Assessment of the accuracy of the conventional ray-tracing technique: Implications in remote sensing and radiative transfer involving ice clouds

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**Abstract**

A fundamental problem in remote sensing and radiative transfer simulations involving ice clouds is the ability to compute accurate optical properties for individual ice particles. While relatively simple and intuitively appealing, the conventional geometric-optics method (CGOM) is used frequently for the solution of light scattering by ice crystals. Due to the approximations in the ray-tracing technique, the CGOM accuracy is not well quantified. The result is that the uncertainties are introduced that can impact many applications. Improvements in the Invariant Imbedding T-matrix method (II-TM) and the Improved Geometric-Optics Method (IGOM) provide a mechanism to assess the aforementioned uncertainties. The results computed by the II-TM + IGOM are considered as a benchmark because the II-TM solves Maxwell’s equations from first principles and is applicable to particle size parameters ranging into the domain at which the IGOM has reasonable accuracy. To assess the uncertainties with the CGOM in remote sensing and radiative transfer simulations, two independent optical property datasets of hexagonal columns are developed for sensitivity studies by using the CGOM and the II-TM + IGOM, respectively. Ice cloud bulk optical properties obtained from the two datasets are compared and subsequently applied to retrieve the optical thickness and effective diameter from Moderate Resolution Imaging Spectroradiometer (MODIS) measurements. Additionally, the bulk optical properties are tested in broadband radiative transfer (RT) simulations using the general circulation model (GCM) version of the Rapid Radiative Transfer Model (RRTMG) that is adopted in the National Center for Atmospheric Research (NCAR) Community Atmosphere Model (CAM, version 5.1). For MODIS retrievals, the mean bias of uncertainties of applying the CGOM in shortwave bands (0.86 and 2.13 \(\mu\)m) can be up to 5% in the optical thickness and as high as 20% in the effective diameter, depending on cloud optical thickness and effective diameter. In the MODIS infrared window bands centered at 8.5, 11, and 12 \(\mu\)m, biases in the optical thickness and effective diameter are up to 12% and 10%, respectively. The CGOM-based simulation errors in ice cloud radiative forcing calculations are on the order of 10 W m\(^{-2}\).
1. Introduction

Ice clouds are known to play an important role in regulating atmospheric radiation through interaction with both solar and infrared (IR) radiation fields [1,2]. Theoretical modeling of ice cloud radiative effects is indispensable in many atmospheric applications. The modeling approach typically begins with the single scattering of light by individual ice particles of different habits (shapes) and sizes contained in a scattering volume element, followed by the multiple-scattering calculations (i.e., the radiative transfer, or RT) involving ice clouds. For example, inferring the microphysical properties of ice clouds from observations by ground- and satellite-based instruments requires comparing the instrument observations to the model-simulated results obtained from the single-scattering properties and relevant multiple-scattering simulations [3–7]. To understand the ice cloud impact in the climate models, an accurate representation of ice cloud bulk-scattering properties is a critical component required by RT models to quantify the cloud radiative effect (CRE), such as radiative forcing of natural cirrus clouds [8,9] and anthropogenic contrails [10,11]. For this reason, much effort has been dedicated to developing physically representative ice cloud models (e.g., ice habits, particle size distributions, and mass-dimension relationships) and accurately computing the single-scattering properties of individual ice particles (i.e., the extinction efficiency, the single-scattering albedo, and the phase matrix).

However, solving Maxwell’s equations for light scattering by ice particles from first principles is much more complex than light scattering by water droplets. Ice particles are nonspherical and morphologically complex, while having a broad range of particle sizes varying from a few microns to centimeters. In a historical context, the majority of simulations [12–19] are based on geometric-optics principles that are asymptotically correct and approximately valid at large size parameters, i.e., when the particle size is sufficiently large relative to the wavelength. The ratio of the particle circumference to the wavelength is known as the size parameter. A number of physical-geometric optics methods have also been proposed to improve the accuracy in geometric-optics approximations [20–26].

Deschamps [27] addressed the importance of the “ray” concept of light in engineering, stating that “if the nature of light and Maxwell’s equations had been known, earlier optical instruments would not have been invented so readily!”. Similarly, the application of ray techniques in solving light scattering by individual ice particles in the atmospheric radiation discipline plays a critical role because it provides a first-order approximation for higher frequency scattering where rigorous solutions are unavailable. In other words, the knowledge of ice cloud radiation would have been severely limited in a modeling perspective without employing the geometric-optics method. A more specific example is that for an ice particle with a size parameter larger than 200, the ability to obtain a rigorous set of optical properties is almost beyond our current first-principle modeling capabilities. For this reason, the geometric-optics methods continue to be popular, but there is a growing awareness of the need to provide an accuracy assessment due to the inherent approximations [e.g.,28,29].

