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Identifiability of Additive, Time-Varying Actuator and Sensor Faults by State Augumentation

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Abstract

Recent work has provided a set of necessary and sufficient conditions for identifiability of additive step faults (e.g., lock-in-place actuator faults, constant bias in the sensors) using state augmentation. This paper extends these results to an important class of faults which may affect linear, time-invariant systems. In particular, the faults under consideration are those which vary with time and affect the system dynamics additively. Such faults may manifest themselves in aircraft as, for example, control surface oscillations, control surface runaway, and sensor drift. The set of necessary and sufficient conditions presented in this paper are general, and apply when a class of time-varying faults affects arbitrary combinations of actuators and sensors.

The results in the main theorems are illustrated by two case studies, which provide some insight into how the conditions may be used to check the theoretical identifiability of fault configurations of interest for a given system. It is shown that while state augmentation can be used to identify certain fault configurations, other fault configurations are theoretically impossible to identify using state augmentation, giving practitioners valuable insight into such situations. That is, the limitations of state augmentation for a given system and configuration of faults are made explicit. Another limitation of model-based methods is that there can be large numbers of fault configurations, thus making identification of all possible configurations impractical. However, the theoretical identifiability of known, credible fault configurations can be tested using the theorems presented in this paper, which can then assist the efforts of fault identification practitioners.

Contents

1 Introduction

1.1 Background

In this paper, the term *fault* denotes a state in a dynamical system, which may result in a malfunction or failure of the system [1]. A *failure* is either an intermittent or permanent interruption of a system's ability to fulfill a desired function [2]. In dynamical systems with actuators and sensors, faults in these components may lead to failures characterized by, for example, instability and loss of control.

In domains such as civil aviation and space operations, such actuator and sensor faults can have particularly serious implications for safety and reliability. For example, actuator faults such as rudder runaway have been implicated in multiple aviation incidents (for example, see [3] and [4]). Other actuator faults such as undesired control surface oscillations (e.g., the *oscillatory failure case*) can increase the structural loads on an aircraft and compromise its structural integrity in flight [5, 6]. As a further example, sensor-bias faults have contributed to the failure of missions such as NASA's Demonstration of Autonomous Rendezvous Technology (DART) [7]. Many more examples of aviation incidents and accidents where actuator or sensor faults were implicated as contributing factors can be found in [8].

In the aerospace industry, safety and reliability concerns associated with actuators and sensors are primarily addressed through techniques based on hardware redundancy [9, 10]. The counterpart to hardware redundancy is generally referred to as *analytical redundancy*, a broad class of techniques which make use of mathematical models of a system to detect and identify actuator and sensor faults. Such model-based fault detection and identification (FDI) methods have received significant attention in the literature over the last several decades. For surveys on a variety of model-based FDI techniques see [11–14].

The model-based technique investigated in this paper uses multiple models, where each model corresponds to the nominal system state augmented by a set of fault configurations of interest. Typically, a bank of detection filters is used to estimate the present state of the aircraft, and multiple-hypothesis testing determines if a fault has occurred [15–17]. Several authors have proposed using state augmentation, e.g., [16–21]. However, a key requirement for state augmentation to be effective is that each model, or fault configuration of interest, be identifiable [22].

Identification of constant bias-type faults was initially treated in [23]. The preliminary conditions for identifiability of these faults in arbitrary combinations of actuators and sensors were presented in [17], and a subsequent, detailed analysis was given in [24]. A complete characterization of a set of necessary and sufficient conditions, including numerical examples, can be found in [22]. This paper extends these results to the broader class of additive, time-varying faults, which includes faults such

as the sinusoidal and ramp faults, while still remaining applicable to the step fault.

This paper treats time-varying faults as outputs of exogenous, linear, time-invariant (LTI) systems which then additively affect a given LTI system of interest. In particular, a set of necessary and sufficient conditions for identifiability of additive, time-varying faults affecting arbitrary combinations of (1) actuators only, (2) sensors only, and (3) combinations of actuators and sensors are presented. These conditions fully characterize the identifiability of such faults using state augmentation, which provides designers a deeper understanding of the theoretical reasons for non-identifiability and have practical implications on the limitations of using state augmentation alone, especially when the theory shows a fault configuration of interest is not identifiable by state augmentation. The approach in this paper is the investigation of the causes of non-identifiability beyond the level of (numerical or analytical) rank tests, i.e., it seeks to provide exactly the reason(s) why such rank tests will fail for a particular system and set of faults.

The remaining sections of this paper are organized as follows: Section 2 gives a short review of observability, detectability, and the Rosenbrock System Matrix; Section 3 develops a state-space representation for time-varying actuator and sensor faults using state augmentation; Section 4 presents a set of necessary and sufficient conditions for identifiability of additive, time-varying faults affecting arbitrary combinations of actuators, sensors, or both, where examples using a practical system are included; Section 5 presents the conclusions of the research and suggestions for future directions.

2 A Short Review of Key Concepts

This section provides a short review of the concepts of observability, detectability, and the Rosenbrock System Matrix (RSM).

2.1 A Review of Observability and Detectability

First, consider a system having state-space representation given by

$$\dot{x}(t) = Ax(t) + Bu(t),\ x(0) = x_0, \tag{1}$$

$$y(t) = Cx(t) + Du(t), \tag{2}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{l\times n}$, and $D \in \mathbb{R}^{l\times m}$. Furthermore, $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^l$ are the state, input, and output vectors, respectively. In many practical systems, certain states may not be directly measurable, so state estimators (i.e., observers) are used to infer the unknown state(s) by making use of the output vector $y(t)$ and the input vector $u(t)$. For an observer to be effective, a system must generally be *observable*, or at the very least *detectable*.

A system is said to be *observable* if there exists a time $t_1 > 0$ such that any initial state x_0 can be uniquely determined from $y(t),\ t \in [0, t_1]$ [25]. Two equivalent tests for observability given in [26] are the *Popov-Belevitch-Hautus (PBH) rank test* and the *PBH eigenvector test.*

Test 1. PBH Rank Test. The system given by Equations (1) and (2) is *observable* if and only if for every eigenvalue λ_i of A, that is for every $\lambda_i \in \Lambda(A)$, where $\Lambda(A)$ denotes the set of eigenvalues of A,

$$\operatorname{rank} \begin{bmatrix} \lambda_i I - A \\ C \end{bmatrix} = n$$

for $i = 1, 2, \ldots, n$.

