Propulsion System Models for Rotorcraft Conceptual Design

Wayne Johnson
Aeromechanics Office
National Aeronautics and Space Administration
Ames Research Center, Moffett Field, California
wayne.johnson@nasa.gov

ABSTRACT

The conceptual design code NDARC (NASA Design and Analysis of Rotorcraft) was initially implemented to model conventional rotorcraft propulsion systems, consisting of turboshaft engines burning jet fuel, connected to one or more rotors through a mechanical transmission. The NDARC propulsion system representation has been extended to cover additional propulsion concepts, including electric motors and generators, rotor reaction drive, turbojet and turbofan engines, fuel cells and solar cells, batteries, and fuel (energy) used without weight change. The paper describes these propulsion system components, the architecture of their implementation in NDARC, and the form of the models for performance and weight. Requirements are defined for improved performance and weight models of the new propulsion system components. With these new propulsion models, NDARC can be used to develop environmentally-friendly rotorcraft designs.

INTRODUCTION

The objectives of rotorcraft design work in a government laboratory are to support research and to support rotorcraft acquisition. Research activities require a robust design capability to aid in technology impact assessments and to provide system level context for research. At the applied research level, it is necessary to show how technology will impact future systems, and to justify the levels of investment required to mature that technology to an engineering development stage. Conceptual design provides one avenue to accomplishing these objectives. NASA research activities requiring rotorcraft design work include concept exploration, concept decision, concept refinement, and technology development. During these activities, performing quantitative evaluation and independent synthesis of a wide array of aircraft designs is necessary.

The design code NDARC (NASA Design and Analysis of Rotorcraft) was developed to fulfill these requirements (Ref. 1). NDARC is a conceptual/preliminary design and analysis computer program for rapidly sizing and conducting performance analysis of new rotorcraft concepts. NDARC has a modular code base, facilitating its extension to new concepts and the implementation of new computational procedures. The theoretical basis and architecture is described in Ref. 2; design results from the development are presented in Ref. 3.

The NDARC code performs design and analysis tasks (figure 1). The design task sizes the rotorcraft to satisfy a set of design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius, as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity.

NDARC was initially implemented to model conventional rotorcraft propulsion systems, consisting of turboshaft engines burning jet fuel, connected to one or more rotors through a mechanical transmission.
The NDARC propulsion system representation has been extended to cover additional propulsion concepts. A major objective is to be able to develop environmentally-friendly rotorcraft designs. The new propulsion elements include electric motors and generators, rotor reaction drive, turbojet and turbofan engines, fuel cells and solar cells, batteries, and fuel (energy) used without weight change. This paper describes these propulsion system elements, and the architecture of their implementation in NDARC. Requirements are defined for improved performance and weight models of the new propulsion system components.

AIRCRAFT DESCRIPTION

Decomposition of the aircraft system into components is critical to achieving the ability to rapidly model a wide array of rotorcraft concepts. Thus the aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated. The aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations are facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered are single main-rotor and tail-rotor helicopter; tandem helicopter; coaxial helicopter; and tiltrotors. Novel and advanced concepts typically are modeled by starting with one of these conventional configurations. For example, compound rotorcraft can be constructed by adding wings and propellers.

The aircraft is formed from components that include a fuselage and landing gear, usually rotors, perhaps wings and tails, and the following components related to the propulsion representation.

Original NDARC Propulsion Representation

NDARC was initially implemented to model conventional rotorcraft propulsion systems, consisting of turboshaft engines burning jet fuel, connected to one or more rotors through a mechanical transmission. The original aircraft model had the following propulsion elements (figure 2):

– Propulsion Groups: A propulsion group represents the drive system, which connects a set of rotors and engine groups. The components (rotors) define the power required, and the engine groups define the power available. There are one or more drive states, with a set of gear ratios for each state. The power required equals the sum of component power, transmission losses, and accessory losses. There are drive system torque limits, and rotor and engine shaft ratings.

– Engine Groups: Each engine group has one or more engines of the same type. For each engine type an engine model is defined. The engine performance information includes mass flow, fuel flow, jet thrust, and momentum drag at the required power.

– Fuel Tank: There is one fuel tank component for the aircraft. The fuel quantity stored and burned is measured in weight. A fuel tank component can include a number of auxiliary tanks.
Forces: The force component is a simple model to generate a force on the aircraft (representing a lift, propulsion, or control subsystem), including a weight and fuel flow description.

Rotors: In addition to main rotors, tail rotors, propellers, and ducted fans can be modeled. A rotor can be tilting, ducted, antitorque, and/or reaction drive. The rotor power required is evaluated using the energy method, as a sum of induced, profile, and parasite power.

Extended NDARC Propulsion Representation

The NDARC propulsion system model has been extended in the present work to cover additional propulsion system concepts, including electric motors and generators, rotor reaction drive, turbojet and turbofan engines, fuel cells and solar cells, batteries, and fuel (energy) used without weight change (figure 3). The components of the propulsion system can be classified as producing or absorbing shaft torque (engine groups), producing force (jet groups), or producing energy (charge groups):

- **Engine Groups**: An engine group transfers power by shaft torque, so is associated with a propulsion group. An engine model describes a particular engine, used in one or more engine groups. Models include turboshaft engines (perhaps convertible, for turbojet operation or reaction drive), reciprocating engines, compressors (perhaps for reaction drive), electric motors (perhaps with fuel cells), electric generators, and generator-motors.

- **Jet Groups**: A jet group produces a force on the aircraft. A jet model describes a particular jet, used in one or more jet groups. Models include turbojet and turbofan engines (perhaps convertible, for reaction drive), reaction drive, and a simple force model. A reaction drive supplies a blade force that provides the rotor power required.

- **Charge Groups**: A charge group generates energy for the aircraft. A charge model describes a particular charger, used in one or more charge groups. Models include fuel cells and solar cells.

- **Fuel Tank System**: There are one or more fuel tank systems for the aircraft. The fuel quantity stored and burned is measured in weight or energy. Each component that uses fuel (engine, jet, and charge groups) is associated with a fuel tank system of the appropriate type. There can be one or more sizes of auxiliary fuel tanks. Tanks that use weight include jet fuel, gasoline, and hydrogen. Tanks that use energy include batteries, capacitors, and flywheels.

The engine, jet, and charge groups introduce an overall classification of the propulsion system components, each
with a general performance characterization, from which the architecture of the code follows. A model consists of a parameterized, surrogate representation of the component performance and weight, applicable to a wide range of operating conditions and component size. It is the models where simplifications and approximations are found, and future improvements developed.

**SIZING TASK**

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. Sizing is an iterative procedure to find a consistent description of the system. Optimization is handled external to NDARC. The aircraft size is characterized by parameters such as design gross weight or weight empty, rotor radius, and engine power available. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. Refs. 1–2 describe the sizing options of NDARC in detail. Here the extended propulsion models introduce additional sizing options.

For each propulsion group, the engine power or the rotor radius can be sized. The engine power is the maximum of the power required for all designated sizing flight conditions and sizing missions. Hence the engine power is changed by the ratio of the propulsion group power required to power available. Alternatively, the rotor radius can be adjusted so power required equals power available; or both engine power and rotor radius can be input rather than sized.

For each jet group, the design thrust can be sized. The design thrust is the maximum of the thrust required for all designated sizing flight conditions and sizing missions. Hence the design thrust is changed by the ratio of the jet group thrust required to thrust available.

For each charge group, the design power can be sized. The design power is the maximum of the power required for all designated sizing flight conditions and sizing missions. Hence the design power is changed by the ratio of the charge group power required to power available.

**PROPELLATION SYSTEM**

The aircraft propulsion system can be constructed from a number of components: propulsion groups, engine groups, jet groups, charge groups, and fuel tank systems. The NDARC propulsion group is a mechanical drive train, which connects designated rotors and engine groups.

Referred performance parameters are used for propulsion system components that operate with air. Referred parameters account for the characteristic velocity, density, and pressure associated with the working fluid. The operating condition and atmosphere give the standard conditions (temperature $T_{std}$ and pressure $p_{std}$) for a specified pressure altitude; the sea-level standard conditions (temperature $T_0$ and pressure $p_0$); and the operating temperature $T$ and pressure $p$. Here the temperatures are in deg R or deg K. The engine characteristics depend on the temperature ratio $\theta = T/T_0$ and pressure ratio $\delta = p/p_0$. The flight Mach number $M$ is obtained from the aircraft speed. The inlet ram air temperature ratio and pressure ratio are obtained from $M$ and the inlet ram recovery efficiency $\eta_{d}$: $\theta_M = (1 + 0.2M^2)$ and $\delta_M = (1 + \eta_d 0.2M^2)^{3.5}$, using the ratio of specific heats $\gamma = 1.4$.

The reference performance is at sea-level-standard static conditions (subscript 0), and maximum continuous power or thrust (subscript C). Referred or corrected engine parameters are used in the model: power $P/\delta \sqrt{\theta}$, mass flow $\dot{m}/\delta \sqrt{\theta}$, specific power $(P/\dot{m})/\theta$, fuel flow $\dot{w}/\delta \sqrt{\theta}$, thrust $F/\delta$, and turbine speed $N/\sqrt{\theta}$.

Engine performance depends on the engine rating. Each engine rating has specific operating limitations, most importantly an operating time limit intended to avoid damage to the engine. The power available from a turboshift engine depends on the engine rating. Engine power is generally specified in terms of SLS static max continuous power (MCP). Takeoff typically uses maximum rated power (MRP). The thrust available from a turbojet or turbofan engine depends on the engine rating, including maximum continuous thrust (MCT).

