainty quantification study, optimization of the active learning for high-dimensional spaces, and application of the framework to other domains.

References