Test 2. PBH Eigenvector Test. The system given by Equations (1) and (2) is observable if and only if there does not exist a nonzero $\gamma \in \mathbb{C}^n$ such that

$$\begin{bmatrix} \lambda_i I - A \\ C \end{bmatrix} \gamma = 0$$

for $i = 1, 2, \ldots, n$. That is, the pair (C, A) is observable if and only if $C\gamma \neq 0$ for every eigenvector γ of A.

If either Test 1 or 2 fails for any value $\lambda_i,\ i = 1, \ldots, n$, then λ_i is an eigenvalue corresponding to an unobservable mode of A. An eigenvalue λ is an *open left-half plane (OLHP) eigenvalue* if and only if $\mathfrak{Re}(\lambda) < 0$.

Let $\Lambda_u(A)$ denote the set of eigenvalues that lie on the closed right half plane. The system given by Equations (1) and (2) is said to be *detectable* if and only if all of the eigenvalues associated with unobservable modes of A lie in the OLHP [25]. Thus, a system is detectable if and only if either Test 1 or 2 is satisfied for $\{\lambda_j :\ \mathfrak{Re}(\lambda_j) \geq 0\},\ j = 1, 2, \ldots, k$, where $k \leq n$ [25].

Since observability requires that Test 1 be satisfied for all eigenvalues of A, observability implies detectability. However, because detectability requires only those eigenvalues with non-negative real parts correspond to observable modes of A, an unobservable system may still be detectable. Finally, a system may be detectable, but if it has eigenvalues with negative real parts associated with unobservable modes of A, the system is still not observable. Thus, observability implies detectability, but detectability does not imply observability.

2.2 Deriving the Rosenbrock System Matrix

The Rosenbrock System Matrix (RSM) and some of its important properties are used in the proofs of conditions for time-varying actuator and sensor fault identifiability. The derivation of the RSM follows.

Consider the system given by Equations (1) and (2). This system can be represented in the frequency domain by its one-sided Laplace

transformation as

$$s\hat{x}(s) - x_0 = A\hat{x}(s) + B\hat{u}(s), \tag{3}$$

$$\hat{y}(s) = C\hat{x}(s) + D\hat{u}(s), \tag{4}$$

where $\hat{x}(s), \hat{u}(s)$, and $\hat{y}(s)$ are the Laplace transforms of the state, input, and output vectors, respectively. Furthermore, x_0 is the initial condition at time $t = 0$, that is, $x(0)$. Here, the one-sided Laplace transform, $\hat{\Upsilon}(s)$, of a function $\Upsilon(t)$ is defined as

$$\hat{\Upsilon}(s) = \int_{0^-}^{\infty} \Upsilon(t)e^{-st}\,dt. \tag{5}$$

Now, the representation given by Equations (3) and (4) can be expressed as

$$\begin{bmatrix} sI - A & -B \\ C & D \end{bmatrix} \begin{bmatrix} \hat{x}(s) \\ \hat{u}(s) \end{bmatrix} = \begin{bmatrix} x_0 \\ \hat{y}(s) \end{bmatrix}. \tag{6}$$

The coefficient matrix in (6) is referred to as the *Rosenbrock System Matrix* of the system having realization $\{A, B, C, D\}$.

3 Time-Varying Fault Modeling

This following development gives a representation for time-varying faults by treating them as outputs of an exogenous LTI system driven only by initial conditions. It is assumed throughout that the fault of interest has a one-sided Laplace transform as defined by Equation (5).

3.1 A Representation for Time-Varying Faults

Let $f(t)$ be a vector of faults. Such faults may be modeled as the output of an LTI system having state-space representation given by

$$\dot{x}_f(t) = A_f x_f(t), \;\; x_f(0) = x_{f_0}, \tag{7}$$

$$f(t) = C_f x_f(t), \tag{8}$$

where $A_f \in \mathbb{R}^{n_f \times n_f}, C_f \in \mathbb{R}^{\mu \times n_f}, x_f(t) \in \mathbb{R}^{n_f}$, and $f(t) \in \mathbb{R}^{\mu}$. It is assumed that C_f has full row rank, that is, $\text{rank}(C_f) = \mu$. Taking the Laplace transform of Equations (7) and (8), and solving for $\hat{x}_f(s)$ in Equation (7) gives

$$\hat{x}_f(s) = (sI - A_f)^{-1} x_{f_0}, \tag{9}$$

$$\hat{f}(s) = C_f \hat{x}_f(s). \tag{10}$$

Now, substituting Equation (9) into Equation (10) gives

$$\hat{f}(s) = C_f (sI - A_f)^{-1} x_{f_0}.$$

Thus, the frequency domain representation of the fault vector is the zero-input response of the system given by $C_f(sI - A_f)^{-1} x_{f_0}$.

3.2 Actuator Fault Modeling

Consider a system given by Equations (1) and (2). If m_k of the m actuators are affected by additive, time-varying faults, the system dynamics can be represented as

$$\dot{x}(t) = Ax(t) + \sum_{j\in\mathcal{F}_{ak}} b_j\overline{u}_j(t) + \sum_{j\notin\mathcal{F}_{ak}} b_j u_j(t) + \omega_p(t), \tag{11}$$

$$= Ax(t) + \overline{B}^k\overline{u}^k(t) + B^k u^k(t) + \omega_p(t), \tag{12}$$

$$y(t) = Cx(t) + \omega_s(t), \tag{13}$$

where $\overline{B}^k \in \mathbb{R}^{n\times m_k}$, $B^k \in \mathbb{R}^{n\times(m-m_k)}$, $\overline{u}^k(t) \in \mathbb{R}^{m_k}$, $u^k(t) \in \mathbb{R}^{m-m_k}$, $\omega_p(t) \in \mathbb{R}^n$, and $\omega_s(t) \in \mathbb{R}^l$. Furthermore, $\mathcal{F}_{ak}$ in Equation (11) denotes the set of indices corresponding to the failed actuators, $\overline{u}_j(t) \in \mathbb{R}$ denotes a faulty input associated with a faulty actuator at time t, $u_j(t) \in \mathbb{R}$ denotes a non-faulty input associated with a non-faulty actuator at time t, and $b_j \in \mathbb{R}^n$ denotes the particular column of B (see Equation (1)) affected by the corresponding faulty or non-faulty actuator. Thus, Equation (12) represents the system subject to a particular actuator fault configuration.

