

Feasibility Criteria for Interval Management Operations as Part of Arrival Management Operations

Ian M. Levitt*

*NextGen Organization, Research and Technology Development Office, Federal Aviation Administration,
Atlantic City, New Jersey, U.S.A.*

Lesley A. Weitz†

The MITRE Corporation, Center for Advanced Aviation System Development, McLean, Virginia, U.S.A.

Bryan E Barmore‡

NASA Langley Research Center Hampton, Virginia, U.S.A.

and

Michael W. Castle§

Aurora Sciences, LLC, Washington, DC, U.S.A.

Interval Management (IM) is a future airborne spacing concept that aims to provide more precise inter-aircraft spacing to yield throughput improvements and greater use of fuel-efficient trajectories for arrival and approach operations. To participate in an IM operation, an aircraft must be equipped with avionics that provide speeds to achieve and maintain an assigned spacing interval relative to another aircraft. It is not expected that all aircraft will be equipped with the necessary avionics, but rather that IM fits into a larger arrival management concept developed to support the broader mixed-equipage environment. Arrival management concepts are comprised of three parts: a ground-based sequencing and scheduling function to develop an overall arrival strategy, ground-based tools to support the management of aircraft to that schedule, and the IM tools necessary for the IM operation (i.e., ground-based set-up, initiation, and monitoring, and the flight-deck tools to conduct the IM operation). The Federal Aviation Administration is deploying a near-term ground-automation system to support metering operations in the National Airspace System, which falls within the first two components of the arrival management concept. This paper develops a methodology for determining the required delivery precision at controlled meter points for aircraft that are being managed to a schedule and aircraft being managed to a relative spacing interval in order to achieve desired flow rates and adequate separation at the meter points.

I. Nomenclature

ADS-B	Automatic Dependent Surveillance - Broadcast
ASI	Actual Spacing Interval
ATA	Actual Time of Arrival
CMP	Controlled Meter Point
d	delay
ETA	Estimated time of Arrival
FAF	Final Approach Fix
IM	Interval Management

* Mathematician, Engineering Development Services (ANG-C33), ian.levitt@faa.gov

† Lead Simulation Modeling Engineer, lweitz@mitre.org

‡ Research Engineer, Crew Systems and Aviation Operations Branch, bryan.barmore@nasa.gov

§ Systems Engineer, mcastle@aurorasciences.com

p.d.f.	probability distribution function
r	required trajectory time
s	nominal trajectory time
STA	Scheduled time of Arrival
TSS	Terminal Sequencing and Spacing
Δ	spacing goal
ϵ	Schedule error
η	Spacing error
σ	delivery precision

II. Introduction

Improvements in communication, navigation, and surveillance systems in the National Airspace System have led to the development of multiple concepts to improve efficiency and enhance safety. For example, the deployment of Automatic Dependent Surveillance-Broadcast (ADS-B) will provide controllers access to more accurate aircraft state information and more frequent update rates than currently available via radar systems. Aircraft equipped with ADS-B transmitters (ADS-B Out) transmit highly accurate Global Navigation Satellite System-based position and velocity information. Aircraft that are additionally equipped with ADS-B receivers (ADS-B In) are able to receive surveillance information about other aircraft in the surrounding airspace.

Interval Management (IM) is an ADS-B-enabled suite of applications that use ground and flight deck capabilities as well as procedures designed to support the relative spacing of aircraft [Barmore04, Murdoch09, Barmore09, Weitz12]. Relative spacing refers to managing the position or time of one aircraft to a time or distance relative to another aircraft, as opposed to a static reference point such as a point over the ground or clock time. This results in improved inter-aircraft spacing precision and is expected to allow aircraft to be spaced closer to the applicable separation standard than current operations. Consequently, if the reduced spacing is used in scheduling, IM can reduce the time interval between the first and last aircraft in an overall arrival flow. Because IM relies on speed changes to achieve precise spacing, it can reduce costly, low-altitude, off-path maneuvering, which increases both efficiency and throughput in capacity-constrained airspace without negatively impacting controller workload and task complexity. This is expected to increase overall system efficiency.

IM is one of three components in the overall arrival flow management system. The first component manages the strategic plan for aircraft arrivals to an airport. This includes runway assignments, a schedule** to the runway, and the associated schedule to the upstream Controlled Meter Points (CMPs). Concepts such as Extended Metering and Coupled Scheduling [Stein11] all contribute to the determination of this plan for the management of aircraft during arrival and approach.

Given this plan, the second component involves the controller managing the traffic to the schedule. Controller experience may be augmented with ground automation tools such as Ground Interval Management – Spacing (GIM-S) [FAA12], Terminal Sequencing and Scheduling (TSS) [Thippavong10], and Relative Position Indicator [Shepley08] to accurately deliver aircraft to frozen Scheduled Times of Arrival (STAs) defined at the CMPs.

Future ground-system deployments will include functionality to help the controller to identify candidate aircraft pairs for an IM operation, information to the controller to initiate the IM operation (e.g., the assigned spacing interval), tools to help the controller monitor the relative spacing, and status information on whether an aircraft is actively conducting an IM operation.

Performance of arrival management systems has been analyzed in various contexts. Ren and Clarke [Ren07] analyzed trajectory uncertainties to determine the minimum targeted spacing at the terminal meter fix on the en-route/terminal boundary that allows aircraft to continue their area navigation (RNAV) arrivals to the runway without controller intervention. In that work, the aircraft were metered to the terminal meter fix by conventional means, and errors in the inter-arrival time were modeled. That methodology was used to establish inter-arrival spacing for flight tests conducted at Louisville International Airport (SDF), and the performance predicted by the model was shown to agree well with measured performance [Ren08]. The delivery precision to STAs for saturated metering operations at Hartsfield-Jackson Atlanta International (ATL) operations was analyzed by Shresta and Mayer [Shresta09]. With a similar motivation to the research in reference [Ren07], Shresta and Mayer determined the delivery precision at the

** A schedule is an ordered list (sequence) of aircraft at a point and the times the aircraft should cross that point.

meter fix required to achieve certain efficient operations (i.e., no vectoring or extended final) in the terminal area. Finally, a tool has been developed and applied by Thipphavong and Mulfinger [Thipphavong10], which combined the effects of metering to the terminal meter fix and the runway, scheduled delay, and controller intervention rates to optimize the performance of a scheduler for the terminal area.

We have previously developed a model to derive performance requirements for determining and meeting STAs for arrival operations [Levitt13]. This paper extends those results into terminal airspace and adds IM operations.

The main contribution of this paper is the development of key mathematical relationships describing the interactions between aircraft being managed to meet STAs at meter points (referred to as the schedule-managed aircraft) and aircraft that are relative spacing (referred to as the spacing-managed aircraft). In particular, two constraints, defining separation and operational feasibility, are developed. From these constraints, the precision with which aircraft need to meet their STAs and the precision with which spacing-managed aircraft need to achieve their relative spacing goals may be determined. The paper is organized as follows: an operational overview is provided in Section II; the constraint relationships are developed in Section III; some preliminary simulation results are presented in Section IV, and lastly, conclusions and future work are described in Section V.