With the development of computational electrodynamics, numerical techniques to solve Maxwell’s equations are now available to obtain numerically exact solutions. The power of these numerical techniques began to have an impact within the past two decades. A few frequently used computational techniques are the finite-difference time-domain (FDTD) [30–33], the pseudospectral time-domain (PSTD) [34–36], the discrete-dipole-approximation (DDA) [37–40], and the T-matrix method [41–45]. Based on the numerical techniques, a much better knowledge has been obtained of the optical properties of particles at small-to-moderate size parameters. The applicability of the geometric-optics in different size parameter domains is investigated whenever possible. For example, Mishchenko and Macke [46] investigated T-matrix computations of light scattering by circular cylinders and found that ice halos, an optical phenomenon predicted from geometric optics, would not be observed when the particle size parameter is less than approximately 100. Unlike the geometric-optics methods, the performance of numerical methods strongly depends on computational resources. As computer power increases, some new domains of rigorous solutions are being conquered. Meanwhile, because of its simplicity, the use of the geometric-optics method is preferred for a complete range of ice particle sizes in the wavelength spectrum from UV to the near-infrared.

To our knowledge, the induced uncertainties due to the nature of the approximations inherent in the ray-tracing technique have not yet been assessed in remote sensing and climate studies involving ice clouds, most likely because the rigorous solution domain is still extremely limited. The Invariant Imbedding T-matrix method (IITM) [47–50] is applicable to a broad range of size parameters from the Rayleigh region up to the geometric optics domain where ice halos are observed. In this study, we use the IITM to assess the accuracy of geometric-optics approximation and the resulting uncertainties in remote sensing and radiative transfer simulations.

The theoretical components of the employed geometric-optics method are delineated before the assessment because a number of modifications in geometric-optics methods exist in the literature. A brief review of available geometric-optics methods is provided in Bi and Yang [51]. The geometric optics methods most frequently used are the conventional geometric-optics method (CGOM) and the improved geometric-optics method (IGOM), although a few more rigorous, but relatively less computationally efficient, methods have also been developed [20–26]. One unique difference between the CGOM and the IGOM is that the latter considers the spreading of scattered beams (a physical-optics effect) when propagating from the near-field to the far-field region. As the particle size parameter increases, the IGOM simulated phase matrix transitions to that from the CGOM when the spreading effect is negligible. With the ray-spreading effect incorporated, the IGOM is applicable to relatively small size parameters (20–100) where the ice halos disappear, but the geometric-optics principles are still more...
or less valid in estimating the internal electromagnetic field. The CGOM and the IGOM also differ in the computation of the extinction efficiency $Q_{\text{ext}}$, defined by the ratio of particle extinction cross section to particle projected area. The CGOM empirically assumes that the extinction efficiency is constant at a value of 2 in conjunction with the blocking effect and the diffraction effect whereas the IGOM computes the extinction efficiency based on a physical-optics approximation that takes into account the interference between the diffraction and the forward scattered rays.

In this study, we begin with the assessment of the accuracy of the single-scattering properties derived from the CGOM and the IGOM in the T-matrix applicable domains. Next, we use a combination of the II-TM and the IGOM for computing the single-scattering properties in a complete range of particle sizes ranging from 2 to 10,000 nm in maximum diameter. Based on the finding that the IGOM and the T-matrix results are very similar at the transition regions from wave optics to geometric optics, we consider the II-TM+IGOM results as a benchmark and assess the uncertainties of applying the CGOM to an entire range of particle size parameters. From an application perspective, we then assess the uncertainty of using the CGOM in retrieving the ice particle effective diameter and optical thickness from Moderate Resolution Imaging Spectroradiometer (MODIS) measurements and the broadband radiative forcing calculations using the general circulation model (GCM) version of the Rapid Radiative Transfer Model (RRTMG), which is the model employed in the Community Atmosphere Model (CAM, version 5.1) developed at National Center for Atmospheric Research. For simplicity, the ice particles are assumed to be smooth-faceted hexagonal columns. Although atmospheric ice habits are diverse, the accuracy of the CGOM, IGOM, and II-TM is not assumed to change for different habits.

The remainder of this paper is organized as follows. In Section 2, we describe the numerical methods used for computing the optical properties of ice particles. The numerical results for the optical properties of hexagonal ice particles are shown in Section 3, including the comparison of the single-scattering properties and bulk-scattering properties computed from the CGOM and the II-TM+IGOM. Section 4 provides sensitivity studies to assess the uncertainties of applying the CGOM in the applications of remote-sensing retrievals. In Section 5, the uncertainties in radiative forcing calculations due to applying the CGOM are quantified through a single-column radiative transfer model and a GCM. The conclusions of this study are given in Section 6.

2. Computational methods

In this section, we briefly outline the principles of the computational methods of the CGOM, the IGOM, and the II-TM. The readers are referred to [51,52] as primary references for the applications of geometric optics. A common theoretical basis of the CGOM is a combination of Fraunhofer diffraction by the particle shadow (Fig. 1a), and the angular distribution of scattered intensity obtained from the ray-tracing technique shown in Fig. 1b. Specifically, the scattering phase function from the ray-tracing process is obtained from the division of the energy $\Delta E$ with respect to the differential solid angle $\Delta \Omega$.

