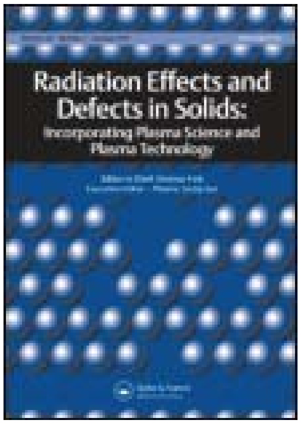

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Restrictions on the quasi-linear description of electron–chorus interaction in the earth's magnetosphere

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The interaction of electrons with coherent chorus waves in the random phase approximation can be described as quasi-linear diffusion for waves with amplitudes below some limit. The limit is calculated for relativistic and non-relativistic electrons. For stronger waves, the friction force should be taken into account.

Keywords: magnetosphere; relativistic electrons; whistlers waves

1. Introduction

The interaction of electrons with whistler mode waves is an important process in the dynamics of the Earth's outer radiation belt (RB) (*1*) and the RBs of other planetary magnetospheres (*2*, *3*), where whistler mode waves can be found as broadband hiss (Jupiter, Saturn, Uranus, and Neptune) and narrowband chorus packets (Jupiter, Saturn, and Uranus) (*4*). The chorus packets consist of apparently randomly separated coherent sub-packets each with a few to few-tens of milliseconds duration (*5*).

Two approaches are used to analyze particle energization and scattering as a consequence of the particle interaction with the chorus wave packets. In the first, "quasi-linear", approach, particle motion is considered to be stochastic because there are multiple interactions with a large number of uncorrelated wave packets. It is believed that the arbitrary sub-packet sequence, medium inhomogeneity, and particle bouncing introduce the required randomization. The observed chorus wave packets are approximated by incoherent broadband noise, and the quasi-linear diffusion theory is applied to the global scale dynamics of the RBs (see Summers et al. (*6*), and references therein). This approach assumes small wave amplitudes, multiple interactions of the waves with the particles, and neglects nonlinear effects.

In the second, "local", approach, different nonlinear mechanisms for particle energization and scattering are analyzed. Particle motion in monochromatic ion cyclotron waves and whistlers (see Albert (*7*, *8*) and references therein) and an inhomogeneous magnetic field is considered, and regular as well as stochastic particle motion is found. Different regimes of particle diffusion are identified. Faith et al. (*9*) showed that stochasticity of particle motion in a strong monochromatic whistler

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wave propagating parallel to the ambient magnetic field results from particle bouncing. Roth et al. (*10*) found that bouncing along the inhomogeneous magnetic field and an oblique monochromatic wave are required for effective stochastic diffusion in energy and pitch-angle. Matsoukis et al. (*11*) established that an electron interacting with two oppositely directed, parallel propagating monochromatic whistler waves exhibits stochastic behavior and energy gain. Wykes et al. (*12*) extended the treatment to broadband whistler wave packets with continuous spectra and showed that the stochastic diffusion mechanism is again present. They estimated the pitch-angle diffusion coefficient for electrons. Khazanov et al. (*13–15*) considered the dynamics of non-relativistic and relativistic particles in a coherent whistler packet with a discrete spectrum and calculated the domain of particle stochastic motion as functions of particle energy and wave packet parameters, energy and pitch-angle diffusion coefficients, and steady-state solutions of diffusion equations.

Cattell et al. (*16*) recently reported whistlers with electric field amplitudes greater than 240 mV/m, which is larger an order of magnitude larger than any previously observed in the Earth's RBs. Cattell et al. (*16*) suggested that the standard quasi-linear approach taken by global scale RB models may be inadequate to understand the dynamics associated with such large amplitude waves.

Both conditions required for the quasi-linear description to be applicable, namely (a) small wave amplitudes and (b) broadband spectra with random phases, can be violated in planetary magnetospheres. This raises a question concerning where and when the limitations on the quasi-linear description of the electron interaction with whistler waves break down.

This paper presents an analysis of the applicability of the quasi-linear approach for the interaction of a strong whistler with relativistic and non-relativistic electrons based on the results obtained by Khazanov et al. (*14*).