III. Operational Overview

For the modeling in this paper, we assume the arrival operation begins with arriving aircraft being sequenced and scheduled to land at the airport of interest. For simplicity, the runways are treated as having independent flows of traffic. There are a set of common arrival routes, such as Standard Terminal Arrival Routes, feeding the runway of interest. There are multiple CMPs along each route including the points where the different routes merge together.

For IM operations, the spacing-managed aircraft is trying to achieve the spacing at one CMP and then maintain that spacing until the operation is terminated. For mature-state IM, this point, called the Achieve-by Point, is expected to be the final approach fix (FAF). Prior to the Achieve-by Point the aircraft will be working towards the spacing goal but is not required to be at the assigned spacing yet.

Figure 1 shows a generic arrival airspace with routes leading to a single runway.

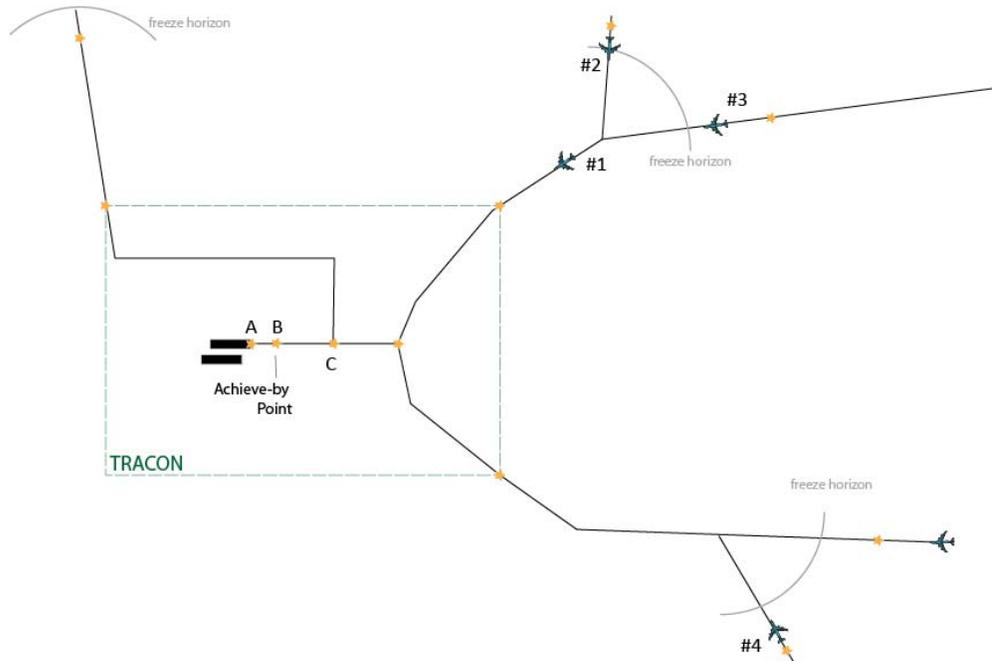


Figure 1 Generic arrival airspace. Yellow stars represent Controlled Meter Points (CMPs).

Similar to the scheduling in the Time-Based Flow Management (TBFM) system [FAA09], aircraft are continually sequenced and scheduled as they approach the arrival airspace. Along each route there is a freeze horizon which is located prior to where aircraft begin their descent. As an aircraft crosses the freeze horizon the sequence and schedule for that aircraft are fixed. The scheduling tool builds the arrival sequence based on a first-come, first-served policy using the estimated times of arrival (ETAs) to the runway. Based on the aircraft's wake category and the expected

operations they will be performing, a spacing goal is defined for each pair that will ensure separation between consecutive aircraft at the CMPs. This defines the schedule which is communicated to the air traffic controller as a set of STAs. During periods of high demand, aircraft will need to be delayed to ensure separation is maintained at downstream CMPs. That is, an aircraft's STA will always match or be later than its ETA (Fig. 2).

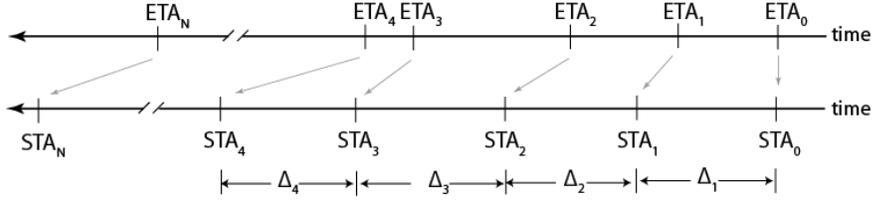


Figure 2 Schematic showing how a sequence of aircraft is converted into a schedule.

There are two methods available to manage an aircraft to the runway. The first is to manage the aircraft to achieve the STAs at each of the CMPs. This is what the proposed controller tool TSS does. The other option is to manage the aircraft to achieve the spacing goal relative to the preceding aircraft in the sequence. This is what IM operations would provide. In the case where speed changes alone are insufficient to achieve the goal, it is assumed that any vectoring of the aircraft occurs soon after the freeze horizon and the following analysis applies once the aircraft is headed back onto the common route structure and under speed control only. The analysis assumes that both types of operations, schedule-managed and spacing-managed, can occur simultaneously in the same airspace, and operations are not segregated. This is referred to as mixed operations. Obviously, any individual aircraft can only be controlled by one of these methods.

IV. Model Development

In this section, we develop analytic models of delivery error distributions for the two different control methods. The models include the constraints for ensuring separation between aircraft at each CMP, as well as the feasibility of the aircraft meeting the required times.

The CMPs are labeled A, B, C, ... starting at the runway threshold, A, and moving backwards to the freeze horizon (see Fig. 1). At each CMP, there is a sequence of aircraft labeled 1, 2, 3, ... When used to label a variable, the CMP label appears as a superscript and the aircraft label as a subscript. For example, the schedule error of the 5th aircraft at CMP B is ϵ_5^B . For quantities that extend between CMPs, the two points are separated by a comma in the label. For example, the nominal trajectory time of the 3rd aircraft between points D and C would be $s_3^{C,D}$. Indices may be omitted when there is no possibility of confusion.

Once the schedule for a CMP is frozen, the scheduled delay for aircraft i is defined to be the difference between the aircraft's ETA and the STA at that meter point.

$$d_i^{CMP} = STA_i^{CMP} - ETA_i^{CMP}$$

It should be noted that ETA_i^{CMP} is fixed when the schedule is frozen and d_i^{CMP} is the cumulative delay from the freeze horizon to the CMP.

