Bayesian Statistics and Uncertainty Quantification for Safety Boundary Analysis in Complex Systems

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Abstract

The analysis of a safety-critical system often requires detailed knowledge of safe regions and their high-dimensional non-linear boundaries. We present a statistical approach to iteratively detect and characterize the boundaries, which are provided as parameterized shape candidates. Using methods from uncertainty quantification and active learning, we incrementally construct a statistical model from only few simulation runs and obtain statistically sound estimates of the shape parameters for safety boundaries.

Introduction

• All spacecraft, aircraft, and other complex systems can only work safely within a given operational envelope (Figure shows the flight path (red) of the ill-fated flight AF447 as altitude over much number important boundaries are shown in gray colors)
• Multiple, non-linear boundaries in a high-dimensional parameter space and small/expensive simulation runs limit the use of current analysis techniques like single-variable and linear techniques
• We use statistical emulation and hierarchical Bayesian modeling to quantify the uncertainties in models and make reliable predictions of complex phenomena like number, location, and shapes of boundaries

Active Learning Architecture

1. Select candidate points with maximum \( D^2 \) (Adaptive)  
2. For selected candidate points, obtain real data  
3. Update the tree at the selected points

Boundary Shapes

• General goal: candidate points should be near boundaries
• Minimum entropy \( Y = \sum_{a \in \{0,1\}} p_a \log p_a \) is too greedy
• Active Learning McKay (ALM): select maximum variance
• Active Learning Cohn (ALC): maximum reduction in predictive variance
• Expected Improvement (EI): maximize posterior expectation of improvement statistic

Future work will focus on further uncertainty quantification study, optimization of the active learning framework for non-linear complex systems.