2. Electron–chorus interaction

Consider a relativistic electron interacting with a circular polarized whistler wave packet propagating along a homogeneous ambient magnetic field. The wave packet $\vec{A}^w(t,z) = \sum_k \vec{A}_k \exp[i(kz-\omega_k t)]$ will be approximated assuming that the amplitudes are constant, $k = k_0 + m\Delta k$, $\omega = \omega_0 + m\Delta\omega$, $\Delta k/k_0 \ll 1$, $\Delta\omega/\omega_0 \ll 1$ and the particle velocity is larger than the wave group velocity. This results in the space-like wave packet representation (*17*)

$$\vec{A}^w(t,z) = \vec{A}\,\mathrm{e}^{\mathrm{i}\phi} \sum_{m=-N}^{m=N} \exp(izm\Delta k) \cong \vec{A}\,\mathrm{e}^{\mathrm{i}\phi} \sum_n \delta\left(\frac{z}{L} - n\right),$$
$$\phi = k_0 z - \omega_0 t, \quad \Delta k = \frac{2\pi}{L}, \quad N = \frac{2\pi k_0}{\Delta k}. \tag{1}$$

Here, $\vec{A}^w$, ω_0, and k_0 are the wave packet characteristic vector potential, frequency, and wave vector, respectively; z is chosen along the ambient magnetic field, L is the characteristic space scale, and $N \gg 1$ is the number of modes in the packet. Therefore, the initial broad ($N \gg 1$) wave packet is reduced to a monochromatic wave interacting with the particle at some points along the trajectory. This representation qualitatively corresponds to the observations, as can be seen from figure 1 of Santolik (*18*).

The Hamiltonian of the problem presented in the action-angle canonical variables of the unperturbed ($A^w = 0$) problem is

$$H(z,p_z,\theta,I,t) = H_0(p_z,I) + ec\sqrt{2m\omega_B I}H_0^{-1}A\cos\psi \sum_n \delta\left(\frac{z}{L} - n\right),$$

$$H_0 = c\sqrt{m^2c^2 + 2m\omega_B I + p_z^2},$$
$$\omega_B = \frac{eB_0}{mc}, \quad \psi = \phi - \theta. \tag{2}$$

Here, $\tan\theta = -p_x/p_y$ and $I = (p_x^2 + p_y^2)/2m\omega_B$. The phase ψ is the angle between the particle momentum component normal to the ambient magnetic field and the wave magnetic field vector. The equations of motion are

$$\frac{dp}{dt} = eck_0\sqrt{2m\omega_B I}H_0^{-1}A\sin\psi\sum_n \delta\left(\frac{z}{L} - n\right),$$
$$\frac{dI}{dt} = ec\sqrt{2m\omega_B I}H_0^{-1}A\sin\psi\sum_n \delta\left(\frac{z}{L} - n\right), \tag{3}$$
$$\frac{d\psi}{dt} = \frac{k_0 p_z c^2 + \omega_B mc^2}{H_0} - \omega_0, \quad \frac{dz}{dt} = \frac{p_z c^2}{H_0},$$

with the first integral

$$p_z - k_0 I = \text{const} = S. \tag{4}$$

The constant S is set to zero below. The integral of motion reduces the dimension of the system and results after mapping into the equations of motion:

$$I_{n+1}^{3/2} = I_n^{3/2} + Q\sin\psi_n, \quad Q = e\frac{3\sqrt{2m\omega_B}}{2k_0 c}AL,$$
$$\psi_{n+1} = \psi_n + F(I_{n+1}), \quad T = \frac{LH_{0,n+1}}{(p_{z,n+1}c^2)}. \tag{5}$$

The phase shift $F(I_{n+1})$ is presented in Equations (3) and (4) of Khazanov et al. (*14*) and for ultra-relativistic particles reduces to $F(I_{n+1}) = 1.5\mathrm{sgn}\, I_{n+1}/|I_{n+1}|$, while $T = L/c$. Equations (5) in this limit have been studied by Khazanov et al. (*14*), who found that these equations describe a strange attractor and the random walk rate can be characterized by a diffusion coefficient

$$D = \left\langle \frac{\left(I_{n+1}^{3/2} - I_n^{3/2}\right)^2}{T_{n+1}} \right\rangle, \tag{6}$$

with the averaging on the ensemble of particles with different initial $I_0^{3/2}$, ψ_0. They also calculated the upper boundary of the particle energy as a function of wave packet parameters for stochastic diffusion.

