Regional Spherical Harmonic Modeling from Near-Surface and Satellite-Altitude Anomalies

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Abstract

The near-surface data and satellite altimetric magnetic measured data are modeled with a regionally concentrated spherical harmonic presentation technique over Australia and Antarctica. Global crustal magnetic anomaly studies have used a spherical harmonic analysis to represent the Earth's magnetic field. This global approach, however, is best applied where the data are uniformly distributed over the entire Earth. Satellite observations generally meet this requirement, but unequally distributed data cannot be easily adapted in global modeling. Even for the satellite observations, data errors spread over the globe, data smoothing is inevitable in the global spherical harmonic presentation. In addition, the global high-resolution modeling requires a great number of spherical harmonic coefficients for the region of interest. This study used the best modeling of the near-surface data and satellite gravity field data for small cap regions of the Earth and Moon (Han et al., 2008). We tested to compare the methods in both global and regional approaches and in cases where the errors are propagated outside of the regions of interest. Observations from the high-resolution modeling in the present study will allow the production of a lesser number of spherical coefficients that are relevant to the region of interest.

Introduction

Studies on geomagnetism have used a spherical harmonic analysis as a tool of representing Earth's magnetic field both internally and externally. Therefore, geomagnetic features have been modeled by spherical harmonics in a global scale. However, this global approach is best applied where the data are uniformly distributed over the entire Earth such as polar-orbiting satellite measurements. Until now, these satellite observations are good for regional or large-scale studies due to the limited resolutions at such altitudes. For the detailed regional studies, the separated features of the magnetic anomalies, unequally distributed data measured at the near-surface are useful but cannot be easily adopted in global modeling. A spherical cap analysis (Harvey, 1985) has been used as an alternative in regional modeling for geophysical and recent this technique was well updated by Thiede et al. (2003) and Thiede (2008), in this study, we introduce a regional approach as a magnetic field modeling that has been effectively implemented to model satellite gravity field data for small cap regions of the Earth and Moon (Han et al., 2008).

Theory of Spherical Slepian basis functions

Slepian (1983) showed that optimally concentrated, band-limited and time-limited signal is considered by maximizing the energy inside the region (in frames) to the whole area. This concentration problem turned out to be the association of the eigenvalues and eigenvectors of an integral matrix form of spherical harmonics. Such basis functions are orthogonal each other and satisfy the same differential equation and boundary condition as the spherical harmonics on a sphere. These are in fact the linear combinations of conventional spherical harmonic functions only concentrating the area of interest.

Given a band-limited signal \( f(\theta, \phi) \) on a sphere in terms of the usual spherical harmonics with a relevant set of coefficients, these signal can also be represented by a set of Slepian basis functions and expansion coefficients, respectively, as follows:

\[
    f(\theta, \phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} S_n^m f_{nm} Y_n^m(\theta, \phi)
\]

where \( S_n^m \) are orthonormal and longitude dependent and latitude and \( Y_n^m(\theta, \phi) \) is spherical harmonic function of degree \( n \) and order \( m \). The coefficients \( f_{nm} \) can also be represented by the A-th Slepian function, \( s_{A}^{nm}(\theta, \phi) \), and its expansion coefficients, \( f_{nm} \), respectively. \( n \) is the highest degree of spherical harmonic expansion used. \( s_{A}^{nm}(\theta, \phi) \) is the function in the expansion of linear combination of spherical harmonic functions so that it can be rewritten as:

\[
    f_{nm} = \sum_{A=1}^{nm} f_{nm} s_{A}^{nm}(\theta, \phi)
\]

The new spherical harmonic coefficients, \( f_{nm} \), of new basis function, \( s_{A}^{nm}(\theta, \phi) \), is now determined by maximizing the energy concentration over the area of interest to the energy over the entire sphere, which is as follows:

\[
    \sum_{A=1}^{nm} f_{nm} \sum_{A=1}^{nm} f_{nm} \overline{S}_{A}^{nm} S_{A}^{nm} \overline{S}_{A}^{nm} = \sum_{A=1}^{nm} f_{nm} \sum_{A=1}^{nm} f_{nm} \overline{S}_{A}^{nm} S_{A}^{nm} \overline{S}_{A}^{nm}
\]

Advantages of Regional over Global: Truncation errors and error distributions

The regional approach is superior to the global approach in terms of truncation errors and error distributions. The regional approaches have the following advantages:

1. The truncation errors are significantly smaller for regional approaches than for global approaches.
2. The error distributions are concentrated in the region of interest for regional approaches.
3. The regional approaches provide more accurate results for the region of interest.
4. The regional approaches require fewer computational resources.
5. The regional approaches are more efficient for modeling the magnetic field.

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Regional magnetic anomaly modeling by localized basis functions over Australia

Regional modeling was done over the Weddell Sea area with the same model parameters. The resulting differences between the different modeling approaches are seen in Map H at the satellite altitude in a global, and in Map D in a regional approach. The anomaly differences are more significant at the surface level such as Maps J (global) and K (region).

High resolution (n = 720) crustal magnetic anomaly modeling in the Antarctic region (Weddell Sea area)

A. Reasonably close results of our modeling, with a 19 arc-minute model resolution. The input data were first low-pass filtered with 50 km long W and resampled at the grid of 12.5 km. Total points of input data in this cap (radius is 11.5°, centered at 72.5° W, 57° W) was amounted to 39,745. The number of gauss coefficients used for regional modeling was 5,578. Note that the number of gauss coefficients to represent the degree up to 120 for global expansions is 5,184.

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