Geometry and control are defined for engine groups, jet groups, and charge groups. The component amplitude and mode are control variables. The amplitude can be power (engine group), thrust (jet group), or power (charge group). The mode can be mass flow (for convertible engines), or power flow (for generator-motor). The group orientation is specified by selecting a nominal direction. The yaw and incidence angles can be connected to the aircraft controls.

The group produces a force acting in the direction of the component; and a drag acting in the wind direction. An aerodynamic model is defined for engine groups, jet groups, and charge groups. The group includes a nacelle,
which contributes to the aircraft drag. The reference area for the nacelle drag coefficient is the nacelle wetted area.

Rotor Reaction Drive
A rotor reaction drive can be modeled using either an engine group or a jet group.

The total rotor power required $P_{\text{rotor}}$ consists of induced, profile, interference, and parasite terms. In most helicopter designs the power is delivered to the rotor by a mechanical drive, through the rotor shaft torque. Such designs require a transmission and a means for balancing the main rotor torque. The shaft power is $P_{\text{sha}} = P_{\text{rotor}}$, which contributes to the propulsion group power required, $P_{\text{propPG}}$, and produces a torque on the aircraft.

An alternative is to supply the power by a jet reaction drive of the rotor, using cold or hot air ejected out of the blade tips or trailing edges. Helicopters have also been designed with ram jets on the blade tips, or with jet flaps on the blade trailing edges that use compressed air generated in the fuselage. Since there is no torque reaction between the helicopter and rotor (except for the small bearing friction), no transmission or anti-torque device is required, resulting in a considerable weight saving. With a jet reaction drive, the propulsion system is potentially lighter and simpler, although the aerodynamic and thermal efficiency are lower. The helicopter must still have a mechanism for yaw control.

With reaction drive the shaft power is zero ($P_{\text{sha}} = 0$), and the reaction power $P_{\text{react}} = P_{\text{rotor}}$ contributes to the engine group or jet group power required. The reaction drive produces a force $F_{\text{react}}$ on the rotor blade at effective radial station $r_{\text{react}}$, so $P_{\text{react}} = \Omega r_{\text{react}} F_{\text{react}}$. Momentum balance gives the total force. The average force in the nonrotating frame is the drag of the inlet momentum ($\dot{m}_{\text{react}} V$), which is accounted for in the engine group or jet group model. The mean in the rotating frame gives the total jet force required $F_{\text{react}} = \dot{m}_{\text{react}} (V_{\text{react}} - \Omega r_{\text{react}})$. The engine group or jet group performance includes the blade duct and nozzle, perhaps even with tip burning. If the reaction drive is turned off, then the rotor must be trimmed such that $P_{\text{rotor}} = 0$.

FUEL TANK
The fuel quantity stored and burned can be measured in weight or energy. Each component (engine group, jet group, charge group) that uses or generates fuel is associated with a fuel tank system of the appropriate type.

The unit of fuel energy is Mega-Joules (MJ). For reference, 1 BTU = 1055.056 Joule.

Fuel Tank Systems that Store and Burn Weight
For fuel use measured by weight, the fuel properties are density $\rho_{\text{fuel}}$ (weight per volume, lb/gal or kg/liter) and specific energy $\epsilon_{\text{fuel}}$ (MJ/kg). Table 1 gives the properties of a number of aviation fuels, based on military and industry specifications. Fuels considered include jet fuel, gasoline, diesel, and hydrogen. From the fuel weight $W_{\text{fuel}}$, the energy is $E_{\text{fuel}} = \epsilon_{\text{fuel}} W_{\text{fuel}}$ (MJ) and the volume is $V_{\text{fuel}} = W_{\text{fuel}} / \rho_{\text{fuel}}$ (gallons or liters). A motive device has a fuel flow $\dot{w}$ (lb/hour or kg/hour), and its specific fuel consumption is $\text{sfc} = \dot{w} / P$ or $\text{sfc} = \dot{w} / T$.

The fuel tank capacity $W_{\text{fuel-cap}}$ (maximum usable fuel weight) is input or determined from designated sizing missions. The corresponding volumetric fuel tank capacity is $V_{\text{fuel-cap}} = W_{\text{fuel-cap}} / \rho_{\text{fuel}}$. The fuel system weight consists of the tank weight (including support) and the plumbing weight.

Fuel Tank Systems that Store and Burn Energy
For fuel use and storage measured by energy, there is no weight change as energy is used. The energy storage (tank) is characterized by specific energy $\epsilon_{\text{tank}}$ (MJ/kg) and energy density $\rho_{\text{tank}}$ (MJ/liter). Table 2 gives the properties of a number of energy storage systems. The tank weight and volume are obtained from the energy capacity $E_{\text{fuel-cap}}$ (MJ). The fuel weight $W_{\text{fuel}}$ is zero. A motive device has an energy flow $\dot{E}$, and its specific fuel consumption is $\text{sfc} = \dot{E} / P$ (inverse of efficiency).

Storage systems considered include batteries, capacitors, and flywheels. A battery (or capacitor) stores charge (A-hr), so the capacity is expressed as energy for a nominal voltage. Variation of the voltage with operation affects the efficiency of the relation between useful power and the rate of change of the energy stored. Efficiency of charge/discharge is accounted for in the model of the device supplying or using the energy. Each fuel tank system that stores and burns energy has a battery model for computation of the efficiency.

The fuel tank capacity $E_{\text{fuel-cap}}$ (maximum usable fuel energy) is input or determined from designated sizing missions. The fuel tank weight is $W_{\text{tank}} = E_{\text{fuel-cap}} / \epsilon_{\text{tank}}$ (lb or kg) and the fuel tank volume is $V_{\text{fuel-cap}} = E_{\text{fuel-cap}} / \rho_{\text{fuel}}$ (gallons or liters).
Table 1. Fuel properties (Refs. 4–6).

<table>
<thead>
<tr>
<th>fuel</th>
<th>specification</th>
<th>density lb/gal</th>
<th>specific energy MJ/kg</th>
<th>energy density MJ/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>gasoline</td>
<td>MIL-STD-3013A</td>
<td>6.0*</td>
<td>43.50</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.719</td>
<td>18700*</td>
<td>31.3</td>
</tr>
<tr>
<td>diesel</td>
<td>nominal</td>
<td>7.0</td>
<td>43.03</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>range</td>
<td>6.84–7.05</td>
<td>43.0</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.839</td>
<td>18500</td>
<td>36.1</td>
</tr>
<tr>
<td>JetA/A-1</td>
<td>MIL-STD-3013A</td>
<td>6.7*</td>
<td>42.80</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>range</td>
<td>6.84/6.71</td>
<td>42.8</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.803</td>
<td>18400*</td>
<td>34.4</td>
</tr>
<tr>
<td>JP-4</td>
<td>nominal</td>
<td>6.5</td>
<td>42.80</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>MIL-DTL-5624U</td>
<td>6.23–6.69</td>
<td>42.8</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>alternate design</td>
<td>6.8*</td>
<td>42.91</td>
<td>32.2</td>
</tr>
<tr>
<td>JP-5</td>
<td>MIL-STD-3013A</td>
<td>6.6*</td>
<td>42.57</td>
<td>33.7</td>
</tr>
<tr>
<td></td>
<td>alternate design</td>
<td>6.8*</td>
<td>42.91</td>
<td>35.0</td>
</tr>
<tr>
<td>JP-8</td>
<td>MIL-STD-3013A</td>
<td>6.5*</td>
<td>42.80</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>alternate design</td>
<td>6.8*</td>
<td>43.19</td>
<td>35.2</td>
</tr>
<tr>
<td></td>
<td>MIL-DTL-83133H</td>
<td>6.45–7.01</td>
<td>42.8</td>
<td>34.6</td>
</tr>
<tr>
<td>hydrogen (700 bar)</td>
<td></td>
<td>0.328</td>
<td>120.</td>
<td>4.72</td>
</tr>
<tr>
<td>hydrogen (liquid)</td>
<td></td>
<td>0.592</td>
<td>120.</td>
<td>8.52</td>
</tr>
</tbody>
</table>

*specification value

Table 2. Energy storage properties.

<table>
<thead>
<tr>
<th></th>
<th>tank specific energy MJ/kg</th>
<th>tank energy density MJ/L</th>
<th>efficiency kW/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>lead-acid battery</td>
<td>0.11–0.14</td>
<td>0.22–0.27</td>
<td>70–90%</td>
</tr>
<tr>
<td>nickel-cadmium battery</td>
<td>0.14–0.20</td>
<td>0.18–0.54</td>
<td>70–90%</td>
</tr>
<tr>
<td>lithium-ion</td>
<td>0.54–0.90</td>
<td>0.90–1.30</td>
<td>-99%</td>
</tr>
<tr>
<td>state-of-art</td>
<td></td>
<td>250–360</td>
<td>1.80</td>
</tr>
<tr>
<td>+5 years</td>
<td>1.26</td>
<td>1.80</td>
<td>500</td>
</tr>
<tr>
<td>+10 years</td>
<td>2.34</td>
<td>2.25</td>
<td>625</td>
</tr>
<tr>
<td>ultracapacitor</td>
<td>0.01–0.11</td>
<td>0.02–0.16</td>
<td>1.00</td>
</tr>
<tr>
<td>flywheel</td>
<td>steel</td>
<td>0.11</td>
<td>-90%</td>
</tr>
<tr>
<td>graphite</td>
<td>0.90</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4. Propulsion group power flow.
**PROPULSION GROUP**

The propulsion group is a set of components and engine groups connected by a drive train. The components (rotors) define the power required. The engine groups define the power available. Figure 4 illustrates the power flow. Power available is evaluated starting from engine installed power, subtracting installation losses and implementing a mechanical limit (engine installed power), accounting for inoperative engines, a power factor for margins, and the engine shaft rating (engine group power), summing over all engine groups and implementing a drive train torque limit (propulsion group power available), subtracting transmission losses and accessory power, and implementing a rotor shaft rating (power available to the rotor). Power required is evaluated starting from component ( rotor) power required, summing over all components and adding transmission losses and accessory power (propulsion group power required), distributing power to engine groups, accounting for inoperative engine (engine installed power required), and adding installation losses (engine uninstall ed power). See Refs. 1–2 for more details.