Now consider the state-space representation for time-varying faults given by Equations (7) and (8). Observe that such a representation may model time-varying faults in the actuators, resulting in a system with fault state vector, $x_a(t)$, and output $\overline{u}^k(t)$, i.e.,

$$\dot{x}_a(t) = A_a x_a(t) + \omega_{p_a}(t),\ x_a(0) = x_{a_0}, \tag{14}$$

$$\overline{u}^k(t) = C_a x_a(t) + \omega_{s_a}(t), \tag{15}$$

where $A_a \in \mathbb{R}^{n_a\times n_a}$, $C_a \in \mathbb{R}^{m_k\times n_a}$, and $x_a(t) \in \mathbb{R}^{n_a}$. Additionally, terms for the exogenous actuator fault process and measurement noise, i.e., $\omega_{p_a}(t) \in \mathbb{R}^{n_a}$ and $\omega_{s_a}(t) \in \mathbb{R}^{m_k}$, respectively, have been added. These terms allow for the modeling of faults which might not manifest themselves as entirely deterministic representations in practice. Now, Equations (12)-(15) can be combined to represent the interconnected system. Thus, the state-augmented system can be represented as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_a(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & \overline{B}^k C_a \\ 0 & A_a \end{bmatrix}}_{A_\xi^k} \underbrace{\begin{bmatrix} x(t) \\ x_a(t) \end{bmatrix}}_{\xi^k(t)} + \underbrace{\begin{bmatrix} B^k \\ 0 \end{bmatrix}}_{B_\xi^k} u^k(t)$$

$$+ \underbrace{\begin{bmatrix} \overline{B}^k\omega_{s_a}(t) + \omega_p(t) \\ \omega_{p_a}(t) \end{bmatrix}}_{\omega_{\xi_p}^k(t)}, \tag{16}$$

$$y(t) = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_\xi^k} \begin{bmatrix} x(t) \\ x_a(t) \end{bmatrix} + \omega_s(t), \tag{17}$$

where, after making the appropriate substitutions indicated by the braces, the system above can be expressed more compactly as

$$\dot{\xi}^k(t) = A_\xi^k \xi^k(t) + B_\xi^k u^k(t) + \omega_{\xi_p}^k(t), \tag{18}$$

$$y(t) = C_\xi^k \xi^k(t) + \omega_s(t). \tag{19}$$

Equations (18) and (19) model the general case of arbitrary, additive, time-varying actuator faults treated in this paper. In the case of step faults, $A_a = 0_{m_k \times m_k}$, and $C_a = I_{m_k}$. Thus, Equation (16) reduces to

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_a(t) \end{bmatrix} = \begin{bmatrix} A & \overline{B}^k \\ 0 & I_{m_k} \end{bmatrix} \begin{bmatrix} x(t) \\ x_a(t) \end{bmatrix} + \begin{bmatrix} B^k \\ 0 \end{bmatrix} u^k(t) \\ + \begin{bmatrix} \overline{B}^k \omega_{s_a}(t) + \omega_p(t) \\ \omega_{p_a}(t) \end{bmatrix}, \end{aligned} \tag{20}$$

which is identical to Equation (4) in [22], where the case of actuator step faults was considered.

3.3 Sensor Fault Modeling

Consider again a system given by Equations (1) and (2). If q of the l sensors for the given system are affected by additive, time-varying faults, the system dynamics can be represented as

$$\dot{x}(t) = Ax(t) + Bu(t) + \omega_p(t), \tag{21}$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \tag{22}$$

$$= Cx(t) + \begin{bmatrix} 0 \\ y_s(t) \end{bmatrix} + \omega_s(t), \tag{23}$$

where $A, B, C, x(t), u(t), y(t), \omega_p(t)$, and $\omega_s(t)$ are as previously defined, and $y_1(t) \in \mathbb{R}^{l-q}$ and $y_2(t) \in \mathbb{R}^q$ represent the vectors containing the fault-free sensor measurements and the faulty sensor measurements, respectively. Finally, $y_s(t) \in \mathbb{R}^q$ is the vector containing the additive time-varying sensor faults affecting the q faulty sensors.

Furthermore, consider the fault dynamics represented by Equations (7) and (8). Such a representation can be modified to address the specific case of sensor faults as

$$\dot{x}_s(t) = A_s x_s(t) + \omega_{p_s}(t), x_s(0) = x_{s0}, \tag{24}$$

$$y_s(t) = C_s x_s(t) + \omega_{s_s}(t), \tag{25}$$

where $A_s \in \mathbb{R}^{n_s \times n_s}$, $C_s \in \mathbb{R}^{q \times n_s}$, $x_s(t) \in \mathbb{R}^{n_s}$, $y_s(t) \in \mathbb{R}^q$, and $\omega_{p_s}(t) \in \mathbb{R}^{n_s}$ and $\omega_{s_s}(t) \in \mathbb{R}^q$ are exogenous sensor fault process and measurement noise, respectively. Observe that Equations (24) and (25) model the sensor faults present in Equation (23). Thus, Equations (21) and (23)

can be viewed as an LTI system with q of its outputs affected by time-varying sensor faults given by the output of the exogenous LTI system in Equations (24) and (25). The interconnected system can be represented in augmented state-space form by letting $\omega_s(t) = \begin{bmatrix} \omega_{s,1}^T(t) & \omega_{s,2}^T(t) \end{bmatrix}^T$ and by substituting Equation (25) into Equation (23) as follows

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_s(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0_{n\times n_s} \\ 0_{n_s\times n} & A_s \end{bmatrix}}_{A_\eta} \underbrace{\begin{bmatrix} x(t) \\ x_s(t) \end{bmatrix}}_{\eta(t)} + \underbrace{\begin{bmatrix} B \\ 0_{n_s\times m} \end{bmatrix}}_{B_\eta} u(t) + \underbrace{\begin{bmatrix} \omega_p(t) \\ \omega_{p_s}(t) \end{bmatrix}}_{\omega_{\eta p}(t)}, \tag{26}$$

$$y(t) = \underbrace{\begin{bmatrix} C_1 & 0_{(l-q)\times n} \\ C_2 & C_s \end{bmatrix}}_{C_\eta} \begin{bmatrix} x(t) \\ x_s(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \omega_{s_1}(t) \\ \omega_{s_2}(t) + \omega_{s_s}(t) \end{bmatrix}}_{\omega_{\eta_s}(t)}, \tag{27}$$

where by making the appropriate substitutions indicated by the braces, the system above can be expressed more compactly as

$$\dot{\eta}(t) = A_\eta \eta(t) + B_\eta u(t) + \omega_{\eta_p}(t), \tag{28}$$

$$y(t) = C_\eta \eta(t) + \omega_{\eta_s}(t). \tag{29}$$

Equations (28) and (29) model the general case of additive, time-varying sensor faults treated in this paper.