![Fig. 1. Conceptual figure of geometric-optics principles.](image-url)
The largest advantage of the CGOM is in its simplicity and flexibility to adapt to particle geometries; however, it suffers from several drawbacks. The solid angle is zero in both the forward and backscattering directions, thus causing a singularity problem. In most CGOM algorithms, values of the phase function in the direct forward and backward directions are obtained through extrapolation of the near-forward and near-backward directions. Moreover, if the particle is in a fixed orientation, the phase function is not a continuous curve but several discrete points. Although the random-orientation condition produces a relatively smooth phase function, an isolated peak is usually observed in the direction of forward scattering, commonly called the delta-transmission energy, that is fundamentally related to parallel facets in a hexagonal ice habit. In the case of distorted or roughened particles, the delta-transmission term is not observed. Mishchenko and Macke specifically explained that the delta-transmission term exists because of the missing physical-optics effect. When the particle is non-absorptive, the angular distribution of scattered energy is evidently size-independent, but this is contradictory to the wave theory of light. Furthermore, the extinction efficiency is an asymptotic value and does not capture the size-dependence for small and moderate size parameters.

To overcome the shortcomings of the CGOM, geometric-optics principles are employed to compute either the internal field or the particle surface field, and the far field is obtained through exact electromagnetic volume integral equations or surface integral equations. Within the new framework, the physical-optics effects are taken into account. For example, the extinction efficiency considers the interference between the diffraction and the waves associated with the forward scattering rays. The diffraction effect associated with outgoing scattered beams (Fig. 1c) is taken into account so that the scattering phase function from the ray-tracing calculation is size-dependent. For example, halos will only appear when the particle size parameter is sufficiently large (more than 100), where the ray-spreading effect is small and the rays are strongly localized. Yang and Liou proposed a simplified algorithm to account for the physical-optics effect in the phase matrix computation that is implemented in IGOM.

When ice particles are absorptive, most localized waves inside the particle are inhomogeneous. In this study, we have considered the inhomogeneous nature of localized waves in the ray-tracing process in both the CGOM and the IGOM, although for simplicity the effect is usually neglected in many studies. Two obvious reasons to justify the consideration of inhomogeneous waves are that: (1) the energy is not conserved at the first-order of refraction if the inhomogeneity of localized waves is omitted; (2) the asymptotic value of the extinction efficiency derived from

![Table 1](image)

<table>
<thead>
<tr>
<th>Component</th>
<th>CGOM</th>
<th>IGOM</th>
</tr>
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<tbody>
<tr>
<td>Extinction efficiency</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Delta transmission</td>
<td>Removed</td>
<td>Removed</td>
</tr>
<tr>
<td>Ray spreading</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Inhomogeneous wave</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
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Fig. 2. Comparison of phase functions computed from the II-TM, the IGOM and the CGOM. The wavelength and the hexagonal column height are indicated in each sub-figure. The size parameters for each sub-figure from (a)-(f) are approximately 126, 110, 118, 129, 121 and 124.
the IGOM differs from 2 by a non-negligible amount if not considering the electric field component along the propagation direction.

In consideration of a number of geometric-optics algorithms, we specifically delineate the theoretical components involved in the CGOM and the IGOM in Table 1. Note that the delta-transmission peak has been removed in the CGOM by considering the physical-optics effect.

For a particle having a small size parameter, where the ray-concept is inappropriate for calculating the electromagnetic field inside the particle or particle surface, the IGOM gives inaccurate results; Maxwell’s equations need to be solved. The transition (or T-) matrix method in conjunction with the extended-boundary condition (EBC) technique in the T-matrix computations pioneered by Waterman [41,42] is a powerful approach for computing the optical properties of nonspherical particles. In principle, the EBC is applicable to an arbitrarily shaped nonspherical particle; however, the EBC is most successful in cases of axially symmetric particles. For example, for a hexagonal particle whose geometry is of 6-fold rotational symmetry, the EBC’s applicable size parameter region is substantially narrowed in comparison to spheroid cases.

Note that in the EBC method, the T-matrix is computed from surface integrals over the particle surface. Alternatively, the T-matrix can be derived from an electromagnetic volume integral equation and computed from an invariant imbedding procedure. One unique advantage of the invariant imbedding approach is that the algorithm is stable and flexible in adapting to a given particle geometry. We have extended the II-TM applicable region to large size parameters for nonspherical particles [47–49]. Here, we will not discuss the theoretical basis of the II-TM as the details can be found in [47–50].

3. Optical properties of ice particles

The optical properties of hexagonal ice particles are computed in a spectral region from 0.2 to 100 μm. The aspect ratio of hexagonal ice particles is the same as the geometry defined in [55] (their Table 1). Six wavelengths in the visible and infrared are selected for remote-sensing studies: 0.65, 0.86, 2.13, 8.5, 11, and 12 μm. The ice refractive indices compiled by Warren and Brandt [56] are used in the simulations.