The drive train defines gear ratios for all the components that it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive train state, in order to model a multiple-speed or variable-speed transmission. Each drive train state corresponds to a set of gear ratios.

The flight state specifies the tip speed of the primary rotor and the drive train state, for each propulsion group. The drive train state defines the gear ratio for dependent rotors and the engine groups. From the rotor radius the rotational speed of the primary rotor is obtained; from the gear ratios, the rotational speed of dependent rotors and the engine groups are obtained; and from the rotor radius, the tip speed of the dependent rotor is obtained.

The component power required $P_{comp}$ is evaluated for a specified flight condition, as the sum of the power required by all the components of the propulsion group. The total power required for the propulsion group is obtained by adding the transmission losses and accessory power: $P_{reqPG} = P_{comp} + P_{a} + P_{acc}$.

The power required for the propulsion group must be distributed to the engine groups. With only one engine group, $P_{reqEG} = P_{reqPG}$. An engine group power can be fixed at an input amplitude, or at a fraction of engine power available, or at a fraction of engine rated power. The power required for the remaining (perhaps all) engine groups is distributed proportional to the engine rated power.

The drive train rating is defined as a power limit, $P_{DS\, \text{limit}}$. The rating is properly a torque limit, $Q_{DS\, \text{limit}} = P_{DS\, \text{limit}}/\Omega$, but is conventionally expressed as a power limit. The drive train rating is a limit on the entire propulsion group. To account for differences in the distribution of power through the drive train, limits are also used for the torque of each rotor shaft and of each engine group.

**ENGINE GROUP**

The engine group consists of one or more engines of a specific type. An engine group transfers power by shaft torque, so is associated with a propulsion group. For each engine type an engine model is defined. The engine model describes a particular engine, used in one or more engine groups. The models include turboshaft engines (perhaps convertible, for turbojet operation or reaction drive), reciprocating engines, compressors, electric motors (perhaps with fuel cells), electric generators, and generator-motors.

The engine size is described by the power $P_{eng}$, which is the sea-level static power available per engine at a specified takeoff rating. The number of engines $N_{eng}$ is specified for each engine group.

The propulsion group power available is obtained from the sum over the engine groups: $P_{avPG} = \sum P_{avEG}$. The propulsion group power required $P_{reqPG}$ consists of the component power, transmission losses, and accessory power. The component power $P_{comp}$ includes compressor power, generator power, and generator-motor power when it is producing energy.

The installed power required $P_{req}$ and power available $P_{a}$ are measured at the engine output shaft. In addition to shaft power, the engine exhaust produces a net jet thrust $F_{N}$, from mass flow that goes through the engine core. The fuel flow and mass flow are the total required to produce the shaft power and jet thrust. The forces produced by mass flow that does not go through the engine core (such as IR suppressor or cooling air) are treated as momentum drag $D_{aux}$.

In general, the engine performance is described by the uninstall ed power available $P_{a}$, at each engine rating and
the specification engine speed $N_{\text{spec}}$; the mass flow $\dot{m}$ and fuel flow $\dot{w}$ required to produce uninstalled power required $P_q$ at engine turbine speed $N$; and the gross jet thrust $F_g$ at $P_q$. The difference between net and gross jet thrust is the momentum drag:

$$F_n = F_g - \dot{m}_{\text{req}} V = \dot{m}_{\text{req}} (V_j - V)$$

where $V_j$ is the engine jet exhaust velocity. The specific power is $SP = P/\dot{m}$, the specific thrust is $ST = F_g/\dot{m}$, and the specific fuel consumption is here $\text{sfc} = \dot{w}/P$.

### Turboshaft Engine

Turboshaft engine performance is obtained from the Referred Parameter Turboshaft Engine Model (RPTEM).

#### Power Available

Given the flight condition and engine rating, the power available $P_a$ is calculated from the specific power $SP_a = P_a/\dot{m}_a$ and mass flow $\dot{m}_a$:

$$\frac{SP_a}{\theta} = SP_0 g_{sp} (\theta, M, n)$$

$$\frac{\dot{m}_a}{\delta/\sqrt{\theta}} = \dot{m}_{0} g_{m} (\theta, M, n)$$

$$\frac{P_a}{\delta/\sqrt{\theta}} = P_{0} g_{p} (\theta, M, n)$$

as functions of temperature ratio $\theta = T/T_0$, Mach number $M$, and referred engine turbine speed $n = N/\sqrt{\theta}$. Installation losses are subtracted ($P_{av} = P_a - P_{\text{loss}}$), and the mechanical limit is applied.

The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines), including a specified power fraction $f_P$: $P_{avEG} = f_P (N_{\text{eng}} - N_{\text{inop}}) P_{av}$ (typically to implement a power margin).

#### Performance at Power Required

The power required of a single engine is obtained from the engine group power: $P_{req} = P_{reqEG} / (N_{\text{eng}} - N_{\text{inop}})$; and installation losses are added, $P_q = P_{req} + P_{\text{loss}}$. The engine performance is calculated for a specified power required $P_q$ and flight condition:

$$\frac{\dot{m}_{req}}{\delta/\sqrt{\theta}} = \dot{m}_{0c} g_m (q, \theta, M, n)$$

$$\frac{\dot{w}_{req}}{\delta/\sqrt{\theta}} = \dot{w}_{0c} g_w (q, \theta, M, n)$$

$$\frac{F_g}{\delta} = F_{g0c} g_f (q, \theta, M, n)$$

as functions of $q = P_q/(P_{0c} \delta \sqrt{\theta})$, $\theta = T/T_0$, $M$, and $n = N/\sqrt{\theta}$.

The performance of the engine group is:

$$\dot{m}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}}) \dot{m}_{req}$$

$$\dot{w}_{reqEG} = (N_{\text{eng}} - N_{\text{inop}}) \dot{w}_{req} K_{f\text{fd}}$$

$$F_{N\text{EG}} = (N_{\text{eng}} - N_{\text{inop}}) F_N$$

$$D_{auxEG} = (N_{\text{eng}} - N_{\text{inop}}) D_{aux}$$

The fuel flow has been multiplied by a factor $K_{f\text{fd}}$ accounting for deterioration of the engine efficiency.

#### Installation

The difference between installed and uninstalled power is the inlet and exhaust losses $P_{\text{loss}}$. The installed gross jet thrust is $F_G = K_{f\text{rg}} F_g$, where $K_{f\text{rg}}$ accounts for exhaust effects. The net jet thrust is $F_N = F_G - \dot{m}_{req} V$. The momentum drag of the auxiliary air flow is a function of the mass flow $\dot{m}_{aux} = f_{aux} \dot{m}_{req}$:

$$D_{aux} = (1 - \eta_{aux}) \dot{m}_{aux} V = (1 - \eta_{aux}) f_{aux} \dot{m}_{req} V$$

where $\eta_{aux}$ is the ram recovery efficiency.

#### Convertible Engine: Turbojet/Turbofan

The engine mode $B$ is the mass flow fraction diverted for a convertible engine: $B = 0$ for all mass flow to the power turbine (turboshaft operation), and $B = 1$ for all mass flow to the jet exhaust or a fan (turbojet/turbofan operation). A separate engine model defines the performance for turbojet/turbofan operation (not all parameters of which are used; in particular, there is only one value for the size $P_{\text{eng}}$). The engine group power $P_{reqEG}$ is a measure of the jet thrust, and this engine does not contribute to the propulsion group shaft power available. The turbojet/turbofan thrust is the engine group net jet thrust, $F_N = F_G - \dot{m}_{req} V$.

#### Convertible Engine: Reaction Drive

The engine mode $B$ is the mass flow fraction diverted for a convertible engine: $B = 0$ for all mass flow to the power turbine (turboshaft operation), and $B = 1$ for all mass flow to the rotor (reaction jet operation). A separate engine model defines the engine performance for reaction jet operation (not all parameters of which are used; in particular, there is only one value for the size $P_{\text{eng}}$). The engine group power is obtained from the rotor power ($P_{reqEG} = P_{\text{react}}$), and this engine does not contribute to the propulsion group shaft power available. The gross jet thrust is zero, so the net thrust is the momentum drag, $F_N = -\dot{m}_{req} V$. 