In the case of sensor step faults, $A_s = 0_{q\times q}$, and $C_s = I_q$. Thus, when all of the sensor faults are step faults, Equations (26) and (27) reduce to

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_s(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_s(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} \omega_p(t) \\ \omega_{p_s}(t) \end{bmatrix}, \tag{30}$$

$$y(t) = \begin{bmatrix} C_1 & 0 \\ C_2 & I_q \end{bmatrix} \begin{bmatrix} x(t) \\ x_s(t) \end{bmatrix} + \begin{bmatrix} \omega_{s_1}(t) \\ \omega_{s_2}(t) + \omega_{s_s}(t) \end{bmatrix}. \tag{31}$$

Equations (30) and (31) are identical to Equations (17) and (18) in [22], where the case of sensor step faults was considered.

3.4 Simultaneous Actuator and Sensor Fault Modeling

Consider the case of time-varying actuator faults represented by Equations (14) and (15) and the case of time-varying sensor faults represented by Equations (24) and (25). Now, in order to represent simultaneous, additive, actuator and sensor faults, it is sufficient to augment the states in the form $\begin{bmatrix} x(t)^T & x_a(t)^T & x_s(t)^T \end{bmatrix}^T$. Thus, when q of the l sensors and m_k of the m actuators are affected by time-varying faults, the aug-

mented system can be expressed as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_a(t) \\ \dot{x}_s(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & \overline{B}^k C_a & 0_{n\times n_s} \\ 0_{n_a\times n} & A_a & 0_{n_a\times n_s} \\ 0_{n_s\times n} & 0_{n_s\times n_a} & A_s \end{bmatrix}}_{A_\varphi} \underbrace{\begin{bmatrix} x(t) \\ x_a(t) \\ x_s(t) \end{bmatrix}}_{\varphi(t)} + \underbrace{\begin{bmatrix} B^k \\ 0 \\ 0 \end{bmatrix}}_{B_\varphi} u^k(t)$$

$$+ \underbrace{\begin{bmatrix} \overline{B}^k \omega_{s_a}(t) + \omega_p(t) \\ \omega_{p_a}(t) \\ \omega_{p_s}(t) \end{bmatrix}}_{\omega_{\varphi p}(t)}, \tag{32}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} C_1 & 0_{(l-q)\times n_a} & 0_{(l-q)\times n_s} \\ C_2 & 0_{q\times n_a} & C_s \end{bmatrix}}_{C_\varphi} \begin{bmatrix} x(t) \\ x_a(t) \\ x_s(t) \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} \omega_{s,1}(t) \\ \omega_{s,2}(t) + \omega_{s_s}(t) \end{bmatrix}}_{\omega_{\varphi_s}(t)}, \tag{33}$$

where after making the appropriate substitutions indicated by the braces, the system above can be expressed compactly as

$$\dot{\varphi}(t) = A_\varphi \varphi(t) + B_\varphi u^k(t) + \omega_{\varphi_p}(t), \tag{34}$$

$$y(t) = C_\varphi \varphi(t) + \omega_{\varphi_s}(t). \tag{35}$$

Equations (34) and (35) model the general case of simultaneous, additive, time-varying actuator and sensor faults treated in this paper.

For the case when all actuator and sensor faults are step faults, $A_a = 0_{m_k\times m_k}, A_s = 0_{q\times q}, C_a = I_{m_k}$, and $C_s = I_q$. Then, Equations (32) and (33) reduce to

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_a(t) \\ \dot{x}_s(t) \end{bmatrix} = \begin{bmatrix} A & \overline{B}^k & 0_{n\times q} \\ 0_{m_k\times n} & 0_{m_k\times m_k} & 0_{m_k\times q} \\ 0_{q\times n} & 0_{q\times m_k} & 0_{q\times q} \end{bmatrix} \begin{bmatrix} x(t) \\ x_a(t) \\ x_s(t) \end{bmatrix} + \begin{bmatrix} B^k \\ 0 \\ 0 \end{bmatrix} u^k(t)$$

$$+ \begin{bmatrix} \overline{B}^k \omega_{s_a}(t) + \omega_p(t) \\ \omega_{p_a}(t) \\ \omega_{p_s}(t) \end{bmatrix}, \tag{36}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} C_1 & 0_{(l-q)\times m_k} & 0_{(l-q)\times q} \\ C_2 & 0_{q\times m_k} & I_q \end{bmatrix} \begin{bmatrix} x(t) \\ x_a(t) \\ x_s(t) \end{bmatrix}$$

$$+ \begin{bmatrix} \omega_{s,1}(t) \\ \omega_{s,2}(t) + \omega_{s_s}(t) \end{bmatrix}. \tag{37}$$

Equations (36) and (37) are identical to Equations (31) and (33) presented in [22], where the case of simultaneous actuator and sensor step faults was considered.

4 Conditions for Identifiability of Time-Varying Actuator and Sensor Faults

The main results of this paper are developed in this section, that is, a set of necessary and sufficient conditions for the identifiability of additive, time-varying faults affecting combinations of

(1) actuators only,

(2) sensors only, and

(3) actuators and sensors, simultaneously.

The conditions are presented as three separate theorems accompanied by a proof for each of the indicated fault configurations. It is further shown through corollaries that, when all of the faults are step faults, a set of necessary and sufficient conditions for items (1), (2), and (3) above reduce to those presented in [22].

Mathematically, a fault is identifiable if the augmented system is detectable. However, in some practical applications, observability is preferred [22]. This paper presents conditions for both detectability and observability, where satisfying either property shall imply identifiability, in the weak and the strong sense, respectively. Before the conditions for identifiability and the associated proofs are given, several relevant assumptions are addressed.

4.1 Assumptions

It is assumed in the proofs in this section that the following rank conditions are satisfied:

$(A1)$ $\text{rank}(\overline{B}^k) = m_k$, where $\overline{B}^k$ corresponds to the columns of B associated with the m_k faulty actuators,

$(A2)$ $\text{rank}(C) = l$,

$(A3)$ $\text{rank}(C_a) = m_k$, where C_a represents the output matrix associated with an exogenous LTI system which generates the m_k time-varying actuator fault signals,

$(A4)$ $\text{rank}(C_s) = q$, where C_s represents the output matrix associated with an exogenous LTI system which generates the q time-varying sensor fault signals,

$(A5)$ $\text{rank}(C_1) = l-q$, where C_1 corresponds to the rows of C associated with the $l-q$ non-faulty sensors, and

$(A6)$ $\text{rank}(C_2) = q$, where C_2 corresponds to the rows of C associated with the q faulty sensors.

