Design, Analysis, and On-Sun Evaluation of Reflective Strips Under Controlled Buckling

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Outline

I. Introduction and Background
   • Solar concentrators for high process heating temperatures:
     1. Single parabola
     2. Spherical sections placed in the approximate shape of a parabola
     3. Series of parabolic strips each directed at a common focal region
   • Controlling geometry of a 2-dimensional reflective strip by buckling offers two advantages:
     1. Light weight construction
     2. Careful control of the axial force to compensate for thermally induced distortions or to control the concentrator's focal pattern

II. Modeling and On-Sun Testing
   • Numerical Study: A Single Strip 152 cm in Length
   • Test Article Selected for Manufacture
   • Predicted Performance
   • On-Sun Testing with a Calorimeter, Observed Performance

III. Future Work
   • The piano wire concept

IV. Conclusions
Objective, Approach, and Innovation

Objective: To model the elastica and its reflections, and to utilize the results from the modeling effort to design and build an elastica strip concentrator.

Approach: A finite element method (FEM) of analysis of the strain energy can be utilized to model the shape of the strip under an axial load. Minimizing the summation of strain energies leads to a FEM solution for an elastica and defines the shape of the strip under a particular axial load.

Innovation: FEM minimization guides the selection of the cross sectional geometry of individual nodes enabling the tailoring of the shape upon buckling. Selecting unique thicknesses per node results in a tapered thickness, which leads to an elastica of parabolic shape.
Background

• Controlling the geometry of a 2-dimensional reflective strip by applying an axial force, was first proposed by White.¹
• A series of strips under compression would comprise a complete solar concentrator.
• An axially loaded member is considered to have failed when subjected to a load greater than its buckling limit. The resulting curve is called an *elastica*.
• A finite element method (FEM) of analysis of the strain energy can be utilized to model the shape of the strip under an axial load.
• Minimizing the summation of strain energies leads to a FEM solution for an elastica and defines the shape of the strip under a particular axial load.
• Selecting unique thicknesses per node results in a tapered thickness, which leads to an elastica of parabolic shape.

(selecting varying material properties per node leads to the same result)

Modeling

• The premise for modeling is based on calculating the buckled strip’s curvature utilizing an energy minimization method.

• The sum of all element strain energies is calculated.

• Excel© contains a tool called “Solver” that works to optimize the value in a selected cell based on varying the values in other specified cells. In this case, the cell that is optimized contains the total value of all element strain energies.

• The iterative process continues until an acceptably low value of total strain energy is reached.
Numerical Study: A Single Strip 152 cm in Length

- Twenty nodes were selected for this study:
  - each 7.62 cm in length, 2.54 cm wide, and 0.318 cm thick
- Model allows the user to apply an axial load at the strip’s endpoint.
- From displacements and tangent slopes:
  - the direction of reflected light from each node was calculated based on each node’s tangent slope
  - a point was charted at its intersection with a user-specified distance from the buckled strip’s midpoint
  - A parabola and focus was created for comparison
- Only one reflected ray, the reflected ray from Node 5, passes through the focus of the parabola.
- Innovation: increasing thickness decreases bending … decreasing thickness increases bending.
Numerical Study: A Single Strip 152 cm in Length

- The location of each ray impinging on the focal line was calculated, and the intensity for that node’s reflected light contribution was accumulated.

- The same process was repeated for accumulating intensity from the parabola used for comparison.

- For both the elastica and the parabola, the total intensity under the curve is 20 units.

- The central nodes having reflected rays passing the centerline before the focus must be thickened and the outer nodes having reflected rays passing the centerline after the focus must be thinned in order to approximate a parabola.
Though diamond turning would have been ideal for the reflective surface, the choice was made to utilize aluminum stock having a number 8 reflective finish.