9
Reciprocating Engine

Reciprocating engine performance is described in Ref. 7. The work per cycle is \( W = P n_c / R \), where \( R \) is the rotational speed (rev/sec), \( n_c \) is the revolutions per cycle (2 for a 4-stroke engine, 1 for a 2-stroke engine), and \( R / n_c \) is the cycles per second. The engine displacement is \( V_d \) (volume). The mean effective pressure is defined as

\[
\text{mep} = \frac{W}{V_d} = \frac{P n_c / R}{V_d} = \frac{Q n_c 2\pi}{V_d}
\]

from the power \( P = NQ = 2\pi RQ \); so mep is the specific torque. The engine output is the brake horsepower (BHP), which equals indicated power (IHP) less friction power (FHP):

\[
\text{BHP} = \text{IHP} - \text{FHP} = \text{IHP} - (\text{MHP} + \text{PHP} + \text{CHP} + \text{AHP} - \text{THP})
\]

The friction power is composed of losses due to mechanical friction (MHP), pumping (PHP, the work of the piston during inlet and exhaust strokes), compressor or supercharger (CHP), auxiliary or accessories (AHP, such as oil pump, water pump, cooling fan, generator), and exhaust turbine (THP, treated as negative friction). The mechanical efficiency is \( \eta = \text{BHP}/\text{IHP} \). The sum of airflow and fuel flow is the charge flow: \( \dot{w}_c = \dot{w}_f + \dot{w}_a \). The fuel-air ratio is \( F = \dot{w}_f / \dot{w}_a \). The mass flow is \( \dot{m} = e_s \dot{m}_{\text{ideal}} = e_s \rho V_d (R / n_c) \); so mep \( \propto (P / \dot{m}) \rho \). The indicated power is the product of the fuel flow, fuel specific energy \( (e_{\text{fuel}} = JQ_c \), from the heat of combustion \( Q_c \) and Joule's constant \( J \) relating work and heat), and thermal efficiency \( P_I = e_{\text{fuel}} \eta_{\text{fg}} \dot{w} \). These equations have constant factors when conventional units are used.

The maximum brake mean effective pressure is typically 125–250 lb/in\(^2\) (850–1700 kN/m\(^2\)). Typical brake specific fuel consumption is \( \text{bsfc} = \dot{w} / P = 0.38 \) to 0.45 lb/hp-hr. Typically the volumetric efficiency \( e_v = \dot{m} / \dot{m}_{\text{ideal}} \) = 0.8 to 0.9.

The power available \( P_a \) is calculated from the specific power \( \text{SP}_a = P / \dot{m}_a \) and mass flow \( \dot{m}_a \):

\[
\frac{\text{SP}_a}{\dot{m}} = \frac{P \rho g_{sp}}{\dot{m}} \propto \text{mep} / \dot{m}_a
\]

\[
\frac{\dot{m}_a}{\dot{m}} = \frac{\dot{m}_{0g_m}}{\dot{m}_a} = \frac{N / N_{\text{spec}}}{\sqrt{\theta}}
\]

Installation losses are subtracted from \( P_a \), and the mechanical limit is applied. The engine performance at \( P_q \) is:

\[
\frac{\dot{m}_{\text{req}}}{\dot{m}} = \frac{\dot{m}_{0g_m}}{\dot{m}_0} = \frac{N / N_{\text{spec}}}{\sqrt{\theta}}
\]

\[
\frac{\dot{w}_{\text{req}}}{\dot{m}} = \dot{w}_{0g_m} = \dot{w}_{0C} q
\]

\[
\frac{F_g}{\delta} = F_{g0C} q_f = 0
\]

where \( q = P_q / (P_{0C} \delta / \sqrt{\theta}) \).

Compressor

A compressor converts input shaft power to a jet velocity and thrust. The shaft power contributes to the propulsion group power required. The compressor does not use fuel.

Given the flight condition and engine rating, the power available \( P_a \) is calculated from the specific power \( \text{SP}_a = P / \dot{m}_a \) and mass flow \( \dot{m}_a \):

\[
\frac{\text{SP}_a}{\dot{m}} = \text{SP}_a \text{g}_{sp}(\theta, M, n)
\]

\[
\frac{\dot{m}_a}{\dot{m}} = \dot{m}_{0g_m}(\theta, M, n)
\]

\[
\frac{P}{\delta} = P_{0g_f}(\theta, M, n)
\]

as functions of \( \theta = T / T_0 \), \( M \), and referred compressor speed \( n = N / \sqrt{\theta} \). The power available of the engine group is \( P_{\text{EG}} = f_p (N_{\text{eng}} - N_{\text{inop}}) P_{\text{av}} \), including a specified power fraction \( f_p \).

The power required of a single compressor is \( P_{\text{req}} = P_{\text{reqEG}} / (N_{\text{eng}} - N_{\text{inop}}) \). Accounting for installation losses gives the uninstalled power required \( P_q = P_{\text{req}} + P_{\text{loss}} \). The compressor performance at \( P_q \) is:

\[
\frac{\dot{m}_{\text{req}}}{\dot{m}} = \dot{m}_{0g_m}(q, \theta, M, n)
\]

\[
\frac{\text{ST}_{\text{req}}}{\sqrt{\theta}} = \text{ST}_{\text{0g}_0}(q, \theta, M, n)
\]

\[
\frac{F_g}{\delta} = F_{g0C} q_f(q, \theta, M, n)
\]

as functions of \( q = P_q / (P_{0C} \delta / \sqrt{\theta}) \), \( \theta = T / T_0 \), \( M \), and \( n = N / \sqrt{\theta} \). The specific thrust gives the gross thrust, \( F_g = (\text{ST}) \dot{m} \). The performance of the engine group is:

\[
\dot{m}_{\text{reqEG}} = (N_{\text{eng}} - N_{\text{inop}}) \dot{m}_{\text{req}}
\]

\[
F_{\text{NG}} = (N_{\text{eng}} - N_{\text{inop}}) F_N
\]

\[
D_{\text{auxEG}} = (N_{\text{eng}} - N_{\text{inop}}) D_{\text{aux}}
\]

\[
P_{\text{comp}} = (N_{\text{eng}} - N_{\text{inop}}) P_q K_{ffd}
\]
The component power is the product of the uninstalled power required and the number of operational engines, and a factor $K_{fd}$ accounting for deterioration of the engine efficiency.

**Compressor for Reaction Drive**

If the compressor supplies the jet force for rotor reaction drive, then the engine group power required is obtained from the rotor power: $P_{reqEG} = P_{react}$. The gross jet thrust is zero, so the net thrust is the momentum drag, $F_N = -\dot{m}_{req}V$.

**Electric Motor or Generator**

A motor converts electrical energy (fuel) to shaft power. A generator converts input shaft power to electrical energy. The uninstalled shaft power contributes to the propulsion group power required.

The power available is related to the size $P_{eng}$. Given the flight condition, the uninstalled power available $P_a$ is calculated. The installed power available is $P_{av} = P_a \eta_i$, where $\eta_i$ is the efficiency. Then the mechanical limit is applied. The power available of the engine group is $P_{reqEG} = f_p (N_{eng} - N_{inop}) P_{av}$, including a specified power fraction $f_p$.

From the engine group power required $P_{reqEG}$, the power required of a single engine is $P_{req} = P_{reqEG}/(N_{eng} - N_{inop})$. The uninstalled power required is $P_q = P_{req}/\eta_i$.

**Motor**

The motor power required determines the energy flow from the fuel tank. The energy flow is calculated for $P_q$ and a specified flight condition: $\dot{E}_{req} = \dot{E}_{oc} g_c (q,n)$ as a function of $q = P_q/P_{eng}$ and engine speed $n = N/N_{spec}$. The performance of the engine group is $\dot{E}_{reqEG} = (N_{eng} - N_{inop}) \dot{E}_{req} K_{fd}$. The energy flow has been multiplied by a factor $K_{fd}$ accounting for deterioration of the engine efficiency.

**Generator**

The generator energy flow to the fuel tank defines the power required. The energy flow is calculated for $P_q$ and a specified flight condition: $\dot{E}_{req} = \dot{E}_{oc} g_s (q,n)$ as a function of $q = P_q/P_{eng}$ and engine speed $n = N/N_{spec}$. The performance of the engine group is

$$\dot{E}_{reqEG} = (N_{eng} - N_{inop}) \dot{E}_{req}$$

$$\dot{P}_{comp} = (N_{eng} - N_{inop}) P_q K_{fd}$$

The component power is the product of the uninstalled power required and the number of operational engines, and a factor $K_{fd}$ accounting for deterioration of the engine efficiency.

**Generator-Motor**

The engine mode $B$ is the direction of power flow for a generator-motor: $B$ positive for motor operation, and $B$ negative for generator operation. Separate motor models are used for the two modes (not all parameters of which are used; in particular, there is only one value for the size $P_{eng}$).

**Motor and Fuel Cell**

A motor with a fuel cell burns a fuel (typically hydrogen) to produce electrical energy, which is converted to shaft power. The device can also be modeled as separate motor and fuel cell components, with a battery (fuel tank) to transfer the electrical energy.

The engine performance is calculated for $P_q$ and a specified flight condition:

$$\dot{m}_{req} = \dot{m}_{oc} g_m (q,n) = K_{mf} \dot{w}_{req}$$

$$\dot{w}_{req} = \dot{w}_{oc} g_w (q,n) = sfc P_q$$

as a function of $q = P_q/P_{eng}$ and $n = N/N_{spec}$. The net thrust is the inlet momentum drag, $F_N = -\dot{m}_{req} V$. The performance of the engine group is

$$\dot{m}_{reqEG} = (N_{eng} - N_{inop}) \dot{m}_{req}$$

$$\dot{w}_{reqEG} = (N_{eng} - N_{inop}) \dot{w}_{req} K_{fd}$$

$$F_{N EG} = (N_{eng} - N_{inop}) F_N$$

$$D_{auxEG} = (N_{eng} - N_{inop}) D_{aux}$$

The fuel flow has been multiplied by a factor $K_{fd}$ accounting for deterioration of the engine efficiency.