The delay between any two meter points, A and B, sharing a freeze horizon will be written $d_i^{B,A}$. Note that for a sequence of CMPs C, B, and A, the delays are related by

$$d^{B,A} = d^A - d^B$$

and

$$d^{C,A} = d^{C,B} + d^{B,A}$$

The nominal trajectory time from the freeze horizon to the CMP is denoted s_i^{CMP} and is equal to the difference between the arrival time at the freeze horizon and ETA_i^{CMP} . The nominal trajectory time between meter points B and A is

$$s_i^{B,A} = ETA_i^A - ETA_i^B$$

The spacing goal for aircraft i at a CMP is denoted by Δ_i^{CMP} and is defined to be the difference between consecutive aircrafts' STAs. The i^{th} spacing goal is

$$\Delta_i^{CMP} = STA_i^{CMP} - STA_{i-1}^{CMP}$$

Claim: For every aircraft, $STA^A - STA^B = s^{B,A} + d^{B,A}$ (1)

Proof:

$$\begin{aligned} STA^A - STA^B &= (ETA^A + d^A) - (ETA^B + d^B) \\ &= (ETA^A - ETA^B) + (d^A - d^B) = s^{B,A} + d^{B,A} \end{aligned}$$

The difference between spacing goals for aircraft $i > 0$ at consecutive CMPs B and A is the difference between the lead and trail's nominal trajectory times plus the difference in their delays.

Claim: $\Delta_i^B - \Delta_i^A = (s_i^{B,A} - s_{i-1}^{B,A}) + (d_i^{B,A} - d_{i-1}^{B,A})$ (2)

Proof:

$$\begin{aligned} \Delta_i^B - \Delta_i^A &= (STA_i^B - STA_{i-1}^B) - (STA_i^A - STA_{i-1}^A) \\ &= (STA_i^B - STA_i^A) - (STA_{i-1}^B - STA_{i-1}^A) \\ &= (s_i^{B,A} + d_i^{B,A}) - (s_{i-1}^{B,A} + d_{i-1}^{B,A}) \\ &= (s_i^{B,A} - s_{i-1}^{B,A}) + (d_i^{B,A} - d_{i-1}^{B,A}) \end{aligned}$$

Define the maximum and minimum *trajectory time control* to be the maximum and minimum time by which an aircraft can adjust its actual arrival time (ATA) to a CMP as δ_{max}^{CMP} and δ_{min}^{CMP} , respectively. Let \mathcal{F}_i^{CMP} be the set of all feasible arrival times for aircraft i to the CMP. Note that $ETA_i^{CMP} \in \mathcal{F}_i^{CMP}$. By definition,

$$\delta_{max}^{CMP} = \max \mathcal{F}_i^{CMP} - ETA_i^{CMP} > 0$$

and

$$\delta_{min}^{CMP} = \min \mathcal{F}_i^{CMP} - ETA_i^{CMP} < 0$$

The values of δ_{max}^{CMP} and δ_{min}^{CMP} depend on the distance-to-go to the CMP, the planned airspeeds and vertical profile, and the wind speeds between CMPs. It is assumed that $\delta_{min}^{CMP} < d_i^{CMP} < \delta_{max}^{CMP}$ for every i , where the margin between scheduled delay and available trajectory time control is balanced with other constraints on the system. If this inequality does not hold then path changes would be needed to implement the delay, d_i^{CMP} .

The *schedule delivery error* for aircraft i at a CMP is denoted by ε_i^{CMP} . It is defined to be the difference between the ATA and the STA.

$$\varepsilon_i^{CMP} = ATA_i^{CMP} - STA_i^{CMP}$$

The actual spacing interval (ASI) that is delivered between consecutive aircraft at a meter point is defined to be the difference between their actual arrival times

$$ASI_i^{CMP} = ATA_i^{CMP} - ATA_{i-1}^{CMP}$$

The *relative spacing error* for aircraft i at a CMP is denoted by η_i^{CMP} . It is defined to be the difference between the actual spacing interval and the spacing goal, and can be related to the schedule delivery error.

$$\eta_i^{CMP} = ASI_i^{CMP} - \Delta_i^{CMP}$$

Claim: $\eta_i^{CMP} = \varepsilon_i^{CMP} - \varepsilon_{i-1}^{CMP}$ (3)

Proof:

$$\begin{aligned}
 \eta_i^{CMP} &= ASI_i^{CMP} - \Delta_i^{CMP} \\
 &= (ATA_i^{CMP} - ATA_{i-1}^{CMP}) - (STA_i^{CMP} - STA_{i-1}^{CMP}) \\
 &= (ATA_i^{CMP} - STA_i^{CMP}) - (ATA_{i-1}^{CMP} - STA_{i-1}^{CMP}) \\
 &= \varepsilon_i^{CMP} + \varepsilon_{i-1}^{CMP}
 \end{aligned}$$

Figure 3 graphically depicts the schedule error, spacing goal, and spacing error.

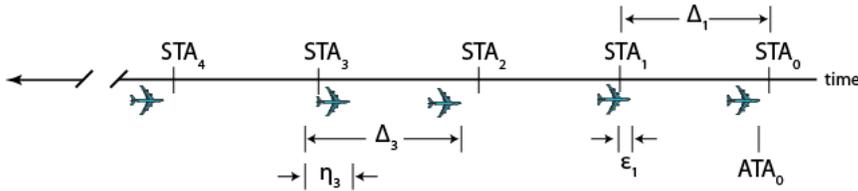


Figure 3: Definition of schedule error, ε , and spacing error, η .

Consider a string of $k + 1$ aircraft, where aircraft 0 is being managed to the schedule, and aircraft 1 through k is being managed to the spacing goal. The schedule delivery error of the k^{th} aircraft, ε_k is a function of ε_0 , the schedule delivery error for aircraft 0, and η_i , the relative spacing error for aircraft i for $i = 1, 2, 3 \dots k$.

Claim: For every CMP, $\varepsilon_k = \varepsilon_0 + \sum_{i=1}^k \eta_i$ (4)

Proof:

One may repeatedly apply Eq. (3) to obtain

$$\varepsilon_k = \varepsilon_{k-1} + \eta_k = \varepsilon_{k-2} + \eta_{k-1} + \eta_k = \dots = \varepsilon_0 + \eta_1 + \eta_2 + \dots + \eta_k$$

Intuitively, $STA_k = STA_0 + \sum_{i=1}^k \Delta_i$ and $ATA_k = ATA_0 + \sum_{i=1}^k (\Delta_i + \eta_i)$, so

$$\begin{aligned}
 \varepsilon_k &= ATA_k - STA_k = ATA_0 - STA_0 + \sum_{i=1}^k \eta_i \\
 &= \varepsilon_0 + \sum_{i=1}^k \eta_i
 \end{aligned}$$

Equation (4) is illustrated in Figure 4.

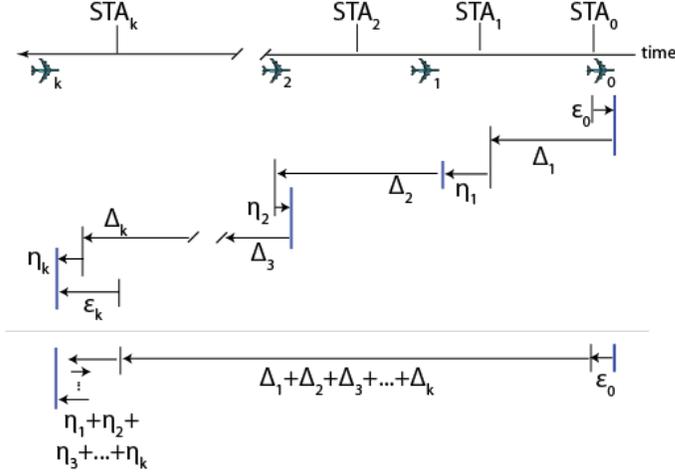


Figure 4 Relationship between schedule error of the k^{th} aircraft and the spacing errors of all preceding aircraft.