Assumptions $(A1)$ and $(A2)$ are taken directly from [22], that is: (1) it is assumed that inputs associated with linearly dependent columns of $\overline{B}^k$ have been aggregated and that $\overline{B}^k$ is full column rank, and (2) it is assumed that outputs associated with linearly dependent rows of C have been aggregated and that C is full row rank. If $(A1)$ or $(A2)$ do not hold, faults associated with linearly dependent columns of $\overline{B}^k$, or linearly dependent rows of C will not be uniquely identifiable. Similarly, Assumptions $(A3)$ and $(A4)$ follow from the representation of time-varying actuator and sensor faults shown in Sections 3.2 and 3.3, respectively. Finally, Assumptions $(A5)$ and $(A6)$ follow from Assumption $(A2)$.

4.2 Time-Varying Actuator Fault Identifiability

Before the main conditions are provided, a preliminary notation is developed, and two useful lemmas are provided. First, let $\Sigma(s)$ denote the RSM given by

$$\begin{bmatrix} sI-A & -\overline{B}^k \\ C & 0 \end{bmatrix},$$

and let $\Gamma^\xi(s) \subset \mathbb{C}^{n+m_k}$ denote the right nullspace of $\Sigma(s)$, where $s \in \mathbb{C}$. Furthermore, let $\Gamma^\xi_{m_k}(s) \subset \mathbb{C}^{m_k}$ denote the subspace spanned by the last m_k components of a basis for $\Gamma^\xi(s)$. The elements of $\Gamma^\xi_{m_k}(s)$ are characterized next in terms of (extended) invariant zeros and input-zero directions. For a recent treatments of these objects in the context intended in this paper, see [27].

1. When s is an (extended) invariant zero but not an output-decoupling zero of $\Sigma(s)$, then $\Gamma^\xi_{m_k}(s) = \mathcal{G}^\xi_{m_k}(s) \cup \{0\}$, where $\mathcal{G}^\xi_{m_k}(s)$ is the set of all input-zero directions of $\Sigma(s)$,

2. When s is an (extended) invariant zero and an output-decoupling zero of $\Sigma(s)$, then $\Gamma^\xi_{m_k}(s) = \mathcal{G}^\xi_{m_k}(s) = \{0\}$ is the only input-zero direction of $\Sigma(s)$, and

3. When s is not an invariant zero of $\Sigma(s)$, then $\Gamma^\xi_{m_k}(s) = \overline{\mathcal{G}}^\xi_{m_k}(s)$ is the subspace spanned by all of non-input-zero directions of $\Sigma(s)$.

Lemma 1. *The pair (C_a, A_a) is detectable (observable) if and only if the pair $(\overline{B}^k C_a, A_a)$ is detectable (observable).*

Proof. Observe that the pair $(\overline{B}^k C_a, A_a)$ is detectable (observable) if and only if

$$\operatorname{rank} \begin{bmatrix} sI - A_a \\ \overline{B}^k C_a \end{bmatrix} = n_a \text{ for } s \in \Lambda_u(A_a) \ (s \in \Lambda(A_a)),$$

$$\operatorname{rank} \begin{bmatrix} I_n & 0 \\ 0 & \overline{B}^k \end{bmatrix} \begin{bmatrix} sI - A_a \\ C_a \end{bmatrix} = n_a \text{ for } s \in \Lambda_u(A_a) \ (s \in \Lambda(A_a)).$$

By Sylvester's inequality (see [28]) and Assumptions ($A1$) and ($A3$),

$$\operatorname{rank}\begin{bmatrix} sI - A_a \\ C_a \end{bmatrix} \leq \operatorname{rank}\begin{bmatrix} I_n & 0 \\ 0 & \overline{B}^k C_a \end{bmatrix}\begin{bmatrix} sI - A_a \\ C_a \end{bmatrix} \leq \operatorname{rank}\begin{bmatrix} sI - A_a \\ C_a \end{bmatrix}.$$

Therefore, $\operatorname{rank}\begin{bmatrix} sI - A_a \\ \overline{B}^k C_a \end{bmatrix} = \operatorname{rank}\begin{bmatrix} sI - A_a \\ C_a \end{bmatrix}$ for $s \in \Lambda_u(A_a)$ ($s \in \Lambda(A_a)$), so that the pair (C_a, A_a) is detectable (observable) if and only if the pair $(\overline{B}^k C_a, A_a)$ is detectable (observable). □

Lemma 2. *The pair (C_ξ^k, A_ξ^k) is detectable (observable) if and only if all of the following conditions are satisfied:*

(i) the pair (C, A) is detectable (observable),

(ii) (C_a, A_a) is detectable (observable), and

(iii) for (λ_a, v) an (eigenvalue, eigenvector) pair of A_a with $\lambda_a \in \Lambda_u(A_a)$ ($\lambda_a \in \Lambda(A_a)$), $C_a v \notin \Gamma_{m_k}^{\xi}(\lambda_a)$.

Proof. Applying the PBH eigenvector test, the pair (C_ξ, A_ξ) is detectable (observable) if and only if

$$\begin{bmatrix} sI - A & -\overline{B}^k C_a \\ 0_{n_a \times n} & sI - A_a \\ C & 0_{l \times n_a} \end{bmatrix}\begin{bmatrix} \zeta \\ v \end{bmatrix} = 0$$

is satisfied only by the trivial solution (that is, $\begin{bmatrix} \zeta^T & v^T \end{bmatrix}^T = 0$) for $s \in \Lambda_u(A) \cup \Lambda_u(A_a)$ (for $s \in \Lambda(A) \cup \Lambda(A_a)$). The first n columns of the PBH test matrix are independent for $s \in \Lambda_u(A)$ (for $s \in \Lambda(A)$) if and only if the pair (C, A) is detectable (observable). The last n_a columns are independent for $s \in \Lambda_u(A_a)$ (for $s \in \Lambda(A_a)$) if and only if the pair (C_a, A_a) is detectable (observable) (see Lemma 1). For $s \notin \Lambda(A_a)$ the last n_a columns are independent of the first n columns. For $s = \lambda_a \in \Lambda_u(A_a)$ ($s = \lambda_a \in \Lambda(A_a)$) the last n_a columns are independent of the first n columns if and only if

$$\underbrace{\begin{bmatrix} \lambda_a I - A & -\overline{B}^k \\ C & 0 \end{bmatrix}}_{\Sigma(\lambda_a)}\begin{bmatrix} \zeta \\ C_a v \end{bmatrix} = 0 \tag{38}$$

does not have a nontrivial solution for (λ_a, v) an (eigenvalue, eigenvector) pair of A_a, that is, if and only if $C_a v \notin \Gamma_{m_k}^{\xi}(\lambda_a)$. □

The following theorem provides a necessary and sufficient condition for the identifiability of additive, time-varying actuator-only faults.