**JET GROUP**

A jet group produces a force on the aircraft, possibly used for lift, propulsion, or control. A jet model describes a particular jet, used in one or more jet groups. The models include turbojet and turbofan engines (perhaps convertible, for reaction drive), reaction drive, and a simple force. A reaction drive supplies a blade force that provides the rotor power required.

The jet size is described by the thrust $T_{jet}$, which is the sea-level static thrust available per jet at a specified takeoff rating. The number of jets $N_{jet}$ is specified for each jet group.

The uninstalled thrust required is $T_q$, and the thrust available $T_a$. The jet model calculates $T_a$ as a function of flight condition and engine rating; or calculates mass flow
and fuel flow at $T_q$. The specific thrust is $ST = T/m$, and the specific fuel consumption is here $sfc = \dot{w}/T$. The forces produced by mass flow that does not go through the core or fan are treated as momentum drag $D_{aux}$.

**Turbojet or Turbofan**

The thrust of a turbojet is

$$ T = \dot{m}(1+f)V_e - V + (p_e - p_{aman})A_e $$

where $\dot{m}$ is the mass flow; $f = \dot{w}/m$ is the fuel-air ratio; and $V_e$, $p_e$, $A_e$ are the velocity, pressure, and area at the exit. The pressure term is zero or small, and the fuel-air ratio is small, so the net thrust is approximately $T = \dot{m}(V_e - V)$, from the gross thrust $T_G = \dot{m}V_e$ and the inlet-momentum or ram drag $\dot{m}V$.

The thrust of a turbofan is

$$ T = \dot{m}(1+f)V_e - V + \dot{m}_{fan}(V_{fan} - V) $$

$$ = \dot{m}(1+f)V_e + \delta V_{fan} - \dot{m}(1+\delta)V $$

with bypass ratio $\beta = \dot{m}_{fan}/\dot{m}$.

The difference between net and gross thrust is the momentum drag: $T_N = T_G - \dot{m}(ST)_{mom}$, where $(ST)_{mom} = (1+\delta)V$ for a turbojet or turbofan, or $(ST)_{mom} = \Omega_{react}$ for a reaction drive.

Turbojet or turbofan performance is obtained from the Referred Parameter Jet Engine Model (RPJEM), based on Refs. 8 and 9. The referred thrust, specific thrust $ST = T/m$, and specific fuel consumption $sfc = \dot{w}/T$ are functions of the flight Mach number $M$ and compressor speed $n = N/(N_0 \sqrt{\theta})$:

$$ \frac{T}{\delta} = G_t(M,n) $$

$$ \frac{T}{\dot{m}} = G_{st}(M,n) $$

$$ \frac{\dot{w}}{T} = \frac{1}{\eta_b} G_{ac}(M,n) $$

The independent variable can be the compressor speed, or the turbine inlet temperature, or the fuel flow. The combustion efficiency $\eta_b$ depends on the atmosphere (altitude and temperature), hence the specific fuel consumption is not a function of just $M$ and $n$.

**Thrust Available**

Given the flight condition and engine rating, the thrust available $T_a$ is calculated from the specific thrust $ST_a = T_a/\dot{m}_a$ and mass flow $\dot{m}_a$:

$$ \frac{ST_a}{\delta} = ST_{0,g_{st}}(\theta,M) $$

$$ \frac{\dot{m}_a}{\delta} = \dot{m}_{0,g_{st}}(\theta,M) $$

$$ T_a = T_{0,g_{st}}(\theta,M) $$

as functions of temperature ratio $\theta = T/T_0$ and Mach number $M$. Installation losses are subtracted ($T_{av} = T_a - T_{loss}$), and the mechanical limit is applied. The thrust available of the jet group is obtained by multiplying the single jet thrust by the number of jets operational (total number of jets less inoperative jets), including a specified thrust fraction $f_T$: $T_{av,JG} = f_T(N_{jet} - N_{inop})T_{av}$ (typically to implement a thrust margin).

**Performance at Thrust Required**

The thrust required of a single jet is $T_{req} = T_{req,JG} / (N_{jet} - N_{inop})$. Installation losses are added to get the uninstalled thrust required ($T_q = T_{req} + T_{loss}$). The jet performance is calculated for a specified thrust required $T_q$ and flight condition:

$$ \frac{\dot{m}_{req}}{\delta} = \dot{m}_{0,g_{st}}(t,\theta,M) $$

$$ \frac{\dot{w}_{req}}{\delta} = \dot{w}_{0,g_{st}}(t,\theta,M) $$

as functions of $t = T_q/(T_{0,c} \delta)$ (or referred gross thrust), $\theta = T/T_0$, and $M$. The performance of the jet group is:

$$ \dot{m}_{req,JG} = (N_{jet} - N_{inop})\dot{m}_{req} $$

$$ \dot{w}_{req,JG} = (N_{jet} - N_{inop})\dot{w}_{req}K_{fhd} $$

$$ D_{aux,JG} = (N_{jet} - N_{inop})D_{aux} $$

The fuel flow has been multiplied by a factor $K_{fhd}$ accounting for deterioration of the jet efficiency.

**Convertible Engine: Reaction Drive**

The jet mode $B$ is the mass flow fraction diverted for a convertible engine: $B = 0$ for all mass flow to the exhaust (turbojet operation), and $B = 1$ for all mass flow to the rotor (reaction jet operation). A separate jet model defines the engine performance for reaction jet operation (not all parameters of which are used; in particular, there is only one value for the size $T_{req}$). The jet thrust required is $T_{req,JG} = F_{react} = P_{rotor} / \Omega_{react}$. The mass flow and fuel flow follow. The jet group net thrust is the inlet momentum drag, $F_N = -\dot{m}_{req}V$. 

12
Reaction Drive

The jet group performance for reaction drive includes the blade duct and nozzle, perhaps even with tip burning. The rotor power required \( P_{\text{rotor}} \) gives the required force on the rotor blade \( F_{\text{react}} = P_{\text{rotor}} / \Omega_{\text{react}} \) at effective radial station \( r_{\text{react}} \). The net jet group thrust required is \( T_{\text{reqJG}} = F_{\text{react}} - \dot{m}_{\text{reqJG}} \Omega_{\text{react}} \). From \( P_{\text{reqJG}} \), the jet performance (mass flow and fuel flow) is calculated. The jet group net thrust is the inlet momentum drag, \( F_N = -\dot{m}_{\text{req}} V \).

Simple Force

For the simple force model, the design maximum thrust is \( T_{\text{max}} \) (per jet). The thrust available is thus \( T_u = T_{\text{max}} \). The force generation can use fuel as weight or as energy.

If the component burns fuel weight, the fuel flow is calculated from an input thrust-specific fuel consumption: \( \dot{w}_{\text{req}} = T_q (\text{sfc}) = \dot{w}_{\text{OC}} q \), where \( q = T_q / T_{\text{max}} \). Then \( \dot{w}_{\text{reqJG}} = (N_{\text{jet}} - N_{\text{inop}}) \dot{w}_{\text{req}} K_{\text{JG}} \).

If the component uses fuel energy, the energy flow is calculated from an input thrust-specific fuel consumption: \( E_{\text{req}} = T_q (\text{sfc}) = \dot{E}_{\text{OC}} q \), where \( q = T_q / T_{\text{max}} \). Then \( E_{\text{reqJG}} = (N_{\text{jet}} - N_{\text{inop}}) E_{\text{req}} K_{\text{JG}} \).

The simple force weight is calculated from specific weight \( S \) plus a fixed increment: \( W = ST_{\text{max}} + \Delta W \).

**CHARGE GROUP**

A charge group generates energy for the aircraft. A charger model describes a particular charger, used in one or more charge groups. The models include fuel cells and solar cells.

The charger size is described by the power \( P_{\text{chrg}} \), which is the sea-level static power available per charger at a specified takeoff rating. The number of chargers \( N_{\text{chrg}} \) is specified for each charge group.

Given the flight condition and the charger rating, the cell power available is \( E_{\text{cell}} = P_0 \). The total cell power available is obtained by multiplying the single charger power by the number of chargers operational (total number of chargers less inoperable chargers):

\[
P_{\text{reqtotal}} = f_P (N_{\text{chrg}} - N_{\text{inop}}) \dot{E}_{\text{cell}}
\]

including a specified power fraction \( f_P \).

The energy flow to the fuel tank gives the charge group power required: \( P_{\text{reqCG}} = \dot{E}_{\text{reqCG}} \). The energy flow required of a single charger is

\[
\dot{E}_{\text{req}} = \dot{E}_{\text{reqCG}} K_{\text{JG}} (N_{\text{chrg}} - N_{\text{inop}})
\]

The energy flow has been multiplied by a factor \( K_{\text{JG}} \) accounting for deterioration of the charger efficiency. The uninstalled energy flow required is \( \dot{E}_{\text{in}} = \dot{E}_{\text{req}} / \eta_s \), where \( \eta_s \) is the installation efficiency. The cell energy flow required \( \dot{E}_{\text{cell}} \) is obtained from the uninstalled energy flow. Then \( P_{\text{reqtotal}} = (N_{\text{chrg}} - N_{\text{inop}}) \dot{E}_{\text{cell}} \) is the total cell power required.