Given the STAs and the delivery errors that have been realized, aircraft i will have a required trajectory time to fly from CMP B to CMP A, denoted $r_i^{B,A}$. In the case of a schedule-managed aircraft, it is a function of the nominal trajectory time, the delay allocated, and the upstream delivery error at CMP B.

Claim: For a schedule-managed aircraft i , $r_i^{B,A} = s_i^{B,A} + d_i^{B,A} - \varepsilon_i^B$

(5)

Proof:

$$r_i^{B,A} = STA_i^A - ATA_i^B = STA_i^A - (STA_i^B + \varepsilon_i^B) = s_i^{B,A} + d_i^{B,A} - \varepsilon_i^B$$

In the case of a spacing-managed aircraft at position k in a string, the required trajectory time from CMP B to CMP A is related to the delivery errors of the preceding $k-1$ spacing-managed aircraft in the string and the lead schedule-managed aircraft.

Claim: For a spacing-managed aircraft that is in position k in the string,

$$r_k^{A,B} = s_k^{A,B} + d_k^{A,B} + (\varepsilon_0^B + \sum_{i=1}^k \eta_i^B) - (\varepsilon_0^A + \sum_{i=1}^{k-1} \eta_i^A) \quad (6)$$

Proof:

$$\begin{aligned} r_k^{A,B} &= (ATA_{k-1}^A + \Delta_k^A) - ATA_k^B = ((STA_{k-1}^A + \varepsilon_{k-1}^A) + (STA_k^A - STA_{k-1}^A)) - (STA_k^B + \varepsilon_k^B) \\ &= (STA_k^A - STA_k^B) + \varepsilon_{k-1}^A - \varepsilon_k^B \end{aligned}$$

Applying Eqs. (3) and (4),

$$= s_k^{B,A} + d_k^{B,A} + \left(\varepsilon_0^A + \sum_{i=1}^{k-1} \eta_i^A \right) - \left(\varepsilon_0^B + \sum_{i=1}^k \eta_i^B \right)$$

Figure 5 shows the relationship between the flight times, allocated delays and the trajectory control times.

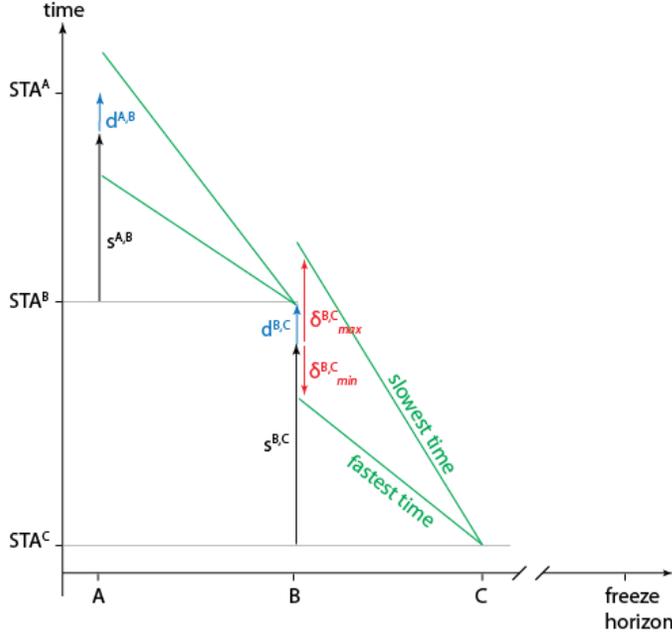


Figure 5: Schematic of how nominal flight time and delay combine to reach the next STA. The minimum and maximum trajectory time controls are also shown.

The performance of delivering a schedule-managed aircraft to a meter fix is measured by the amount of schedule error at the meter fix. The *delivery precision* for schedule-managed operations, σ_{sch} , is defined to be the standard deviation of the schedule error for the schedule-managed aircraft in the assumed operating environment.

The performance of delivering a spacing-managed aircraft to a meter fix is measured by the amount of relative spacing error at the meter fix. The *delivery precision* for spacing-managed operations, σ_{spa} , is defined to be the standard deviation of the relative spacing error for the spacing-managed aircraft in the assumed operating environment. For this analysis, the delivery precision is still considered for spacing-managed aircraft at CMPs which are not the achieve-by point. While IM would not be actively controlling to the assigned spacing goal upstream of the Achieve-by Point, the relative spacing should still be considered there and bounded.

A. Mathematical Modeling of Constraints at a CMP

Extending the results in Ref. [Levitt13], there are two types of constraints modeled for mixed equipage operations.

- *Separation Constraint*: the delivery precision for schedule-managed and spacing-managed operations must not lead to interruptions to the operation too often, because of a pending separation violation. This constraint must be satisfied while also supporting sufficiently high throughput.
- *Feasibility Constraint*: the delivery precision at the upstream CMP must be such that there is sufficient control authority for the aircraft to correct the absolute or relative spacing error to within the precision requirements of the downstream CMP.

B. Separation constraints

Let $M = M^{CMP}(v)$ be the applicable minimum separation requirement between aircraft in-trail or merging at a CMP, which is a function of the groundspeed, v . M is in units of time, converted from a distance-based separation requirement using the expected groundspeed of the trail aircraft at the CMP.

It is assumed that the scheduler assigns STAs that are de-conflicted at all CMPs. This is modeled as there being a minimum spacing goal, which is equal to $M + b$, where b is a fixed spacing buffer added to the minimum separation. For simplicity, we assume that all STAs are set to provide minimum spacing, therefore providing maximum throughput.

The size of the buffer is a function of the underlying distributions for schedule-managed and spacing-managed delivery errors. Define b_{sch} and b_{spa} to be the buffers associated with each control method. First, we express the value of b_{spa} , the buffer size for the spacing-managed aircraft. Let X be the random variable taking the value of the

relative spacing error η^A at a CMP A for a spacing-managed aircraft. Let γ be the allowable interruption rate for the mixed operations. The buffer is determined such that the probability of the spacing-managed aircraft delivering a relative spacing error smaller than $-b_{spa}$ is less than γ .

Let f_X be the probability distribution function (p.d.f.) for the random variable X . Let F_X be the cumulative distribution function, so that the buffer size is constrained by $b_{spa} < |F_X^{-1}(\gamma)|$. For example, in the case that X is a Gaussian random variable, $b_{spa} < 2\sigma_{spa}$ for $\gamma = 0.025$. Figure 6 illustrates F_X and the approach for selecting $-b_{spa}$.