- each of eleven elasticas were 2.5 cm wide and 33 cm long
- stock aluminum of ample thickness was utilized in order to remove material from the non-reflective side

The Excel model was exercised utilizing the lessons learned from the numerical study. The node pattern shown below was generated and submitted for milling.
Errors in Manufacturing

- The Excel® spreadsheet that was utilized to design the elastica also provided theoretical values of deflection.
- The fabrication process resulted in dimensions that were slightly above their specified values.
  - The fabricated strip was 0.005 to 0.015 cm thicker than intended
- Applying a slightly greater axial force served to compensate for the errors in manufacturing.

### Comparison of Design and As-Manufactured Dimensions

<table>
<thead>
<tr>
<th>Element</th>
<th>Design Thickness (cm)</th>
<th>Actual Thickness (cm)</th>
<th>y-Deflection (cm)</th>
<th>y-Deflection (cm)</th>
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<tbody>
<tr>
<td></td>
<td>Design with 6.35 kg</td>
<td>Actual with 7.41 kg</td>
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<td></td>
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<td>0.080</td>
<td>0.094</td>
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<td>0.25</td>
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</table>
Predicted Performance

- Estimates of the various efficiencies throughout the system, multiplied together, generated an overall predicted efficiency value.
  - an estimate of the fraction of light successfully entering the calorimeter, i.e. the reflectivity and specularity of the #8 aluminum finish
  - an estimate of the concentrator area not shadowed by the calorimeter
  - an estimate of the energy lost from the calorimeter, via radiation out of the opening

- Multiplying the estimates yielded an accumulated efficiency of the as-built system of 54%.

<table>
<thead>
<tr>
<th>Item</th>
<th>Efficiency estimate</th>
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<tr>
<td>Reflectivity &amp; Specularity of the #8 aluminum finish</td>
<td>0.74</td>
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<tr>
<td>Shadowing of the concentrator</td>
<td>0.75</td>
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<tr>
<td>Radiant heat losses</td>
<td>0.98</td>
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<tr>
<td>Accumulated efficiency:</td>
<td>0.54</td>
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</table>
On-sun Testing with a Calorimeter, Observed Performance

• The angle and then the compression of the individual mirrors (while on sun) were adjusted, with the goal of focusing each strip of light onto a single band at the opening of the calorimeter.

• Light that successfully entered the calorimeter was absorbed by black inner walls and heated a copper cylinder of known mass.

• The time-varying temperature of the copper along with its mass was used to provide a measure of the solar energy collected by the system.
  – And a calibrated solar cell at the base of a diffuse light-blocking rectangular tube was used to provide a measure of the solar energy impinging on the system.
Average Thermocouple Temperatures and Measured Solar Flux

![Graph showing average temperature and solar flux over time.](image-url)
The Copper Calorimeter
instrumented with thermocouples and enclosed in insulation

• The average solar flux was 773 W/m², the total solar power on the mirror was 44.9 W, and the power to the calorimeter was 19.7 W.
• The overall measured efficiency was 43.8%.
  (about 10% of the light splashed on the outside of the calorimeter)
Future Work

- A novel variation on the elastica concentrator would use piano wire to aid in compression.

- In an array of wires, each wire would serve as a means to place an individual elastica under compression and to rotate the elastica to direct sunlight to a focus.

- In the aggregate the bulky rectangular frame could be replaced with a frame to hold the wires.
Conclusions

• The concept of utilizing a FEM analysis of the strain energy to predict elastica shape was introduced.
  – *Minimizing the summation of strain energies* leads to a solution that defines the shape of a concentrator strip.

• A test article was manufactured by exercising the model to identify cross sectional thickness of numerous nodes.
  – The prescribed amount of material *from the back side* of 11 aluminum strips having a #8 finish was removed.
  – Milling left a slight excess of material, however, increasing axial load *compensated* for the milling error.

• The elastica concentrator performance was evaluated on-sun using a sun tracker equipped with an existing calorimeter and calibrated solar cell.
  – *Predicted performance*, based on reflectivity, specularity, shadowing, and energy loss was 54%.
  – The *actual performance* was 44% (about 10% of the light splashed outside of the calorimeter).

• Minimizing shadowing from the component at the focus using the process heating and improving the finish of the surface should improve performance.