**Fuel Cell**

A fuel cell burns a fuel (typically hydrogen) and generates electrical energy. The cell energy flow is calculated for \( \dot{E}_{\text{qcell}} \) and a specified flight condition: \( \dot{E}_{\text{qcell}} = \dot{E}_0 g_s(q) \), where \( q = E_{\text{q}} / P_0 \). The fuel cell performance is obtained from \( \dot{E}_{\text{qcell}} \), or

\[
\dot{m}_{\text{req}} = \dot{m}_{\text{OC}} g_s(q)
\]

\[
\dot{w}_{\text{req}} = \dot{w}_{\text{OC}} g_w(q)
\]

The net thrust is the inlet momentum drag, \( F_N = -\dot{m}_{\text{req}} V \). The performance of the charge group is

\[
\dot{m}_{\text{reqCG}} = (N_{\text{chrg}} - N_{\text{inop}}) \dot{m}_{\text{req}}
\]

\[
\dot{w}_{\text{reqCG}} = (N_{\text{chrg}} - N_{\text{inop}}) \dot{w}_{\text{req}}
\]

\[
F_N \text{CHG} = (N_{\text{chrg}} - N_{\text{inop}}) F_N
\]

\[
D_{\text{auxCG}} = (N_{\text{chrg}} - N_{\text{inop}}) D_{\text{aux}}
\]

The momentum drag of the auxiliary air flow is a function of the mass flow \( \dot{m}_{\text{aux}} = f_{\text{aux}} \dot{m}_{\text{req}} \):

\[
D_{\text{aux}} = (1 - \eta_{\text{aux}}) \dot{m}_{\text{aux}} V = (1 - \eta_{\text{aux}}) f_{\text{aux}} \dot{m}_{\text{req}} V
\]

where \( \eta_{\text{aux}} \) is the ram recovery efficiency.

**Solar Cell**

The power available from solar radiation is approximately 1.36 kW/m², reduced by atmospheric effects (absorption, reflection, scattering) to about 1.00 kW/m². The average solar radiation in the continental United States is 3.5–7.0 (kW/m²)(hour/day), hence approximately 15 to 25% of the available power. Typical efficiencies of solar cells are 10–20%.

\[
\dot{E}_{\text{qcell}} = \dot{E}_0 g_s(q) \]

where \( q = E_{\text{q}} / P_0 \).

**COMPONENT MODELS**

The engine group, jet group, and charge group provide a general framework for the propulsion system components. For each group, an engine, jet, or charger model is used to
evaluate performance and weight. A model can be used by more than one group. The performance of each component is described by a number of quantities (g) that depend on power or thrust, atmosphere, speed, and other parameters. A component model has a specific functional form for each quantity, typically piecewise linear or polynomial functions of the independent parameters. The objective is to find a parameterized, surrogate representation of the component performance and weight, applicable to a wide range of operating conditions and component size.

The engine group models implemented are the Referred Parameter Turboshaft Engine Model (RPTEM), compressor model, and motor/generator model. The jet group model implemented is the Referred Parameter Jet Engine Model (RPJEM), in addition to the simple force model. The charge group models implemented are fuel cell and solar cell models. There is also a battery model, associated with fuel tank systems that store and burn energy.

**REFERRED PARAMETER TURBOSHAFT ENGINE MODEL**

Aircraft gas turbine engine performance capabilities are formally specified by computer programs known as engine decks, which are created by engine manufacturers in an industry-standard format. Engine decks are typically based on thermodynamic cycle analysis using real engine component performance maps. The most important performance maps for turboshaft engines are compressor, gas generator turbine, and power turbine. These component performance maps are critical to obtaining realistic off-design engine performance. Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. Thus engine performance must be realistic even far from the engine design point. A simple thermodynamic cycle analysis that assumes design point component efficiencies everywhere is not realistic for such an application. Rather than developing models for component performance, the approach taken is to use a model for the total engine performance. The engine is not being designed.

The Referred Parameter Turboshaft Engine Model (RPTEM) is based on curve-fits of performance data for existing or projected engines over a range of operating conditions. The curve-fits are typically obtained by exercising an engine deck. The use of referred parameters tends to collapse the data, and provides a basis for scaling the engine. The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, power turbine speed, exhaust nozzle area, and either engine rating or engine power required. These curve-fits, typically based on real engines, are scaled to the required size and adjusted to the appropriate technology level to represent a notional engine. Engine size is represented by mass flow. Engine technology is represented by specific power available and specific fuel consumption at maximum continuous power (MCP), sea level standard day (SLS), static (zero airspeed) conditions. Engine installation effects (inlet and exhaust losses) are also modeled.

The power available \( P_q \) is calculated for the flight condition and engine rating. The specific power and referred mass flow (at \( N_{\text{spec}} \), relative to \( SP_0 \) and \( m_0 \) for this rating) are approximated by functions of the ambient temperature ratio \( \theta \) and inlet ram air ratios:

\[
SP_q(N_{\text{spec}}) = SP_0 \theta K_{\text{spec}} \left[ \delta M \sqrt{\theta_M} \right]^{X_{\text{spa}}}
\]

\[
m_q(N_{\text{spec}}) = m_0 (\delta / \sqrt{\theta}) e^{K_{\text{mfa}} \left[ \delta M \sqrt{\theta_M} \right]^{X_{\text{mfa}}}}
\]

where the static lapse rate \((K_{\text{spa}}, K_{\text{mfa}})\) and ram air exponents \((X_{\text{spa}}, X_{\text{mfa}})\) are piecewise linear functions of \( \theta \). The power available is then \( P_q = SP_q m_q \). The uninstalled power available at \( N \) is calculated, installation losses are subtracted, and the mechanical limit is applied.

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for the power required \( P_q \), flight condition, and engine rating. Uninstalled power at specification turbine speed \( P_q(N_{\text{spec}}) \) is obtained from \( P_q(N) \). The referred quantities (relative to SLS static MCP quantities) are approximated by cubic functions of \( q = P_q(N_{\text{spec}})/(P_{0C} e^{\delta f (\theta)}) \):

\[
\dot{w}_{\text{req}} = \dot{w}_{0C} (\delta f (\theta))(K_{f_{\text{flq}}} + K_{f_{\text{flq}}q})
\]

\[
\dot{m}_{\text{req}} = m_{0C} (\delta / \sqrt{\theta})(K_{m_{\text{flq}}} + K_{m_{\text{flq}}q})
\]

\[
F_g = F_{g0C} \delta(K_{f_{\text{flq}}} + K_{f_{\text{flq}}q})
\]

at \( N_{\text{spec}} \), with \( \dot{w}_{0C} = sc_{\text{Wc}} P_{0C} \). The mass flow and fuel flow are primarily functions of the gas power, and are assumed to be independent of turbine speed. Then the installed net jet thrust \( F_N \) and momentum drag \( D_{\text{aux}} \) are calculated.

The parameters of the engine model can be defined for a specific engine, but scaling the parameters as part of the aircraft sizing task is also necessary, in order to define an engine for a specified power. In addition, advanced
technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific fuel consumption (at SLS static conditions, MCP, and \( N_{\text{spec}} \)).

The engine weight can be calculated as a function of power, or scaled with engine mass flow. As a function of power, the weight of one engine is

\[
W_{\text{eng}} = K_{0\text{eng}} + K_{1\text{eng}} P + K_{2\text{eng}} P X_{\text{weng}}
\]

where \( P \) is the installed takeoff power per engine. Alternatively, the specific weight \( SW = P/W \) can be scaled with the mass flow \( m_{\text{gc}} \).

**COMPRESSOR MODEL**

A compressor converts input shaft power to a jet velocity and thrust. The shaft power contributes to the propulsion group power required. The compressor does not use fuel.

The operating condition is described by pressure altitude, ambient air temperature, flight Mach number, and either compressor rating or power required. The parametric model is scaled to the required size and adjusted to the appropriate technology level to represent a notional compressor. Compressor size is represented by mass flow. Technology is represented by specific power available at maximum continuous power (MCP), sea level standard day (SLS), static (zero airspeed) conditions.

The specific power and referred mass flow (relative to \( SP_0 \) and \( m_0 \) for this rating) are approximated by functions of the ambient temperature ratio \( \theta \), here just:

\[
SP_a = SP_0 \left[ \delta M \sqrt{\theta_M} \right]^{X_{\text{spe}}} \\
m_a = m_0 \left( \delta / \sqrt{\theta} \right) \left[ \delta M \sqrt{\theta_M} \right]^{X_{\text{spe}}}
\]

The power available is then \( P_a = SP_a m_a \). The influence of compressor rotational speed is not considered.

The compressor performance (mass flow and gross jet thrust) is calculated for a specified power required \( P_q \), flight condition, and rating. The referred quantities (relative to SLS static MCP quantities) are approximated by functions of \( q = P_q / (P_{\text{gc}} \delta \sqrt{\theta}) \):

\[
\dot{m}_{\text{req}} = \dot{m}_{\text{gc}} (\delta \sqrt{\theta}) (K_{mfq^2} + K_{mfq^1} q) + K_{mfq^2} q^2 + K_{mfq^3} q^3 \left[ \theta_M \right]^{X_{\text{wq}}} \\
ST_{\text{req}} = ST_{\text{gc}} \sqrt{\theta} \left[ \theta_M \right]^{X_{\text{wq}}}
\]

The gross jet thrust is then

\[
F_G = ST_{\text{req}} \dot{m}_{\text{req}} = F_{\text{gc}} \delta (K_{mfq^2} + K_{mfq^1} q) + K_{mfq^2} q^2 + K_{mfq^3} q^3 \left[ \theta_M \right]^{X_{\text{wq}}} + X_{\text{wq}}
\]

Then the installed net jet thrust \( F_N \) and momentum drag \( D_{\text{aux}} \) are calculated. The influence of compressor rotational speed is not considered.

