Bayesian Statistics and Uncertainty Quantification for Safety Boundary Analysis in Complex Systems

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Abstract

The analysis of a safety-critical system often requires detailed knowledge of safe regions and their high-dimensional non-linear boundaries. We present a statistical approach to iteratively detect and characterize boundaries, which are provided as parameterized shape candidates. Using methods from uncertainty quantification and active learning, we incrementally construct a statistical model from only few simulation runs and obtain statistically sound estimates of the shape parameters for safety boundaries.

Introduction

• All spacecraft, aircraft, and other complex systems can only work safely within a given operational envelope (Figure shows the flight path (red) of the ill-fated flight AF447 as altitude over much number important boundaries are shown in grey colors)
• Multiple, non-linear boundaries in a high-dimensional parameter space and story-expensive simulation runs limit the use of current analysis techniques like single-variable and linear techniques.
• We use probabilistic emulation and hierarchical Bayesian modeling to quantify the uncertainties in models and make reliable predictions of complex phenomena like number, location, and shapes of boundaries.

Shape set (solid)
ground truth shapes (dotted)
input pointset (squares) from ground truth plus noise

Architecture Overview

• We use DynaTrees: dynamic regression trees and sequential tree model for online applications [Taddy.Gramacy.Polson 2011]
  – Recursive partition of input space
  – Particle-learning for posterior simulation

p(T, S)[x, y] = \frac{1}{N} \prod_{i=1}^{N} \left( \int_{-\infty}^{\infty} p(x, y | T, S, x_i) p(T, S | x_i) dx_i \right)

solved with reoptimization and propagation
• High efficiency through tree-based partitioning in higher dimensions
• Particle mechanism suitable for active learning and experimental design

Active Learning Architecture

• Task: estimate shapes of boundaries given points X_n near boundaries
• Boundary shapes can incorporate physics and domain knowledge
• Shape dictionary can be provided by domain expert

Metrics for shape estimation

Completeness \( \gamma \)
Minimality \( \theta \)
Summary \( \tau \)

Our Extension: Boundary-EI

• Focus on \( x \) with \( 0.5 - \epsilon \leq y(x) \leq 0.5 + \epsilon \) for \( 0 < \epsilon \)
• Improvement [Jones 1998, Ranjan 2008]: \( I(x) = \mathcal{E}(x) - \min_{y(x) \in \mathcal{Y}(x)} \mathcal{E}(y(x)) \)
• Active Learning Mc-Kay (ALM): select maximum variance
• Active Learning Cohn (ALC): maximize reduction in predictive variance
• Expected Improvement (EI): maximize posterior expectation of improvement statistic

Summary

We developed a statistical framework to support analysis and uncertainty quantification of non-linear complex systems.
We used Bayesian statistical methodology in combination with active learning techniques for efficient detection and characterization safety regions and their boundaries.
Case studies include NASA Intelligent Flight Control System (IFCS) and Terminal Tactical Separation Assured Flight Environment (TTSAFE) for Next Generation Air Traffic Control.
Future work will focus on further uncertainty quantification study, optimization of the active learning for high-dimensional spaces, and application of the framework to other domains.

Experimental Result

Uncertainty in TTSafe (Terminal Tactical Separation Assured Flight Environment) track data. We analyzed TTSafe behavior with respect to bias in the measured Radar data.

References