![Fig. 3. Comparison of the extinction efficiency, the single-scattering albedo, and the asymmetry factor at wavelengths (a) 0.65 μm (left column), (b) 2.13 μm (middle column), and (c) 12 μm (right column) computed from the II-TM+IGOM and the CGOM.](image-url)
Fig. 2 shows a comparison of phase functions computed from the CGOM, the II-TM and the IGOM at six representative wavelengths. The particle sizes for each phase function are indicated in the figure. The phase functions computed from the II-TM and IGOM are very similar. At wavelengths of 0.65 and 0.86 μm (Figs. 2(a) and (b)), the ice particles are non-absorptive and ice halos at 22° and 46° are beginning to emerge for the selected size in the II-TM and IGOM calculations, but they are much too prominent in the CGOM results. However, a peak that is predicted by the IGOM and the CGOM at approximately 150° is not observed in the II-TM simulated phase function. The size parameter is most likely insufficiently large, so that the ray-concept in the CGOM and the IGOM fails for higher-order rays. Additionally, a small peak of phase function at the wavelength of 8.5 μm predicted from the CGOM is not observed from the II-TM simulation. Because the II-TM is computationally expensive and the II-TM and the IGOM results are very close as shown in Fig. 2, we only employ the IGOM in the following simulations when the particle sizes are larger than the numbers indicated in the figure. The IGOM results of the extinction efficiency and the absorption efficiency are adjusted with respect to the II-TM for a smooth transition through a semi-empirical estimation of the edge effect contribution [46].

Fig. 3 shows a comparison of the extinction efficiency (Q_{ext}), the single-scattering albedo (ω), and the asymmetry factor (g) of hexagonal columns at three wavelengths (0.65, 2.13, and 12 μm), computed from the CGOM and II-TM+IGOM, and for particle sizes ranging from 2 to 10,000 μm. The differences between the results from the two techniques are evident, particularly for small particle sizes. Note that the extinction efficiencies and the asymmetry factors at the wavelengths of 0.65 and 2.13 μm oscillate, but the CGOM results do not because the interference effect is not taken into account. While the single scattering albedo is well approximated by the CGOM at 2.13 μm, large errors for small particles occur at 12 μm. Note that the size parameter for a given particle size at 12 μm is approximately 5.6 times smaller than that at 2.13 μm and the particle is more absorptive at 12 μm than at 2.13 μm. As is evident from the comparison, the T-matrix and IGOM solutions cover a size parameter range where the conventional geometric-optics principles break down.

Based on the independent scattering assumption, ice cloud bulk-scattering properties are obtained by integrating the single-scattering properties of ice particles with the particle-size distributions (PSD). The Gamma size distribution is adopted in this study, and the PSD can be represented by the following function:

$$n(L) = N_o (L/\nu)^{-1} \exp(-L/\nu),$$  

where \(N_o\) is a normalization factor and \(L\) is the maximum dimension (length of the column). The PSD is determined by \(a\) and \(\nu\), the shape and scale parameters. To determine the two parameters, we use the values from ice cloud in situ measurements [57] and from retrievals [58] for contrails. Fig. 4 shows the relationship between \(a\) and \(\nu\). The cirrus PSD data compiled by Baum et al. [57] are illustrated by blue dots in the figure, and the retrieved data from Iwabuchi et al. [58] are shown as green circles. The data show a large dispersion. Different from the linear fit given by Iwabuchi et al. [58], this study uses a hyperbolic tangent function to fit the observations, and the solid red curve in the figure follows:

$$\log(\alpha+2) = 0.44 - 0.32 \times \tanh\left[2 \times \log(\nu) - 4\right].$$  

(2)

A total of 18 pairs of \(a\) and \(\nu\) values corresponding to the effective particle diameter \(D_{eff}\) ranging from 10 to 180 μm at an interval of 10 μm are used in this study. \(D_{eff}\) is defined by

$$D_{eff} = \frac{3\langle V \rangle}{2\langle A \rangle},$$  

(3)

where \(\langle V \rangle\) and \(\langle A \rangle\) are the averaged volume and projected area of the particles associated with a given size distribution. The dashed curves in the figure indicate that most data lay in the range either twofold smaller or larger than the fitting curve.

Fig. 5 compares the bulk-scattering phase functions computed from the CGOM and the II-TM+IGOM at 0.65, 2.13, and 12.0 μm. The differences are evident when the \(D_{eff}\) is small, but become negligible when \(D_{eff}\) becomes sufficiently large. Indicated in the figure are the bulk extinction efficiency, the bulk single-scattering albedo, and the bulk asymmetry factor. The edge-effect contribution to the extinction efficiency is observable. In the case of a large \(D_{eff}\), small particles contribute less to the total cross section because of their smaller projected area and relatively low weight of probability in the particle size distribution. The difference between the extinction/scattering cross-sections computed by the II-TM+IGOM and the CGOM decrease as the \(D_{eff}\) increases.

From the comparison of the single-scattering properties computed from the CGOM and the “benchmark”, we identify errors in optical properties of small ice crystals calculated using CGOM. In the following sections, we quantify the radiative uncertainties if the CGOM is applied to all particle size parameters.
4. Remote sensing implications

The previous section compared both the single- and bulk-scattering properties of hexagonal columns computed from the II-TM+IGOM and the CGOM. The effect of the bulk-scattering property differences on the remote sensing implementations is assessed in this section. Two independent retrieval methods are used that employ the hexagonal column model and the scattering properties obtained from the II-TM+IGOM and the CGOM. One retrieval method is based on two solar reflectance bands and the other on three infrared (IR) bands. The accuracy of the CGOM is evaluated by comparing the retrieved ice cloud properties (i.e., cloud optical thickness $\tau$ and $D_{\text{eff}}$) with those obtained from the II-TM+IGOM simulations for these two methods.