The compressor weight can be calculated as a function of power:

\[
W_{\text{comp}} = K_{0\text{comp}} + K_{1\text{comp}} P + K_{2\text{comp}} P X_{\text{wcomp}}
\]

where \( P \) is the installed power (SLS static, specified rating) per compressor.

**MOTOR MODEL**

A motor converts electrical energy (fuel) to shaft power. A generator converts input shaft power to electrical energy. The model follows Refs. 10 to 12.

The power available is \( P_a = P_{\text{eng}} \). The motor power required determines the energy flow: \( E_{\text{req}} = P_a \eta \), where the efficiency \( \eta = \eta_{\text{ batt}} \eta_{\text{motor}} \) includes the battery discharging losses. The generator energy flow to the fuel tank is related to the power required: \( E_{\text{req}} = P_a \eta \), where the efficiency \( \eta = \eta_{\text{ batt}} \eta_{\text{motor}} \) includes the battery charging losses.

For a motor and fuel cell, the performance is calculated for a specified power required:

\[
\dot{m}_{\text{req}} = \dot{m}_{\text{gc}} (q / \eta) = K_{mf} \dot{w}_{\text{req}} \\
\dot{w}_{\text{req}} = \dot{w}_{\text{gc}} (q / \eta) = s \dot{F}c_{\text{gc}} (P_q / \eta)
\]

The efficiency \( \eta = \eta_{\text{cell}} \eta_{\text{motor}} \) includes both motor and fuel cell losses. The fuel cell efficiency as a function of power is

\[
1 / \eta_{\text{cell}} = 1 + P / P_{\text{ref}} - (1) + c / \eta + P_{\text{eng}} / P
\]

The ratio of mass flow and fuel flow (\( K_{mf} \)) follows from the chemistry of the reaction.

Motor loss sources include copper (internal resistance, proportional to current-squared hence torque-squared), iron core (eddy current and hysteresis, proportional to rotational speed), and mechanical (friction, proportional to speed, and windage, proportional to speed-cubed). The power loss is described as a polynomial in the motor torque and rotational speed:

\[
P_{\text{loss}} = P_{\text{eng}} \sum_{i=0}^{3} \sum_{j=0}^{3} C_{ij} t^n j
\]
where \( q = \frac{P_q}{P_{eng}}, \) \( n = \frac{N}{N_{spec}}, \) \( t = \frac{q}{n}. \) Controller losses, including power conversion and conditioning, are represented by an efficiency \( \eta_{cont}. \) Then

\[
\eta_{motor} = \frac{\eta_{cont}}{P_q + P_{eng}} \eta_{cont} \frac{q}{q + P_{eng}}
\]

is the motor or generator efficiency. Constant efficiency \( (P_{loss} = \eta P_q) \) implies just \( C_{11} = 1/\eta - 1. \) The copper, iron, and windage losses imply

\[
P_{loss} = K_lQ^2 + K_lN + K_wN^3 + K_0
\]

but more terms are required in order to represent a peak in efficiency as a function of torque and speed.

The motor weight can be calculated as a function of efficiency as a function of torque and speed.

The jet performance is calculated for a specified thrust

\[
T_{a} = \text{ST}_0 \sqrt{\theta} \left[ \frac{\delta M}{\sqrt{\theta M}} \right]^{X_{mo}}
\]

Then

\[
T_{a} = \text{ST}_0 m_a - m_a (ST)_{mom} = T_0 \delta \left[ \frac{\delta M}{\sqrt{\theta M}} \right]^{X_{mo}} + X_{mo} - m_a (ST)_{mom}
\]

is the thrust available.

The jet performance is calculated for a specified thrust

\[
t = \frac{(T_q + \dot{m}(ST)_{mom})}{(T_{0C} \delta)}:
\]

\[
\dot{w}_{req} = \dot{w}_{OC} (\delta \sqrt{\theta}) (K_{f1} + K_{f1t} + K_{f2}t_{\theta}^2)(\theta M)^{X_g}
\]

\[
\dot{m}_{req} = \dot{m}_{OC} (\delta \sqrt{\theta}) T_{\theta}^{K_{off} \left[ \theta M \right]^{X_{mo}}}
\]

The mass flow given \( T_q / (T_{0C} \delta) \) can be found analytically for \( K_{mf} = 0, 1, \) or \( 1/2. \)

The parameters of the jet model can be defined for a specific turbojet or turbofan, but scaling the parameters as part of the aircraft sizing task is also necessary. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific thrust and specific fuel consumption.

The jet weight can be calculated as a function of thrust:

\[
W_{onejet} = K_{0jet} + K_{1jet}T + K_{2jet}T^{X_{jet}}
\]

where \( T \) is the installed takeoff thrust (SLS static, specified rating) per jet.

### FUEL CELL MODEL

A fuel cell burns a fuel (typically hydrogen) and generates electrical energy, which is stored in a fuel tank system or used directly by a motor. The energy flow defines the power required. The power available is related to the size \( P_{chrg}. \) The model follows Ref. 12.

Given the flight condition and the charging rate, the cell power available is \( E_{cell} = P_0. \) The uninstalled power required \( E_q \) is defined by the charge group energy flow. The cell power required is \( E_{cell} = E_q / \eta, \) where the efficiency \( \eta = \eta_{batt} \eta_{chrg} \) includes the battery charging losses.

The fuel cell performance is calculated from the cell power required:

\[
\dot{m}_q = \dot{m}_{OC} (q / \eta) = K_{mf} \dot{w}_{req}
\]

\[
\dot{w}_{req} = \dot{w}_{OC} (q / \eta) = \text{sfc}_{OC} E_{cell}
\]

where \( q = E_q / P_0, \) \( \dot{w}_{OC} = \text{sfc}_{OC} P_{chrg}, \) \( \dot{m}_{OC} = K_{mf} \dot{w}_{OC}. \) The specific fuel consumption is given by the fuel specific energy and the fuel cell thermal efficiency: \( \text{sfc} = \epsilon_{fuel} / \eta_{ph}. \)

The ratio of mass flow and fuel flow follows from the chemistry of the reaction. For hydrogen and air

\[
K_{mf} = \frac{\lambda_A}{\lambda_H} \frac{m_A}{x_O m_H} = \frac{\lambda_A}{\lambda_H} 68.59
\]

The molar masses of hydrogen and air are \( m_H = 2.016 \) and \( m_A = 28.97 \) g/mole; \( x_O = 0.2095 \) is the molar fraction of oxygen in air. The supply ratio is \( \lambda_A / \lambda_H = 1/2 \) from stoichiometry, and typically \( \lambda_A / \lambda_H = 1.25 \) in practice.

The fuel cell efficiency as a function of power is estimated considering an equivalent circuit, defined by internal
resistance $R$ and current $I_0$. The voltage is $V = V_o - IR$. The efficiency is $\eta_{chrg} = P/(P + P_{loss})$, from the power loss $P_{loss} = I^2R + P_0$. For small loss, $I \approx P/V_o$. In terms of $P_{chrg}$, let

$$\frac{R}{V_o^2} = \frac{1}{P_{chrg}} \left( \frac{1}{\eta_{ref}} - 1 \right)$$

Then

$$P_0 = c P_{chrg}$$

So

$$\frac{1}{\eta_{chrg}} = 1 + P(R/V_o^2) + P_0/P = 1 + \frac{P}{P_{chrg}} \left( \frac{1}{\eta_{ref}} - 1 \right) + \frac{P_{chrg}}{P}$$

So $\eta_{chrg} = (1/\eta_{ref} + c)^{-1}$ at $P = P_{chrg}$. The efficiency decreases with $P$ because of the internal resistance, but is zero at $P = 0$ because of the internal current term. Alternatively, the efficiency can be a fixed value.

The fuel cell weight can be calculated as a function of power:

$$W_{one\ chrg} = K_{0cell} + K_{1cell}T + K_{2cell}T^{X\ cell}$$

where $P = P_{chrg}$.

**SOLAR CELL MODEL**

A solar cell generates electrical energy, which is stored in a fuel tank system. The energy flow defines the power required. The power available is related to the size $P_{chrg}$. The solar cell is characterized by power density $e_{solar}$ (W/m$^2$) and weight density $\sigma_{solar}$ (kg/m$^2$). From the size $P_{chrg}$, the area is $A_{solar} = P_{chrg}/e_{solar}$; and then the weight is $W_{one\ chrg} = W_{solar} = A_{solar} \sigma_{solar}$.

Given the flight condition and the charger rating, the cell power available is $\dot{E}_{cell} = P_0$. The unassembled power required $\dot{E}_q$ is defined by the charge stored group flow energy. The cell power required is $\dot{E}_{cell} = \dot{E}_q / \eta$, where the efficiency $\eta = \eta_{bat} \eta_{chrg}$ includes the battery charging losses.