The diffusion coefficient is not sensitive to the exact expression for the phase shift. The difference between the diffusion coefficients calculated from the general expression for the phase shift, $F(I_{n+1})$ in Equations (5), and the simplified ultra-relativistic expression is less than 5%.

Khazanov et al. (*14*) set the integral of motion (Equation (4)) equal to zero, a restriction implying a specific choice for the particle's initial conditions. The choice of the integral of motion defines the permitted lower boundary of the momentum modulus, $|p_z|$, in the process interaction.

The interaction is effective only if the resonance condition is satisfied. For a wave with a frequency below the cyclotron frequency, this condition holds only when the wave vector and particle velocity are oppositely directed. For positive k, this means negative velocities, $p_z < 0$. The choice of $S = 0$ means that the particle can move along a straight line in the p_z – I plane until the point (0,0) and all possible resonance velocities for the particle are available. For $S > 0$, positive p_zS are permitted, but they lie outside the resonance region. For $S < 0$ the region of negative p_zS is not available to the particle. This domain is smaller than the initial p_z and can somewhat change the diffusion time compared to the case $S = 0$. Therefore, both restrictions, the ultra-relativistic phase shift and the choice of the initial conditions leading to $S = 0$ do not affect the applicability of the results found by Khazanov et al. (*14*), and they should be valid for a relativistic particle with arbitrary initial conditions.

To determine how well the random phase assumption of quasi-linear theory holds, the first order diffusion coefficient was calculated from Equations (5) and (6). We also calculated the diffusion coefficient in the random phase approximation from these equations. This coefficient is found by substituting $I_{n+1}^{3/2} - I_n^{3/2}$ from the first Equation (5) into Equation (6) and averaging over the phase ψ to obtain $D_{\mathrm{rand}} = Q^2/2T$. The diffusion coefficient found from the simulations and normalized by D_{rand} is close to one for the stochasticity parameter Q in the range from 0 to 0.0025 with an accuracy of a few percent. Therefore, the assumption of phase randomness of the quasi-linear theory is valid.

The second assumption of the quasi-linear approach is that the wave amplitude is small. This condition is equivalent to the assumption of particle motion along an unperturbed trajectory in the derivation of the quasi-linear diffusion coefficient. Corresponding to this requirement, a mapping and diffusion coefficient can be obtained from Equations (3), if the right-hand side in the first of these equations is kept constant and the random phase approximation is used. This coefficient can also be found from Equations (5) and (6) [Albert, private communication, 2007] by the perturbation method. We introduce a variable $\delta = (I_{n+1} - I_n)/I_n$, and keep the three first terms in the Taylor expansion of $I^{3/2}$ in the first equation in the mapping (5) assuming that for diffusion δ is small. Then, this equation gives

$$\delta\left(1 + \frac{\delta}{4}\right) = \frac{3Q}{2I_n^{3/2}} \sin\psi_n. \tag{7}$$

With the second brackets set to one the diffusion coefficient is the same as the quasi-linear diffusion coefficient found by Albert (*19*)

$$D_{\mathrm{QL}} = \left(\frac{1}{2}\frac{\omega_B}{kc}\frac{p_\perp c}{H_0}\frac{B^w}{B_0}\right)^2 \frac{\pi}{\Delta k|v_{\mathrm{II}} - v_{\mathrm{grII}}|}, \tag{8}$$

if the limit of relativistic particles is taken. Here, v_{gr} is the wave group velocity.