Both retrieval methods use measurements from the Moderate Resolution Imaging Spectroradiometer (MODIS) on the National Aeronautics and Space Administration (NASA) Terra and Aqua platforms. MODIS has 36 spectral bands ranging from 0.41 to 14.2 $\mu$m. In the current study, we employ the Aqua/MODIS level-1B (L1B) collection-5.
The MODIS-retrieved cloud phase, optical thickness, and top height from the Aqua/MODIS level-2 cloud product (MYD06) are also used. The MODIS pixels that are over the ocean and identified as ice clouds in the MODIS level-2 products are used for our retrievals. For the IR retrieval, which is inherently less sensitive to optically thick clouds, the cases with MODIS optical thicknesses larger than 6 are neglected. The atmospheric profiles used in the RT calculations are provided by the Modern Era Retrospective-Analysis for Research and Applications (MERRA) model, specifically the 3-hourly profiles (“Int3_3d_ams_CP”) of temperature, water vapor density, and ozone density defined for 42 pressure levels at a grid resolution of 1.25° × 1.25°.

The first retrieval algorithm is based on two solar reflectance bands: specifically, a weakly absorbing, visible or near-infrared window band (VIS/NIR) (e.g., 0.64 or 0.86 μm) that is sensitive mainly to τ, and an ice absorbing shortwave infrared (SWIR) band (e.g., 1.6 or 2.13 μm) that is sensitive to both τ and Deff [59]. This study uses the MODIS observations at 0.86 and 2.13 μm bands, and the fast radiative transfer model (RTM) developed by Wang et al. [60] to simulate the reflectance of the modeled ice clouds. The fast RTM uses pre-computed bidirectional reflectance/transmittance distribution functions (BRDF/BTDF) for single layer clouds, and lookup tables (LUTs) for different τ and Deff are calculated using the discrete ordinates radiative transfer program (DISORT). Fig. 6 shows the comparisons of the LUTs built on the optical properties calculated by the II-TM+IGOM and the CGOM, respectively. The solar zenith angle is assumed to be 20°.

Two viewing zenith angles, 37° and 70°, are selected for radiative transfer simulations to build LUTs as shown in Fig. 6a and b, respectively. Substantial differences exist in the Deff isolines, because the differences between the optical properties given by the II-TM+IGOM and the CGOM become larger as Deff decreases. Furthermore, as shown in Fig. 6, the relative differences between the LUTs are sensitive to the solar-satellite geometry, because the relative magnitude between the CGOM and the II-TM+IGOM phase functions change with respect to the scattering angles (see Fig. 5). For the case in Fig. 6a, the Deff isolines computed from the CGOM optical properties are lower than the II-TM+IGOM counterparts, which implies that the retrieved Deff based on the CGOM in all probability be smaller than those based on the II-TM+IGOM. However, for the case in Fig. 6b, the CGOM optical properties result in higher reflectivities at the same Deff, suggesting that the retrieved Deff is mostly likely to be overestimated. The two LUTs of the CGOM and the II-TM+IGOM in Fig. 6a and b and c as Deff reaches 10 μm and 30 μm, respectively. In terms of the τ isolines shown in Fig. 6, the results for the II-TM+IGOM and CGOM are similar with noticeable differences for small Deff.

To retrieve the cloud optical thickness and effective particle diameter, the following cost function is defined

\[ \chi^2 = \left( \frac{R_{\tau_{0.86}} - R_{\tau_{0.86}}^\text{II-TM+IGOM}}{R_{\tau_{0.86}}} \right)^2 + \left( \frac{R_{\tau_{2.13}} - R_{\tau_{2.13}}^\text{II-TM+IGOM}}{R_{\tau_{2.13}}} \right)^2, \]

where \( R_{\tau_{0.86}} \) and \( R_{\tau_{2.13}} \) are the MODIS bidirectional reflectivities and the fast RTM simulations at wavelength \( \lambda \), respectively. The τ and Deff values are obtained from the simulated reflectivities that minimize Eq. (4). Fig. 7 illustrates the cloud properties retrieved from the MODIS measurements. A MODIS granule at 07:45 UTC on August 2, 2010 is used in the retrieval. Fig. 7a and b shows the solar zenith and viewing zenith angles, respectively. Fig. 7c and d provides the retrieved cloud τ and Deff based on the LUT developed from the scattering properties given by the II-TM+IGOM. As illustrated from the retrieved results, the clouds in this granule have a wide range of τ and Deff values.