The solar cell efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance $R$ and current $I_0$. The voltage is $V = V_o - IR$. The efficiency is $\eta_{chrg} = P/(P + P_{loss})$, from the power loss $P_{loss} = I^2R + P_0$. For small loss, $I \approx P/V_o$. In terms of $P_{chrg}$, let

$$\frac{R}{V_o^2} = \frac{1}{P_{chrg}} \left( \frac{1}{\eta_{ref}} - 1 \right)$$

Then

$$P_0 = c P_{chrg}$$

The battery capacity is $E_{fuel-cap}$ (maximum usable fuel energy). The battery is characterized by specific energy $e_{tank}$ (MJ/kg) and energy density $\rho_{tank}$ (MJ/liter), so the tank weight and volume are obtained from the capacity. The current amount of energy stored is $E_{fuel}$. The state-of-charge is $s = E_{fuel}/E_{fuel-cap}$. The useable state-of-charge is typically 20–30% of capacity, and can be accounted for either as a factor on specific energy and energy density, or as a factor on capacity.

The model follows Ref. 12. A battery stores charge (A-hr), so the capacity is expressed as energy for a nominal voltage. The rated capacity is described as $C$ (A-hr) at current $I = xC$ (A), hence for a discharge time of 1/x hours. A typical specification is for 20-hour discharge. The capacity depends on the discharge current. The Peukert model assumes $I^{k}T = I^{k-1}C = constant$, where $T$ is the discharge time for current $I$. The Peukert coefficient $k = 1.2$ to 1.3 for lead-acid batteries, and $k = 1.01$ to 1.05 for lithium-ion batteries (weak dependence). Thus the capacity $C = C_{ref} (I_{ref}/I)^{k-1}$ (A-hr). The charge capacity does not change, but for a larger current the battery reaches a specified discharge voltage sooner (due to internal resistance), hence effectively has a reduced capacity as long as that current is maintained. This effect is assumed to be accounted for in the discharge efficiency value.

The battery power limit ($P_{bat} = \pi_{bat} W_{bat}$) is compared to the charge or discharge energy rate. No other limits are considered.
The discharge or charge efficiency as a function of power is estimated considering an equivalent circuit, defined by internal resistance $R$ and current $I_0$. The voltage is $V = V_o - IR$. The open circuit voltage $V_o$ is a function of the state-of-charge $s = E_{\text{fuel}}/E_{\text{fuel-cap}}$: $V_o = f_v(s)V_{100}$, where $V_{100}$ is the voltage at $s=100\%$. The efficiency is $\eta_{\text{batt}} = P/(P + P_{\text{loss}})$, from the power loss $P_{\text{loss}} = I^2R + P_0$. For small loss, $I = P/V_o = I/f_vV_{100}$. In terms of a reference power, let

$$R/V_{100}^2 = \frac{1}{P_{\text{ref}}} \left( \frac{1}{\eta_{\text{ref}}} - 1 \right)$$

Then

$$\frac{1}{\eta_{\text{batt}}} = 1 + P(R/V_o^2) + P/P_{\text{ref}} = 1 + \frac{P}{P_{\text{ref}}}f_v^2 \left( \frac{1}{\eta_{\text{ref}}} - 1 \right) + cP_{\text{ref}}$$

So $\eta_{\text{batt}} = (1/\eta_{\text{ref}} + c)^{-1}$ at $P = P_{\text{ref}}$ and 100% charge. The efficiency decreases with $P$ because of the internal resistance, but is zero at $P = 0$ because of the internal current term. Alternatively, the efficiency can be a fixed value.

The efficiency is $\eta_{\text{batt}} = P_{\text{opt}}/P_{\text{store}} = P/(P + P_{\text{loss}})$ for discharge. For energy rate $\dot{E}$, the power available is $P = \eta_{\text{batt}} \dot{E}$. For charge, the efficiency is $\eta_{\text{batt}} = P_{\text{store}}/P_{\text{in}} = P/(P + P_{\text{loss}})$. For input power $P$, the energy rate is $\dot{E} = \eta_{\text{batt}} \dot{P}$.

**CONCLUDING REMARKS**

The propulsion system representation in the rotorcraft conceptual design code NDARC has been extended. A major objective is to be able to develop environmentally-friendly rotorcraft designs, for example aircraft utilizing electric motors, or electrical energy storage, or hydrogen-burning engines.

Engine groups, jet groups, and charge groups provide a general framework for the propulsion system components in NDARC. For each group, an engine, jet, or charger model is identified. The following models have been implemented.

**Engine group models:**
- Referred Parameter Turboshaft Engine Model (RPTEM)
- Reciprocating engine
- Convertible turboshaft: turbojet/turbofan; reaction drive
- Compressor, including reaction drive
- Motor, generator, generator-motor

**Jet group models:**
- Referred Parameter Jet Engine Model (RPJEM)
- Reaction drive
- Convertible turbojet: reaction drive

**Charge group models:**
- Fuel cell
- Solar cell
- Battery

Each model is a surrogate representation of the component performance and weight. The Referred Parameter Turboshaft Engine Model (RPTEM) has proven to be a good representation for the design code NDARC. The other models are new to the design code. The models described in this paper are sufficient to start designing rotorcraft and airplanes, but the needed for further development and additional information is expected.

For air-breathing propulsion components, the new models have been implemented with simple dependence on atmospheric conditions and flight speed. Based on the turboshaft engine model, more elaborate models likely will be needed in order to adequately cover the full range of operating conditions. Reaction drive and convertible engines may ultimately require further complexities in the model. Surrogate models likely will be developed from detailed engine performance data, as for RPTEM.

For electrical propulsion components appropriate to aircraft, a database of performance characteristics is needed in order to identify the parameters required by the models.

For all components, a database is needed that will support development of parametric models of weights. Also, the influence of technology on performance and weights must be characterized.

Separate tools, not part of the aircraft design code, are needed to design the propulsion components. The Numerical Propulsion System Simulation (NPSS, Ref. 13) is currently used for turboshaft engine design, and NPSS can design turbojet and turbofan engines. Reaction drives for rotorcraft have been extensively investigated, so there are resources on which a design tool can be based (Refs. 14 and 15). Extending design tools for electrical machinery to rotorcraft applications should be possible.

**REFERENCES**


**NOMENCLATURE**

**Acronyms and Subscripts**

- **CG** charge group
- **EG** engine group
- **ISA** International Standard Atmosphere
- **JG** jet group
- **MCP** maximum continuous power
- **MCT** maximum continuous thrust
- **MRP** maximum rated power
- **OEI** one engine inoperative
- **PG** propulsion group
- **RPJEM** referred parameter jet engine model
- **RPTEM** referred parameter turboshaft engine model
- **SLS** sea level standard

**Propulsion**

- **P** power
- **E** energy
- **T** thrust
- **F** force
- **\( \dot{m} \)** mass flow
- **\( \dot{w} \)** fuel flow
- **\( \dot{E} \)** energy flow
- **W** weight
- **\( P_{reqPG} \)** power required, propulsion group
- **\( P_{reqEG} \)** power required, engine group
- **\( T_{reqJG} \)** thrust required, jet group
- **\( P_{reqCG} \)** power required, charge group
- **\( P_{avPG} \)** power available, propulsion group
- **\( T_{avJG} \)** thrust available, jet group
- **\( P_{avCG} \)** power available, charge group
- **\( P_{comp} \)** component power required
- **B** component mode
- **\( K_{ffd} \)** efficiency deterioration factor

**Engine**

- **\( P_{eng} \)** sea level static takeoff power per engine
- **\( N_{eng} \)** number of engines in engine group
- **\( N_{inop} \)** number of inoperative engines
- **\( P_{av} \)** power available, installed
- **\( P_{a} \)** power available, uninstalled
- **\( P_{req} \)** power required, installed
- **\( P_{d} \)** power required, uninstalled
- **\( P_{loss} \)** installation losses
- **\( SP \)** specific power, \( P / \dot{m} \)
- **sfc** specific fuel consumption, \( \dot{w} / P \)
- **\( F_{G} \)** gross jet thrust
- **\( F_{N} \)** net jet thrust
- **\( D_{aux} \)** momentum drag
$N$  
  turbine speed

**Jet**

$T_{jet}$  
  sea level static takeoff thrust per jet

$N_{jet}$  
  number of jets in jet group

$N_{inop}$  
  number of inoperative jets

$T_{av}$  
  thrust available, installed

$T_a$  
  thrust available, uninstalled

$T_{req}$  
  thrust required, installed

$T_q$  
  thrust required, uninstalled

$\eta$  
  installation efficiency

$ST$  
  specific thrust, $T / \dot{m}$

$sfc$  
  specific fuel consumption, $\dot{w} / T$

$D_{aux}$  
  momentum drag

$\beta$  
  turbofan bypass ratio

**Charger**

$P_{chrg}$  
  sea level static takeoff power per charger

$N_{chrg}$  
  number of engines in charge group

$N_{inop}$  
  number of inoperative chargers

$P_{rvtotal}$  
  total cell power available

$P_{reqtotal}$  
  total cell power required

$\eta$  
  installation efficiency

**Fuel Tanks**

$W_{fuel-cap}$  
  fuel capacity, maximum usable fuel weight

$E_{fuel-cap}$  
  fuel capacity, maximum usable fuel energy

**Environment and Operation**

$\rho$  
  density

$p$  
  pressure

$T$  
  temperature, deg R or deg K

$\delta$  
  pressure ratio $p / p_0$

$\theta$  
  temperature ratio $T / T_0$

$V$  
  aircraft velocity magnitude

$M$  
  aircraft Mach number

$W_D$  
  design gross weight

$W_E$  
  empty weight

$W_{fuel}$  
  fuel weight