Theorem 1. *The pair (C_ξ^k, A_ξ^k) is detectable (observable) if and only if all of the following conditions are satisfied:*

(i) for (λ_a, v) an (eigenvalue, eigenvector) pair of A_a with $\lambda_a \in \Lambda_u(A_a)$ $(\lambda_a \in \Lambda(A_a))$, when $l < m_k$ and when λ_a is not an invariant zero of $\Sigma(\lambda_a)$, then $C_a v \notin \overline{\mathcal{G}}_{m_k}^{\xi}(\lambda_a)$,

(ii) the pair (C, A) is detectable (observable),

(iii) the pair (C_a, A_a) is detectable (observable), and

(iv) for (λ_a, v) an (eigenvalue, eigenvector) pair of A_a with $\lambda_a \in \Lambda_u(A_a)$ $(\lambda_a \in \Lambda(A_a))$, when λ_a is an invariant zero of $\Sigma(\lambda_a)$, then $C_a v \notin \mathcal{G}_{m_k}^{\xi}(\lambda_a)$.

Proof. The pair (C_ξ^k, A_ξ^k) is detectable (observable) if and only if Conditions (i)-(iii) of Lemma 2 are satisfied. Thus, Conditions (ii) and (iii) of Theorem 1 are established. It remains only to be shown that Conditions (i) and (iv) of Theorem 1 now hold if and only if Condition (ii) of Lemma 2 holds. Now, suppose that Conditions (i) and (iv) of Theorem 1 hold and that Condition *(iii)* of Lemma 2 does not. Then Equation (38) has a nontrivial solution. The following two possible cases will be considered and shown to result in a contradiction:

1. $\mathrm{rank}\{\Sigma(\lambda_a)\} < \mathrm{rank}\{\Sigma(s)\}$, and
2. $\mathrm{rank}\{\Sigma(\lambda_a)\} = \mathrm{rank}\{\Sigma(s)\}$ with
   (a) $\mathrm{rank}\{\Sigma(s)\} = n + \min\{m_k, l\}$, or
   (b) $\mathrm{rank}\{\Sigma(s)\} < n + \min\{m_k, l\}$.

Case 1 leads to a nontrivial solution if and only if $(\lambda_a, C_a v)$ is an (invariant zero, input-zero direction) pair of $\Sigma(\lambda_a)$, a contradiction of Condition (iv) of Theorem 1, that is, $C_a v \in \mathcal{G}_{m_k}^{\xi}(\lambda_a)$. Case 2(a) leads to a nontrivial solution if and only if $l < m_k$ and $C_a v \in \overline{\mathcal{G}}_{m_k}^{\xi}(\lambda_a)$, a contradiction of Condition (i) of Theorem 1. Case 2(b) implies that $\Sigma(s)$ is degenerate, that is, every $\lambda_a \in \mathbb{C}$ is an invariant zero $\Sigma(s)$. Thus, Case 2(b) leads to a nontrivial solution if and only if $C_a v \in \mathcal{G}_{m_k}^{\xi}(\lambda_a)$, a contradiction of Condition (iv) of Theorem 1. Thus, Conditions (i)-(iv) of Theorem 1 are necessary and sufficient for identifiability of the pair (C_ξ^k, A_ξ^k). □

The following corollary considers the case when the geometric multiplicity of λ_a is equal to n_a. One such instance is when $A_a = 0_{n_a \times n_a}$, (a step fault in $n_a = m_k$ actuators).

Corollary 1. *Let $\lambda_a \in \Lambda_u(A_a)$ $(\lambda_a \in \Lambda(A_a))$ have geometric multiplicity γ_a. If $\gamma_a = n_a$ then Condition (iii) in Lemma 2 becomes*

(i) $l \geq m_k$, and

(ii) $\Sigma(s)$ *has no invariant zeros at* $s = \lambda_a$.

Proof. Let $g \in \Gamma^{\xi}_{m_k}(\lambda_a)$, and consider whether there exists $v \in \mathbb{R}^{n_a}$ such that $C_a v = g$, where v is an eigenvector of A_a in the associated eigenspace W, with $\dim(W) = \gamma_a$. Next, observe that $C_a : \mathbb{R}^{n_a} \to \mathbb{R}^{m_k}$ is a surjective linear transformation since C_a has full row rank, that is, $\mathrm{rank}(C_a) = m_k$ (see Assumption A3). Therefore, for any $g \in \mathbb{R}^{m_k}$, there always exists a vector $\tau \in \mathbb{C}^{n_a}$ such that $C_a \tau = g$. Now, when $\gamma_a = n_a$, $W = \mathbb{C}^{n_a}$, and any nonzero vector in $\mathbb{C}^{n_a}$ is an eigenvector. Thus, for any nonzero solution τ, let $v = \tau$, and it follows that $C_a v = g$. Therefore, when λ_a has geometric multiplicity $\gamma_a = n_a$, $\Sigma(\lambda_a)$ must have full column rank so that $\Gamma^{\xi}_{m_k}(\lambda_a) = \{0\}$, implying that $l \geq m_k$ and that λ_a is not an invariant zero of $\Sigma(s)$ □

Corollary 2. *For the special case when all of the faults are constant biases, that is, a step fault in each of the* m_k *faulty actuators, Conditions (i)-(iv) of Theorem 1 reduce to:*

(i) $l \geq m_k$,

(ii) the pair (C, A) *is observable, and*

(iii) $\Sigma(s)$ *has no invariant zeros at* $s = 0$.

Proof. First, observe that for step faults, $n_a = m_k$, $A_a = 0_{n_a \times n_a}$ and $C_a = I_{n_a \times n_a}$. Thus, A_a has one distinct eigenvalue at zero with geometric multiplicity n_a, that is, any nonzero vector $v \in \mathbb{R}^{n_a}$ is an eigenvector of A_a, and by Corollary 1, Conditions (i) and (iii) of Corollary 2 are established. Furthermore, since (C_a, A_a) is observable, Condition (iii) of Theorem 1 is not needed. The detectability (observability) requirement for the pair (C, A) is now strictly an observability requirement, since $s = 0$ does not lie in the open, left-half plane, thus establishing Condition (ii) of Corollary 2. □

Remark 1. The conditions in Corollary 2 are identical to the conditions for constant, bias-type actuator fault identifiability presented in Theorem 1 in [22].