The relative differences between the II-TM+IGOM and CGOM cases are shown in Fig. 7e and f, and the relative differences are given by

\[ RE = \frac{\tau_{\text{CGOM}} - \tau_{\text{II-TM+IGOM}}}{\tau_{\text{II-TM+IGOM}}} \times 100\%. \]
and
\[
RE_{\text{Deff}} = \frac{\text{Deff, CGOM} - \text{Deff, II-TM + IGOM}}{\text{Deff, II-TM + IGOM}} \times 100\%
\]  

(6)

where \(\tau_{\text{II-TM + IGOM}}\) and \(\text{Deff, II-TM + IGOM}\) and \(\tau_{\text{CGOM}}\) and \(\text{Deff, CGOM}\) are the values retrieved with the scattering properties given by the II-TM+IGOM and the CGOM, respectively. As shown in Fig. 7e and f, the relative differences caused by the CGOM are as large as 15% and 30% for \(\tau\) and \(\text{Deff}\), respectively. To be more specific, the relative differences of \(\tau\) generally under 10%, show weak dependence on the satellite viewing geometry, and the largest differences occur at those pixels with small \(\tau\). However, it should be noted that the relative differences on the retrieved \(\text{Deff}\) shown in Fig. 7f are much larger, and strongly depend on the viewing zenith angle. The correlation between Fig. 7b and f can be explained by the LUTs shown in Fig. 6. The CGOM-based \(\text{Deff}\) isolines in the LUTs can be lower or higher than the II-TM+IGOM counterparts because the reflectivity for a specific solar-satellite configuration is highly related to the magnitude of the phase functions at the corresponding scattering angles where the CGOM phase function can be larger or smaller than the II-TM+IGOM phase function. Consequently, the differences of retrieved effective diameters from the two LUTs can be smaller or larger with a similar pattern as the viewing zenith angle.

Fig. 8 shows the mean biases (solid line) and the standard deviations (shaded areas) of the retrieved \(\tau\) and \(\text{Deff}\) values for the CGOM-derived scattering properties. It can be seen that the retrieved \(\tau\) is quite accurate for \(\tau \leq 30\) with mean biases of less than 1% and standard deviations
less than 5%, but the CGOM-based retrievals underestimate $\tau$ by approximately 5% and have standard deviations that can exceed 5% as $\tau$ increases. Note that, the reflectance at 0.86 $\mu$m becomes less sensitive to $\tau$ for thicker clouds. For example, the reflectivity increases by an amount $< 0.1$ as the $\tau$ increases from 30 to 100 for all $Deff$. Low sensitivity indicates the existence of large retrieval differences for thick clouds. Meanwhile, the relative differences for the retrieved $Deff$ are more significant with mean biases and standard deviations of approximately up to 20% and 40%, respectively, for the particles with $Deff \leq 50 \mu m$. However, the errors become much smaller as $Deff$ increases.

Fig. 9 shows the relative differences in CGOM-retrieved $Deff$ within different ranges of $\tau$. As evident from the figure, the errors of $Deff$ are very sensitive to $\tau$. For example, the patterns in Fig. 9a and b are quite different and very large bias are obtained for optically thin clouds with small particle sizes. Note that the pixels for $Deff > \sim 85$ are not available when $\tau < 1$. In comparison of Fig. 9c and d, the errors of $Deff$ are relatively larger for $\tau < 2$ than those for $\tau > 2$. As shown in Fig. 9e and f, the errors in $Deff$ are most significant for cases where $\tau < 4$, and much smaller as $\tau > 4$. Because Fig. 9e is similar to Fig. 8b, the large errors shown in Fig. 8b primarily occur when $\tau < 4$. Different from the above findings, the retrieved $\tau$ is found not to be very sensitive to $Deff$. For example, the pattern of the relative differences of $\tau$ are very similar for the pixels with $Deff < 50 \mu m$ as for those with $Deff \geq 50 \mu m$ (figure not shown).

The second retrieval method is based on the three MODIS IR bands at 8.5, 11, and 12 $\mu$m. The fast high-spectral-resolution RTM (HRTM) developed by Wang et al. [61,62] is used to simulate the radiances and resulting brightness temperatures for the three bands. Gaseous absorption is taken into account by a pre-computed transmittance database using a rigorous line-by-line radiative transfer model (LBLRTM), and the scattering properties given by the II-TM+IGOM or the CGOM are used to calculate the angular-dependent ice cloud reflectance and transmittance, the effective emissivity, and the effective temperature functions. The spectral response functions of the three MODIS bands are considered in the HRTM. The $\tau$-$Deff$ pair that leads to the least root mean square (RMS) value is chosen as the most appropriate inference of the ice cloud properties, and the RMS is defined as

$$RMS = \left[ \left( BT_{\lambda, 8.5} - BT_{\lambda, 8.5}^H \right)^2 + \left( BT_{\lambda, 11} - BT_{\lambda, 11}^H \right)^2 + \left( BT_{\lambda, 12} - BT_{\lambda, 12}^H \right)^2 \right]^{1/2},$$

(7)

where $BT_{\lambda, \lambda}$ and $BT_{\lambda, \lambda}^H$ are the brightness temperatures from MODIS observations and the HRTM simulations at wavelength $\lambda$. Fig. 10 is the same as Fig. 8 but for $\tau$ and $Deff$ from the IR retrieval. The retrievals carried out using the scattering properties from the CGOM systemically overestimate the $\tau$ by approximately 12% with standard deviations over 3%. For $Deff$, the mean biases caused by the inaccuracy of the CGOM range between about $-10\%$ and $10\%$ with standard deviations up to 20%.