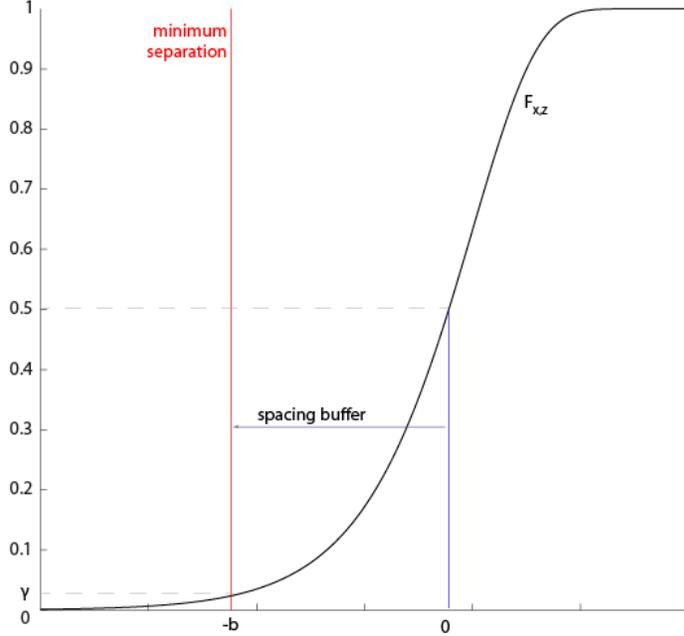


Figure 6 Determining the spacing buffer based on the cumulative distribution function for spacing errors.

The buffer size for the schedule-managed aircraft, b_{sch} , depends on the length of the string of spacing-managed aircraft coming before it. Let the schedule-managed aircraft under study be number $k + 1$. Let the aircraft string preceding the schedule-managed aircraft be numbered $0, 1, 2, \dots, k$, where a string of length 0 indicates a schedule-managed aircraft. By Eq. (3), the relative spacing error for the $(k + 1)^{th}$ aircraft is the difference between its own absolute spacing error and the absolute spacing error of the lead aircraft in the sequence,

$$\eta_{k+1} = \varepsilon_{k+1} - \varepsilon_k \quad (7).$$

Let $Y^{(k)}$ be the random variable taking the value of the schedule delivery error, ε_k , for a spacing-managed aircraft that is number k in the string. From (4), $Y^{(k)} = Y + X_1 + X_2 + \dots + X_k$, where $Y = Y^{(0)}$ is the random variable taking the value of the schedule delivery error for a schedule-managed aircraft and X_i is the relative delivery error for the i^{th} aircraft. From (9), let $Z^{(k)} = Y - Y^{(k)}$ be the random variable taking the value of the relative spacing error for the schedule-managed aircraft under study. Let $g_Y^{(k)}$ be the p.d.f. for $Y^{(k)}$, and $g_Y = g_Y^{(0)}$ be the p.d.f. for Y . Then $f_Z^{(k)} = g_Y * (-g_Y^{(k)})$ is the p.d.f. for Z . Let $F_Z^{(k)}$ be the corresponding cumulative distribution function.

A conservative approach would be to constrain the probability of interruption to be less than γ for every value of k less than some limit, k_{max} . In that case, b_{sch} would be set so that $b_{sch} < \left| [F_Z^{(k)}]^{-1}(\gamma) \right|$ (see Fig. 6) for every $k \leq k_{max}$. Under the assumption that the standard deviation of the schedule delivery error does not decrease with k , it is enough to apply the constraint for $k = k_{max}$.

In the case where X and Y are independent Gaussian distributions with zero mean and standard deviations σ_{spa} and σ_{sch} , respectively, $Z^{(k)}$ is also Gaussian with zero mean and standard deviation $\sqrt{2\sigma_{sch}^2 + k\sigma_{spa}^2}$. Hence, we have

$$b_{sch} < 2\sqrt{2\sigma_{sch}^2 + k\sigma_{spa}^2} \text{ for } \gamma = 0.025.$$

However, short strings of spacing-managed aircraft are more likely than long strings and should be taken into account. Let p_i be the probability that the preceding string of spacing-managed aircraft is of length i , where p_0 is the probability that an aircraft is being managed to the schedule. Then for a given buffer size, the probability of an interruption is

$$P(b_{sch}) = \sum_{i=0}^{k_{max}} p_i F_Z^{(i)}(-b_{sch}) \quad (8)$$

Since P is the sum of monotonic increasing functions of b_{sch} , then P is itself a monotonic increasing function and therefore invertible. Since P^{-1} exists, the constraint is

$$b_{sch} < |P^{-1}(\gamma)|.$$

C. Feasibility Constraint

Consider the k^{th} aircraft of a spacing-managed string. The deviation from the nominal trajectory time that is flown by the k^{th} spacing-managed aircraft is limited by what is feasible. This deviation is a function of the delay allocated, and the delivery errors of all of the k aircraft in the string.

The trajectory time flown by the k^{th} aircraft of a spacing-managed string is given by $t = r_k^{A,B} + \eta_k^A$. Applying equation (6), the deviation from nominal trajectory time is

$$t - s_k^{A,B} = d_k^{A,B} + (\varepsilon_0^B + \sum_{i=1}^k \eta_i^B) - (\varepsilon_0^A + \sum_{i=1}^k \eta_i^A) \quad (9).$$

Note that for $k = 0$, Eq. (9) reduces to the equation for a single schedule-managed aircraft as follows from (5) and as was derived in [Levitt13].

Define the random variable $W^{(k)} = D + Y_A^{(k)} - Y_B^{(k)}$, where D is the delay from CMP B to CMP A; $Y_B^{(k)}$ is the random variable for the relative spacing error of the k^{th} aircraft at CMP B, and, $Y_A^{(k)}$ is the random variable for the relative spacing error of the k^{th} aircraft at CMP A. Let $f_W^{(k)}(x)$ be the joint p.d.f. which gives the probability that the required trajectory time of the k^{th} aircraft in an IM string deviates from nominal by x seconds. If the range of trajectory times available to the aircraft are $[s_k^{B,A} - \delta_{min}^{B,A}, s_k^{B,A} + \delta_{max}^{B,A}]$, then the following inequality defines the feasibility constraint.

$$F_W^{(k)}(\delta_{max}^{B,A}) - F_W^{(k)}(\delta_{min}^{B,A}) > 1 - \gamma \quad (10)$$

Equation (10) defines the probability that the trajectory time of the k^{th} spacing-managed aircraft is within the bounds on the trajectory time control. Equation (10) is sufficiently satisfied if $[F_W^{(k)}]^{-1}(\frac{\gamma}{2}) > \delta_{min}^{B,A}$ and $[F_W^{(k)}]^{-1}(1 - \frac{\gamma}{2}) < \delta_{max}^{B,A}$.