Substitution of δ taken in the linear approximation into Equation (7) leads to

$$\delta\left(1 + \frac{Q\sin\psi_n}{6I_n^{3/2}}\right) = Q\sin\psi_n.$$

Therefore, the limit of quasi-linear approximation is defined by the condition $Q/(6I_n^{3/2}) \ll 1$ or

$$\left(\frac{\omega_B}{\omega_0}\frac{mc^2}{H_0}\right)^{3/2}\left(\frac{\mathrm{v}_{\mathrm{ph}}}{2c}\right)^{0.5}\frac{B^w}{B_0}\frac{L\omega_0}{c} \ll 1. \tag{9}$$

Here, v_{ph}, B^w, B_0 whistler's phase velocity, wave magnetic field intensity, and local magnetic field, correspondingly. This analysis can be performed for an arbitrary choice of the first integral, but the results remain practically the same.

For typical parameters in the Earth's RBs, $\omega_0/\omega_B = 0.5$, $v_{\mathrm{ph}} = 0.25c$, $H_0 = 1\,\mathrm{MeV}$, magnetic shell $L_{\mathrm{sh}} = 5$, and scale size $L = 10^9\,\mathrm{cm}$, the quasi-linear approximation is valid for $B^w/B_0 \ll 0.005$. Therefore, for whistlers comparable to these observed by Cattel et al. (*16*), the quasi-linear limit can be violated even if stochasticity develops.

A similar analysis for the non-relativistic particles leads to diffusion coefficient (8) for $v_{\mathrm{II}} \ll v_{\mathrm{grII}}$. The restriction on the applicability of the quasi-linear approach is

$$\left(\frac{\omega_B}{\omega_0}\right)^2 \frac{c}{\mathrm{v}_\perp} \frac{B^w}{B_0} \frac{L\omega_0}{c} \ll 1. \tag{10}$$

For the typical Earth's RB parameters listed above and electron energies $\sim$15 keV, this condition is more severe, $B^w/B_0 \ll 10^{-4}$.

The meaning of the terms neglected in the quasi-linear limit can be clarified from the Fokker–Plank equation that can be written for the distribution function $f(u, \psi, t)$ in the phase space region of particle stochastic motion. In the random phase approximation, the equation is

$$\frac{\partial f}{\partial t} = \frac{\partial^2}{\partial u^2}(Df), \tag{11}$$

where $u = I^{3/2}$; and the diffusion coefficient, calculated from Equations (5) and (6), $D = Q^2/2T$, is independent on u. Transforming Equation (11) to variable I, we obtain

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial I}\left[D\left(\frac{\mathrm{d}I}{\mathrm{d}u}\right)^2 \frac{\partial F}{\partial I}\right] + \frac{\partial}{\partial I}\left[F\frac{D}{2}\frac{\mathrm{d}}{\mathrm{d}I}\left(\frac{\mathrm{d}I}{\mathrm{d}u}\right)^2\right]. \tag{12}$$

Here, $f\,\mathrm{d}u = F\,\mathrm{d}I$ and $D(\mathrm{d}I/\mathrm{d}u)^2 = D_{\mathrm{QL}}$ with D_{QL} that has been found from Equation (7) in the linear approximation. Therefore, the correction to the quasi-linear diffusion due to the dependence of the right-hand side in equation of motion (4) on the particle magnetic momentum results in the friction (drag) force. The role of this term depends on the characteristic scale of the problem and can be of the same order as the diffusion term.

3. Conclusion

Particle motion in the whistler wave packet given by Equation (1) can be described as diffusion. The diffusion coefficient can be calculated in the random phase approximation and coincides with the standard quasi-linear diffusion coefficient for weak waves. The limits on the wave amplitude that permits quasi-linear description are found. For stronger waves not only diffusion, but also drag force effects should be considered. This can be the case for some of the observed whistlers that are discussed by Cattell et al. (*16*).

The results presented in this paper are based on an assumption that the observed chorus can be represented as a monochromatic wave and their interaction with the particles can be described as instantaneous kicks. The model neglects the observed frequency drift. The role of these factors requires additional studies. Another restriction on the applicability of these results occurs in the case of very strong waves when the particle dynamics is intermittent, combining chaotic, and regular motion (*14*, *15*).

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