5. Uncertainties in radiative forcing calculations

The differences in the ice cloud optical properties between the II-TM+IGOM and CGOM cases also influence the parameterization schemes of ice cloud bulk optical properties used in the single-column radiative transfer models (RTMs), as well as in general circulation models (GCMs), thereby affecting the simulations of ice cloud broadband radiances. Results are shown for bands in the shortwave (SW) spectrum ($\lambda < 5 \mu m$) and in the longwave (LW) spectrum ($5 < \lambda < 100 \mu m$). The cloud radiative effect (CRE) is defined as the difference in the net TOA/surface flux (SW, LW or total) between the cloudy sky and the clear sky.

Fig. 8. The mean and standard deviations for the relative difference in the retrieved optical thickness (a) and effective diameter (b).
The optical properties of ice clouds used in numerical model simulations include the mass extinction coefficient \( k_{\text{ext}} \), the mass absorption coefficient \( k_{\text{abs}} \), the bulk single-scattering albedo \( \omega \), and the mean asymmetry factor \( g \), which are defined by

\[
k_{\text{ext}} = \frac{\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} Q_{\text{ext}}(L, \lambda) A(L) S(\lambda) n(L) dL d\lambda}{\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} V(L) S(\lambda) n(L) dL d\lambda}
\]  

(Fig. 9. The mean and standard deviations for the relative difference in the retrieved effective diameter within different ranges of the optical thickness.

Fig. 10. The mean and standard deviations for the relative difference in the retrieved optical thickness (a) and effective diameter (b) for the IR retrieval.)
\[
\frac{1}{\omega} = \frac{1}{C_0 k_{\text{abs}}} = \frac{1}{k_{\text{ext}}},
\]
\[
k_{\text{abs}} = \frac{\int_{\lambda_{\min}}^{\lambda_{\max}} Q_{\text{abs}}(L, \lambda) A(\lambda) S(\lambda) n(L) dL d\lambda}{\int_{\lambda_{\min}}^{\lambda_{\max}} V(L) S(\lambda) n(L) dL d\lambda},
\]
\[
\bar{m} = 1 - k_{\text{abs}} / k_{\text{ext}},
\]
\[
g = \frac{\int_{\lambda_{\min}}^{\lambda_{\max}} g(L, \lambda) Q_{\text{scat}}(L, \lambda) A(\lambda) S(\lambda) n(L) dL d\lambda}{\int_{\lambda_{\min}}^{\lambda_{\max}} Q_{\text{scat}}(L, \lambda) A(\lambda) S(\lambda) n(L) dL d\lambda},
\]

where \(L\) is the height of the hexagonal column in a particle size range \([L_{\min}, L_{\max}]\), \(V\) and \(A\) are the volume and the average projected area, \(\lambda_{\min}\) and \(\lambda_{\max}\) specify the wavelength limits of a given SW/LW band, and \(S(\lambda)\) is the solar spectrum for SW bands and the Planck function (the cloud temperature is assumed to be 233 K) for LW bands. All are parameterized as functions of ice cloud \(D_{\text{eff}}\).

Fig. 11 shows the ice cloud bulk-scattering properties for selected spectral bands of the RRTMG radiative transfer model. For the CGOM case, the mass extinction coefficient is invariant with the SW spectral bands, and will cause a difference in the mass extinction coefficient of up to 0.05 m\(^2\) g\(^{-1}\) (14%) for the 2.15–2.5 \(\mu\)m band for small \(D_{\text{eff}}\). Similarly, the mass absorption coefficient for a given LW band (e.g., the 14.3–15.9 \(\mu\)m band) can differ by about 0.04 m\(^2\) g\(^{-1}\) (24%). The largest differences in the asymmetry factor occur for small \(D_{\text{eff}}\) in the longer wavelength bands. Relatively small differences are found for the single-scattering albedo (SSA), where the II-TM+IGOM case has slightly lower SSA than the CGOM case for the near IR band.

Although differences in optical properties are evident, it is useful to understand how these differences affect the simulated ice cloud CRE, especially for single bands in the SW and LW spectrum. Here, the RRTMG SW/LW model [64] is implemented with a standard mid-latitude summer atmospheric profile, assuming a solar zenith angle of 60\(^\circ\), a single-layer ice cloud located at 12 km, a surface albedo of 0, and an emissivity of 1 for all wavelengths. The left panel in Fig. 12 shows the differences of TOA SW, LW, and total (SW + LW) ice CRE obtained from CGOM and II-TM+IGOM databases as functions of \(D_{\text{eff}}\) and \(r\) due to the difference in optical properties. The difference is obtained by subtracting the CGOM-based CRE from the II-TM+IGOM-based CRE. The SW radiative effect is negative, meaning that II-TM+IGOM case results in a stronger ice cloud SW radiative effect than the CGOM case. The largest differences can be up to 7 W m\(^{-2}\) for \(r\) ranging from 5 to 10 and \(D_{\text{eff}} = 10 \mu\)m (i.e., small values). The LW radiative effect difference is positive, and twice as large as that of the SW at small \(D_{\text{eff}}\) when \(r\) is near 1. The resulting total CRE
Differences are dominated by the LW feature, resulting in positive differences for small optical thickness values ($\tau < 5$) but negative differences for large values of optical thickness ($\tau > 5$). The right panel shows relevant difference percentages. Although the differences of TOA SW tend to be small in the cases of low $\tau$ and $D_{eff}$ values, the relevant percentage differences may be significant. A similar feature is observed in the LW case.