D. Correlations

Relative spacing errors for the k^{th} spacing-managed aircraft are correlated to the relative spacing errors of the preceding aircraft 0 through $k - 1$. Therefore, the distributions governing the behavior $F_Z^{(i)}$ are not straightforward. To see that there is correlation, consider Eq. (6). The trajectory time $r_k^{A,B}$ that is required of a spacing-managed aircraft at position k in the string:

$$r_k^{A,B} = s_k^{A,B} + d_k^{A,B} + \left(\varepsilon_0^B + \sum_{i=1}^k \eta_i^B \right) - \left(\varepsilon_0^A + \sum_{i=1}^{k-1} \eta_i^A \right)$$

Note here that $r_k^{A,B}$ has a dependence on the delivery errors at CMP B and the delivery errors at CMP A. It is clear that $r_k^{A,B}$ increases with upstream delivery errors and decreases with downstream delivery errors. The performance of the relative spacing error η_k^A is clearly conditional on the value of $r_k^{A,B}$, since the flight time, $t = r_k^{A,B} + \eta_k^A$, is limited by the requirement that $t \in [\delta_{min}^{CMP}, \delta_{max}^{CMP}]$. Furthermore, it is believed that when $r_k^{A,B} > s_k^{A,B}$, it is increasingly likely

that $t < \tau_k^{A,B}$ due to the tendency to move to the nominal trajectory time. Given these observations, it is hypothesized that schedule drift at CMP A for a string of spacing-managed Aircraft is limited due to the nature of the negative correlation to the preceding delivery errors at A. If the lead aircraft has a positive delivery error, and is therefore likely ahead of schedule, then the trail is more likely to have a negative delivery error, and therefore more likely to correct back to schedule.

Therefore, we expect less dependence on the string length, k , in Eqs. (8) and (10) than if there was no correlation. A detailed analysis of this correlation will be the subject of future work.

V. Simulation Results

A. Separation Constraint

The separation constraint from equation (7) is used to set a buffer size for the schedule-managed aircraft that is following another schedule-managed aircraft or a spacing-managed string of length k . As described in section III.D, the analytical result derived assuming independent, Gaussian distributions, is conservative as the correlation between relative spacing errors will limit unbounded schedule drift. The probability of a separation violation is explored via simulation for different values of σ_{spa} , σ_{sch} , the probability of an aircraft being managed to spacing, and a spacing buffer factor. The spacing buffer factor is used to determine the buffer size as a multiple, c_{buf} of the standard deviation of the relative spacing error. For a schedule-managed aircraft, the STA difference is determined using the following, where the $\sqrt{2}$ term is to convert σ_{sch} to relative spacing assuming Gaussian independence of schedule-managed delivery errors:

$$STA_i - STA_{i-1} = M + c_{buf} \cdot \sqrt{2} \cdot \sigma_{sch}$$

The STA difference for a spacing-managed aircraft has a similar form, but does not include the $\sqrt{2}$ term because the spacing-managed aircraft manages its spacing relative to its target aircraft rather than the schedule.

$$STA_i - STA_{i-1} = M + c_{buf} \cdot \sigma_{spa}$$

Simulation results are presented here to reveal some of the tradeoffs between throughput and interventions for different spacing buffer factors given different equipage rates and different absolute and relative spacing performance. In the simulation results, relative spacing errors are modeled as correlated, where the mean error of η_i is a linear function of η_{i-1} . More specifically, η_i is assumed to be Gaussian distributed with standard deviation σ_{spa} and mean $-0.2\eta_{i-1}$. As described in Section III.D, the correlation of relative spacing errors is a topic of future work and simulation results will continue to be refined as an understanding of the correlated distributions matures.

Figure 7 shows the frequency of interventions for a spacing-managed rate of 50% and the spacing buffer factor of two as a function of σ_{spa} ; each line represents a different value of σ_{sch} . For a given absolute spacing performance, the frequency of interventions increases as the precision in the spacing performance decreases. However, the frequency of interventions decreases as σ_{sch} increases. Larger values of σ_{sch} lead to larger buffers due to the spacing buffer factor for the schedule-managed aircraft, which help to prevent interventions when a schedule-managed aircraft follows a long string of spacing-managed aircraft. Allowing a larger value of σ_{sch} to achieve fewer interventions does come at a cost to the throughput because of the larger buffers. Figure 8 shows the average throughput for the same parameters. Increasing σ_{sch} from 5 to 12 seconds results in a throughput reduction of about 8 ac/hr (based on a minimum time-based separation standard of 60 seconds).

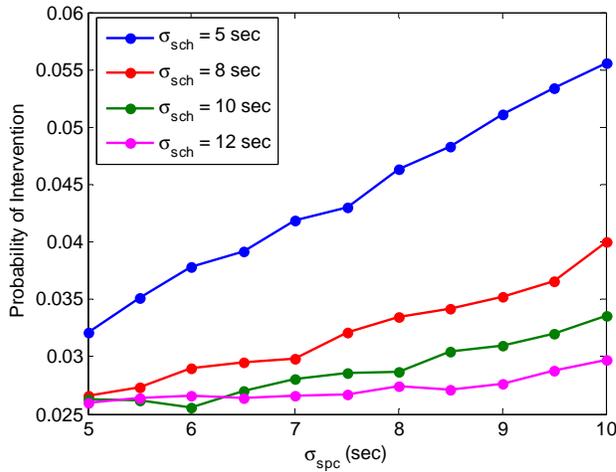


Figure 7. Probability of interventions for spacing-managed equipage rate of 50% and spacing buffer factor = 2.

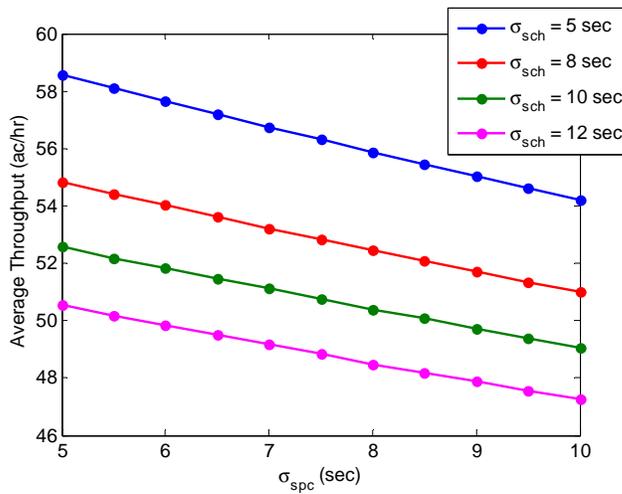


Figure 8. Average throughput for spacing-managed equipage rate of 50% and spacing buffer factor = 2.

Figure 9 shows the interventions when the spacing buffer factor is increased to three. While increasing the spacing buffer factor to three results in fewer interventions (e.g., for $\sigma_{sch} = \sigma_{spa} = 5$ seconds, $c_{buf} = 2$ results in 32 interventions per 1,000 operations and $c_{buf} = 3$ results in 4 interventions per 1,000 operations), the throughput is reduced by about 5 ac/hr. This tradeoff space can be optimized to best meet operational objectives for a given airport and may ultimately help in the derivation of requirements on ground automation and avionics in order to meet throughput and efficiency objectives.