Example 1. Consider the fourth-order, linearized longitudinal dynamics model used in [17] and fully examined for additive, bias-type fault identifiability in [22]. The model represents a large, transport aircraft in

wings-level, cruise condition and is given by

$$A = \begin{bmatrix} -0.6803 & 0.0115 & -1.0490 & 0 \\ -0.0026 & -0.0062 & -0.0815 & -0.1709 \\ 1.0050 & -0.0344 & -0.5717 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} -44.5192 & 0 \\ -11.4027 & 0 \\ 0.88254 & 1.3287 \\ -0.0401 & 0 \end{bmatrix}. \tag{39}$$

The state vector consists of the pitch rate, forward speed, angle of attack, and pitch angle, that is, $x(t) = \begin{bmatrix} q(t) & v(t) & \alpha(t) & \theta(t) \end{bmatrix}^T$. The control vector consists of the elevator deflection and engine thrust, that is, $u(t) = \begin{bmatrix} u_e(t) & u_T(t) \end{bmatrix}^T$.

Suppose that one must identify sinusoidal faults having frequency $\omega = 1$ radian per second in the elevator actuator and bias-type faults having any amplitude in the thrust actuator (i.e., a "stuck actuator" fault). Observe that there are 45 possible fault configurations for the particular faults of interest, where either or both actuators are faulty ($\sum_{kl=1}^{4} \binom{4}{k}$ sensor configurations, where for each configuration there are 3 actuator fault configurations possible). Out of these, there are two cases of non-identifiability, both due to violation of Condition (iv) of Theorem 1. The discussion of these two cases follows.

Case 1: $y(t) = q(t)$ and both actuators are affected by their respective fault type of interest, $\lambda_a = 0$ is both an eigenvalue of A_a and an invariant zero of the triple $(C, A, \overline{B}^k)$. Furthermore, it can be verified that $\left\{ \begin{bmatrix} 1 & 0 \end{bmatrix}^T, \begin{bmatrix} 0 & 1 \end{bmatrix}^T \right\}$ is a basis for $\mathcal{G}^{\xi}_{m_k}$, therefore for any nonzero vector $C_a v$ in $\mathbb{R}^2$ it follows that $C_a v \in \mathcal{G}^{\xi}_{m_k}$.

Case 2: $y(t) = q(t)$ and the thrust actuator becomes stuck. Again $\lambda_a = 0$ is both an eigenvalue of A_a and an invariant zero of the triple $(C, A, \overline{B}^k)$, and in this case it can further be verified that for any $C_a v \in \mathbb{R}$, $C_a v \in \mathcal{G}^{\xi}_{m_k}$.

An interesting observation is that in three of the 45 possible fault cases, both actuators are faulty and $l < m_k$, yet the faults are still identifiable, that is, all of the conditions of Theorem 1 hold. This situation is different from the case when all the faults are "stuck actuators" as described in [22], where it is required that $l \geq m_k$.

4.3 Time-Varying Sensor Fault Identifiability

This section presents conditions for time-varying sensor fault identifiability. Consider the augmented system given by Equations (26) and (27), the model of an arbitrary LTI system subject to time-varying sensor faults. The identifiability of such a fault requires that the pair (C_η, A_η)

be detectable (observable). The following theorem gives a necessary and sufficient condition for identifiability of time-varying sensor faults.

Theorem 2. *The pair (C_η, A_η) is detectable (observable) if and only if all of the following conditions are satisfied:*

(i) the pair (C, A) is detectable (observable),

(ii) the pair (C_s, A_s) is detectable (observable), and

(iii) when (λ_s, ζ) and (λ_s, ψ) are eigenvalue, eigenvector pairs of A and A_s, respectively, and λ_s is not a detectable (observable) eigenvalue of the pair (C_1, A), then $C_2\zeta \neq \alpha C_s\psi$, where $\alpha \in \mathbb{C}$.

Proof. The pair (C_η, A_η) is detectable (observable) if and only if

$$\operatorname{rank}\left(\begin{bmatrix} sI - A & 0_{n\times n_s} \\ 0_{n_s\times n} & sI - A_s \\ C_1 & 0_{(l-q)\times n_s} \\ C_2 & C_s \end{bmatrix}\right) = n + n_s$$

for $s \in \Lambda_u(A) \cup \Lambda_u(A_s)$ (for $s \in \Lambda(A) \cup \Lambda(A_s)$). The first n columns of the PBH test matrix are linearly independent for all $s \in \Lambda_u(A)$ (for all $s \in \Lambda(A)$) if and only if (C, A) is detectable (observable). The last n_s columns are linearly independent for $s \in \Lambda_u(A_s)$ (for $s \in \Lambda(A_s)$) if and only if (C_s, A_s) is detectable (observable). Furthermore, whenever $s \notin \Lambda(A) \cup \Lambda(A_s)$ the last n_s columns are linearly independent of the first n columns. Now, let $s = \lambda_s \in \Lambda_u(A_s) \cup \Lambda_u(A)$ ($s = \lambda_s \in \Lambda(A_s) \cup \Lambda(A)$). The last n_s columns are linearly independent from the first n columns if and only if

$$\begin{bmatrix} \lambda_s I - A & 0_{n\times n_s} \\ 0_{n_s\times n} & \lambda_s I - A_s \\ C_1 & 0_{(l-q)\times n_s} \\ C_2 & C_s \end{bmatrix} \begin{bmatrix} \zeta \\ \psi \end{bmatrix} = 0 \tag{40}$$

only for $\begin{bmatrix} \zeta^T & \psi^T \end{bmatrix}^T = 0$. Now, suppose there exists $\begin{bmatrix} \zeta^T & \psi^T \end{bmatrix}^T \neq 0$ such that (40) is satisfied. Then

$$(\lambda_s I - A)\zeta = 0 \tag{41}$$
$$(\lambda_s I - A_s)\psi = 0 \tag{42}$$
$$C_1\zeta = 0 \tag{43}$$
$$C_2\zeta + C_s\psi = 0, \tag{44}$$

must hold. Since the pair (C, A) is detectable (observable) $\psi \neq 0$, and since the pair (C_s, A_s) is detectable (observable) $\zeta \neq 0$. Observe that Equation (42) is satisfied since ψ is nonzero and, thus, an eigenvector of A_s associated with λ_s. The remaining equations may be expressed as

$$\begin{bmatrix} \lambda_s I - A \\ C_1 \end{bmatrix} \zeta = 0, \tag{45}$$

$$C_2\zeta + C_s\psi = 0. \tag{46}$$

If λ_s is not a detectable (observable) eigenvalue of (C_1, A) then Equation (45) is satisfied. Now, Equation (46) is satisfied if and only if $C_2\zeta \neq \alpha C_s\psi$, where $\alpha \in \mathbb{C}$. Thus, Conditions (i)-(iii) of Theorem 2 are necessary and sufficient for identifiability of the pair (C_η, A_η). $\square$

The following corollary considers the special case when λ_s has geometric multiplicity equal to n_s.