To identify the contribution of CRE from various spectral bands of RRTMG RTM, the band-by-band CRE is calculated. Fig. 13 shows four SW/LW spectral bands with the largest contributions to the CRE difference between the II-TM+IGOM and the CGOM case. The corresponding differences in the optical properties are clearly shown in the CRE at the spectral bands.

Following Yi et al. [65], the bulk ice cloud optical properties derived from the II-TM+IGOM and the CGOM cases are implemented as parameterizations into the NCAR Community Atmospheric Model (CAM, version 5.1). A ten-year climatology is derived from the model runs with the two parameterizations, and the global annual averaged CRE are calculated and shown in Fig. 14. The SWCRE (Fig. 14a) is negative; whereas, the SWCRE difference (Fig. 14d) is generally positive, indicating that the II-TM+IGOM case results in a larger SWCRE. At the same time, as shown in Fig. 14b and e, the II-TM+IGOM case also has a stronger LWCRE. While the SW/LW CRE difference can be up to 8 W m$^{-2}$ regionally, for example, in the tropical area, the global averaged SW and LW CRE differences are 0.44 W m$^{-2}$ and $-1.35$ W m$^{-2}$, respectively.

Fig. 12. Left panel: RRTMG RTM simulated TOA cloud radiative effect difference between the II-TM+IGOM and CGOM cases: solar band CRE (a), IR band CRE (b), and total (solar + IR) CRE (c). Right panel: the percentage of the relative difference in comparison with the II-TM+IGOM reference.
The II-TM+IGOM-based net CRE (NTCRE) and the differences between the CGOM and the II-TM+IGOM cases are shown in Fig. 14c and f, respectively.

Based on the simulated results, the biases caused by the CGOM in the LW are significantly larger than in the SW. The physical reason is that for the longwave spectrum, the particle size parameters for a relatively large range of particle sizes are small so that the accuracy of the CGOM is rather poor.

6. Summary and conclusions

The numerical techniques based on geometric-optics principles and its modifications provide approximate solutions for light scattering by ice crystals. In reality, the geometric-optics approximation is widely used but generally without an awareness of its uncertainties. Here, following [51], the geometric-optics methods are classified into the conventional geometric optics method (CGOM) and the improved geometric optics method (IGOM). A combination of the IGOM and the invariant imbedding T-matrix method (II-TM) is employed to derive the “benchmark” single-scattering properties of hexagonal particles with diameters from 2 to 10,000 μm over a spectral range from 0.2 to 100 μm. Based on the single-scattering properties, we compute the ice cloud bulk-scattering properties by using the particle size distributions obtained from in situ measurements. We begin with an assessment of the uncertainties of the CGOM in the computations of single- and bulk-scattering properties. Significant biases are identified in the optical properties of small individual ice particles calculated by the CGOM. Ice cloud bulk-scattering properties are applied in MODIS satellite retrieval and ice cloud forcing simulations. In the MODIS satellite retrievals using the visible and shortwave infrared, mean biases in optical thickness caused by applying the CGOM are found to be up to 5% with standard deviations of about 5%. Mean biases and standard deviations for the retrieved effective particle diameters can be over 20% and 30%, respectively, and generally decrease with particle size. However, mean biases in effective diameters for clouds with optical thicknesses above 4 are found to be below about 5%. For retrievals using IR bands, use of CGOM leads to systematic overestimation of the optical thickness by approximately 12% and biases in effective particle diameter ranging from −10% to 10%. In the ice cloud radiative forcing calculations, biases are also found, in particular, from longwave radiative effect where the difference can be as high as 14 W m⁻², although global averaged SW and LW CRE differences are 0.44 W m⁻² and −1.35 W m⁻², respectively.

The sources of uncertainties in the retrieval and climate forcing calculation are primarily two-fold: the microphysical model and the accuracy of the computational
techniques. In the present study, we only focus on the latter by assuming a simplified ice habit (i.e., smooth hexagonal column). In recent studies [63,66–68], the relative differences in retrieved optical thickness and effective particle diameter for different ice models can be comparable or larger than the maximum values found here. Similarly, in CRE calculations, differences in short-wave fluxes resulting from different microphysical models can be on the order of 10–20 W/m² [53,65] for ice particle roughness, and −70 to −30 [53] and −100 to −30 W/m² [53,69] for non-unity particle aspect ratios. Actually, the uncertainties from the aforementioned two sources are not independent. For example, in the analysis of model uncertainties, the techniques used in the calculation of the single-scattering properties are assumed to be accurate. Further analysis of the uncertainties will be performed in future studies.

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