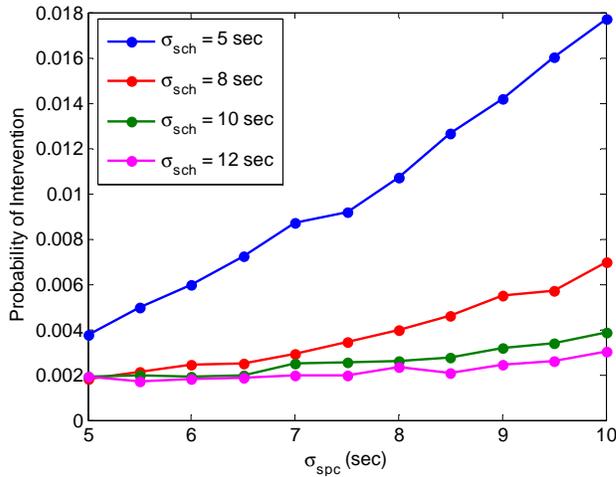


Figure 9. Probability of interventions for spacing-managed equipage rate of 50% and spacing buffer factor = 3.

B. Feasibility Constraint

The feasibility constraint in Eq. (9) describes the trajectory time from CMP B to CMP A for a schedule-managed aircraft when $k = 0$. The probability of the required trajectory time being within the maximum and minimum trajectory control times is explored through simulation. Given a desired precision at the downstream CMP, bounds on the trajectory times, and an allocation of the maximum trajectory time control to the maximum delay, the simulation determines the delivery precision required at the upstream CMP.

The upstream delivery precision is assumed to be zero-mean, Gaussian distributed with standard deviation σ_{sch}^B . Again, as discussed in Section III.B, the downstream delivery errors are correlated to the upstream errors. A simple model of correlation is simulated to present results, and a deeper exploration of the correlation effects will be the subject of future research.

Using equation (5), the downstream delivery precision is modeled to be conditioned on the required trajectory time, $r = s_0^{B,A} + d_0^{B,A} - \epsilon_0^B$. If $\delta_{min}^{B,A} < d_0^{B,A} - \epsilon_0^B < \delta_{max}^{B,A}$, then it is assumed that the aircraft has sufficient trajectory time control to achieve the STA at CMP A and ϵ_0^A is assumed to be zero-mean, Gaussian distributed with standard deviation σ_{sch}^A . If $d_0^{B,A} - \epsilon_0^B$ exceeds one of the trajectory time control bounds, the mean of ϵ_0^A is assumed to be the earliest or latest that the aircraft can arrive at CMP A given the values of $d_0^{B,A}$ and ϵ_0^B , and the distribution is again assumed to be Gaussian with standard deviation σ_{sch}^A .

The probability that the trajectory time will be feasible is explored for $\sigma_{spc}^A = 5$ seconds and two sets of trajectory control times. Trajectory control times were determined from a simulation of a Boeing 737-700 flying the EAGUL5 arrival route at Phoenix Sky International Airport (KPHX). The trajectory control times shown in Table 1 represent the smallest and largest values of $\delta_{max}^{B,A}$ over the last 10 nmi of the procedure to the FAF and for a set of 60 randomly-chosen wind conditions; the slow and fast speeds used to generate $\delta_{max}^{B,A}$ and $\delta_{min}^{B,A}$, respectively, were assumed to be 10% slower and 10% faster than the procedural speed constraints.

Table 1. Trajectory Time Control Values.

Case	Slow Trajectory: $\delta_{max}^{B,A}$ (sec)	Fast Trajectory: $\delta_{min}^{B,A}$ (sec)
1	14.9	-44.5
2	13.9	-33.3

Figure 10 shows the probability that the trajectory time will be feasible for Case 1 with a downstream delivery precision goal of $\sigma_{sch}^A = 5$ seconds and for three different allocations of the maximum trajectory time control to the delay. Given the small window on the trajectory time control in Case 1, the upstream delivery precision needs to be as precise as the downstream delivery precision with no allocation to delay over the last 10-nmi prior to the FAF in order to achieve a feasibility frequency of 0.95.

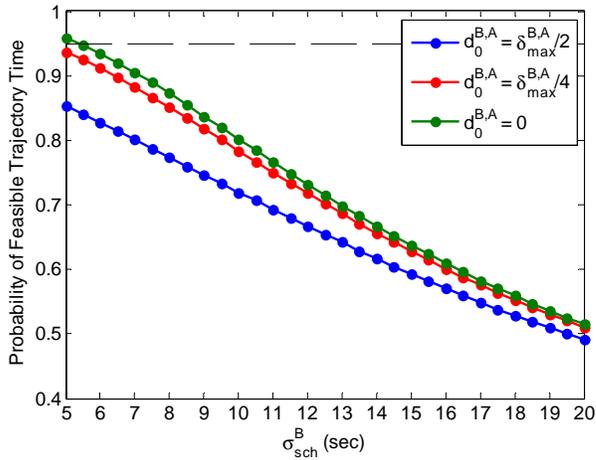


Figure 10. Probability of feasible trajectory time for schedule-managed aircraft ($\sigma_{sch}^A = 5$ seconds and trajectory control time values from Case 1).

The results for Case 2 in Figure 11 show that the upstream delivery precision can be much less accurate when there is a larger trajectory time control window. Furthermore, some delay can be allocated to the last 10 nmi prior to the FAF.

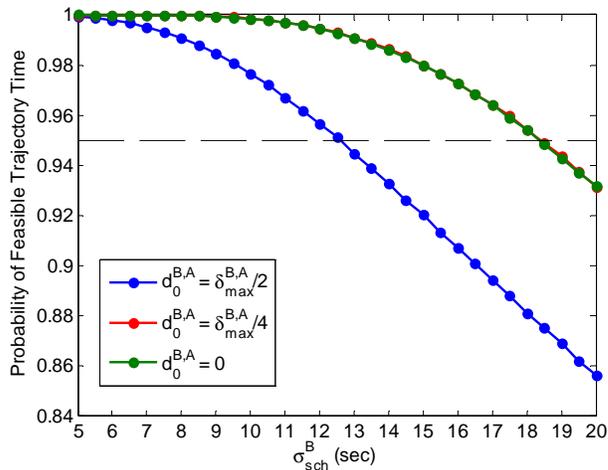


Figure 11. Probability of feasible trajectory time for schedule-managed aircraft ($\sigma_{sch}^A = 5$ seconds and trajectory control time values from Case 2). The red and green curves overlay.

Figure 12 shows the results with Case 2 when the downstream delivery precision is 10 seconds (compared to 5 seconds) in Figure 5. Less precise delivery means that more trajectory time control is needed to ensure that the desired performance is met. Therefore, the delay allocation to the last 10-nmi segment prior to the FAF must be less in this case.

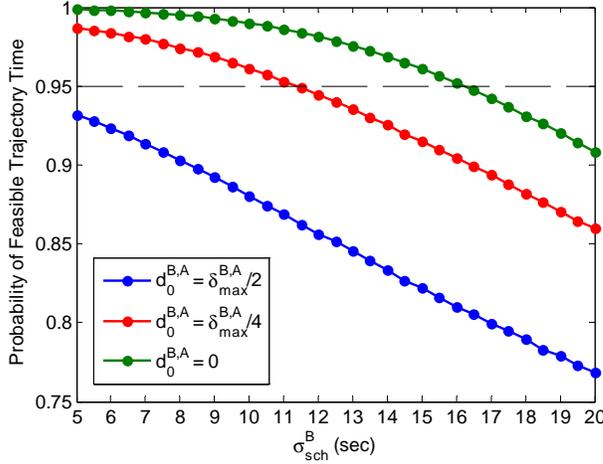


Figure 12. Probability of feasible trajectory time for schedule-managed aircraft ($\sigma_{sch}^A = 10$ seconds and trajectory control time values from Case 2).

As described in equation (6), the feasibility of the k^{th} spacing-managed aircraft in a string providing the desired delivery precision depends on the absolute error of aircraft 0 (a schedule-managed aircraft) at the upstream and downstream CMPs, the relative errors of the $k - 1$ preceding spacing-managed aircraft at the CMP A and B, and the relative error of the k^{th} spacing-managed aircraft at CMP A. The absolute errors of aircraft 0 at CMPs A and B are distributed as described in the previous simulation example. As discussed in Section III.B, the value of η_k^A is correlated both to delivery errors of aircraft k to the upstream CMP B, and the delivery errors of aircraft 1 through k at CMP A. The simulation results below are generated assuming that the mean relative error of aircraft j is correlated with the relative error of aircraft $j-1$ as in section D above.

To model the behavior of η_k^A based on η_k^B , the marginal distributions of the relative errors are assumed to be Gaussian with a standard deviation of σ_{spa}^B at the upstream meter point and σ_{spa}^A at the downstream meter point. When the required trajectory time is within the trajectory time control window, i.e. when

$$[-\delta_{min}^{B,A} < d_0^{B,A} + (\epsilon_0^A + \sum_{i=1}^{k-1} \eta_i^A) - (\epsilon_0^B + \sum_{i=1}^k \eta_i^B) < \delta_{max}^{B,A}],$$

then it is assumed that the aircraft has sufficient trajectory time control to achieve the spacing goal at CMP B and there is no further correlation modeled for η_k^A . That is, η_k^A is assumed to be Gaussian with standard deviation σ_{spa}^A and mean $-0.2\eta_{k-1}^A$. If the required trajectory time exceeds a bound on the trajectory time control, then relative error η_k^A is assumed to be Gaussian with standard deviation σ_{spa}^A and mean determined to be the earliest or the latest that the aircraft can arrive at CMP B given the values of $d_0^{B,A}$, ϵ_0^B , ϵ_0^A , $\sum_{i=1}^k \eta_i^B$, and $\sum_{i=1}^{k-1} \eta_i^A$.

Figure 13 shows the feasibility probability for $k = 5$ and given different values of σ_{spa}^B and σ_{sch}^B . The desired delivery accuracies for schedule and spacing management are assumed to be $\sigma_{sch}^A = 10$ and $\sigma_{spa}^A = 5$ seconds, respectively. The results in Figure 13 are for the trajectory time control values in Case 1 with $d_0^{B,A} = 0$ seconds. In this case, the limited trajectory time control leads to lower feasibility probabilities, which will result in worse than desirable performance at CMP A. Furthermore, Figure 13 indicates that the relative errors at the upstream meter point need to be as good or better than the downstream delivery to maximize feasibility.

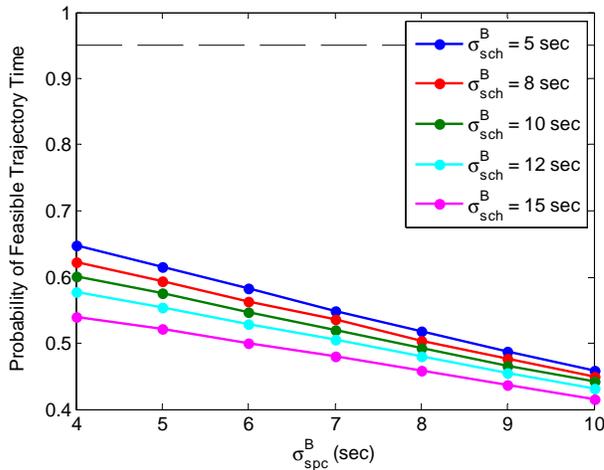


Figure 13. Probability of feasible trajectory time for k^{th} spacing-managed aircraft (trajectory time control values for Case 1).

In comparison, Figure 14 shows the feasibility probability for the Case 2 trajectory time control values with $k = 5$, $\sigma_{sch}^A = 10$ and $\sigma_{spa}^A = 5$ seconds, and $d_0^{B,A} = \delta_{max}^{B,A}/4$ seconds. The larger trajectory time control increases the probability of a feasible trajectory time, but also enables some portion of the trajectory time control to be allocated to delay between CMPs B and A. However, Figure 13 also shows that the upstream delivery precision should be close to the desired delivery precision at the downstream point.

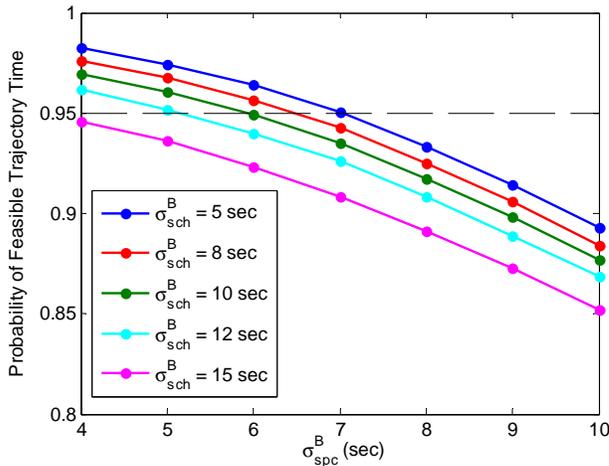


Figure 14. Probability of feasible trajectory time for k^{th} spacing-managed aircraft (trajectory time control values for Case 2).

VI. Conclusion

We extended the results of Ref. [Levitt13] to include mixed equipage operations and to apply generally to a set of Controlled Meter Points with corresponding aircraft in sequence. The relationships derived between the fundamental quantities—such as delay, trajectory times, and delivery errors—reveal important aspects of the performance of integrated operations. This work furthers the understanding of such operations and future development will help guide how they are constructed, adapted, and managed.

In next steps, we will explore the hypotheses on correlation between delivery errors and delay. From this, the distributions of the random variables will be modeled and fast-time simulation may be used to define and validate the curves. We will then apply the analysis to a network of CMPs at an airport, starting at the runway and working backwards to the En-Route Meter Point to determine delivery precision and delay requirements for the airspace. This

analysis will also reveal other aspects of adaptation that should be considered, such as the dependency of delay allocation to the current wind conditions.

VII. Acknowledgments

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