Corollary 3. *Let $\lambda_s \in \Lambda_u(A_s) \cap \Lambda_u(A)$ $(s \in \Lambda(A_s) \cap \Lambda(A))$ have geometric multiplicity equal to γ_s, and let ψ be an eigenvector of A_s from the associated eigenspace V corresponding to the eigenvalue λ_s. Observe that* $\dim(V) = \gamma_s$. *If $\gamma_s = n_s$ then Condition (iii) in Theorem 2 becomes*

(i) *the pair (C_1, A) is detectable (observable) with respect to $s = \lambda_s$.*

Proof. Observe that $C_s : \mathbb{R}^{n_s} \to \mathbb{R}^q$ is a surjective linear transformation since $\text{rank}(C_s) = q$ (see Assumption (A4)). Therefore, there always exists a vector, $p \in \mathbb{C}^{n_s}$, such that $C_s p = -C_2\zeta$. Now, when $\gamma_s = n_s$, $V = \mathbb{C}^{n_s}$, and any vector in $\mathbb{C}^{n_s}$ is an eigenvector. Then, for any solution p let $\psi = p$, and it follows that $C_s\psi = -C_2\zeta$. That is, whenever $\gamma_s = n_s$, Equation (46) is satisfied. Thus, it is required that λ_s be a detectable (observable) eigenvalue of the pair (C_1, A). $\square$

Corollary 4. *For the special case when all of the faults are constant biases, that is, step faults, Conditions (i)-(iii) in Theorem 2 reduce as follows:*

(i) *the pair (C, A) is observable, and*

(ii) $s = 0$ *corresponds to an observable mode of the pair (C_1, A).*

Proof. First, observe that Condition (i) of Theorem 2 is identical to Condition (i) of Corollary 4. Now, $A_s = 0$ and $C_s = I$. Thus, Condition (ii) of Theorem 2 is no longer necessary, since the pair (C_s, A_s) is observable. Furthermore, note that the only eigenvalue of A_a is $s = 0$, having algebraic and geometric multiplicity $q = n_s$, and by Corollary 3, Condition (ii) of Corollary 4 is established, that is, if (C_1, A) is not observable when $s = 0$ then the pair is not detectable. $\square$

Note that Conditions (i) and (ii) of Corollary 4 are exactly those presented in Theorem 2 of [22] for the sensor step fault case.

Example 2. Consider the 6th-order linearized longitudinal dynamics for the Cranfield A3 Observer, a fixed-wing research UAV presented in [29]. The UAV is in cruise condition, and the airframe is in a gust-insensitive

configuration. The dynamics are given by

$$A = \begin{bmatrix} -0.146 & -0.016 & 0.557 & -9.809 & 0 & 0.001 \\ -0.63 & -4.487 & 34.57 & 0.161 & 0 & 0 \\ 0.001 & 0.039 & -0.894 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -0.016 & -1 & 0 & 35.2 & 0 & 0 \\ 665.7 & -6.89 & 0 & 0 & 0 & -8.57 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & -1.368 \\ 0 & -19.96 \\ 0 & -15.96 \\ 0 & 0 \\ 0 & 0 \\ 45910 & 0 \end{bmatrix},$$

and the output is specified as

$$C = \begin{bmatrix} 1 & -0.014 & 0.019 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.984 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The state vector consists of the forward speed, vertical speed, pitch rate, pitch angle, altitude, and engine rpm, that is, $x(t) = \begin{bmatrix} v(t) & w(t) & q(t) & \theta(t) & h(t) & N_E(t) \end{bmatrix}^T$. The control vector consists of engine thrust and elevator deflection, that is, $u(t) = \begin{bmatrix} u_T(t) & u_e(t) \end{bmatrix}^T$. The output vector consists of the measured speed error, pitch rate, pitch angle, perturbed altitude, and engine rpm, that is, $y(t) = \begin{bmatrix} v_e(t) & q(t) & \theta(t) & h_e(t) & N_E(t) \end{bmatrix}^T$. It can be verified that $\Lambda(A) = \{0, -4.8345, 1.9641 \cdot 10^{-1}, -4.0534 \cdot 10^{-1} \pm i2.0428 \cdot 10^{-1}, -8.6482\}$ and that the system realization $\{A, B, C, 0\}$ is minimal, that is, the system is both controllable and observable.

Suppose that it is of interest to identify drift faults in the sensors, which may be modeled as ramp faults. In the absence of actuator faults, there are 31 possible sensor fault configurations, excluding the fault-free case. Of these, there are 16 cases of non-identifiability, and all 16 cases of non-identifiability are associated with at least a drift fault in the perturbed altitude measurement. Furthermore, any drift faults in any combination of the other measurements are always identifiable, provided that the altitude measurement is not faulty.

It can be verified that the pairs (C, A) and (C_s, A_s) are always observable. Furthermore, it can be verified that zero is a common eigenvalue of A and any form of A_s for the fault configurations of interest, and that the eigenvalue at zero corresponds to an unobservable mode of the pair (C_1, A) whenever $y_4(t)$ is faulty. The test of whether or not $\alpha C_s \psi = C_2 \zeta$

for $\alpha \in \mathbb{C}$ is unnecessary, since the eigenspace of A_s associated with $\lambda_s = 0$ is such that there always exists ψ which satisfies the equality. Thus, for the 16 cases of non-identifiability, Condition (iii) of Theorem 2 fails.

4.4 Simultaneous Time-Varying Actuator and Sensor Fault Identifiability

Before presenting the conditions for identifiability for simultaneous actuator and sensor faults, a preliminary notation is given. First, let $\Sigma(s)$ denote the RSM given by

$$\begin{bmatrix} sI - A & -\overline{B}^k \\ C_1 & 0 \end{bmatrix}.$$

Thus, the RSM associated with $\Sigma(s)$ is to be understood in the context of the fault configuration of interest, i.e., whether it is of interest to identify faults in the actuators alone or in combination with sensors. Furthermore, let $\Gamma^{\varphi}(s) \subset \mathbb{C}^{n+m_k}$ denote the right nullspace of $\Sigma(s)$, where $s \in \mathbb{C}$, and let $\Gamma^{\varphi}_{m_k}(s) \subset \mathbb{C}^{m_k}$ denote the subspace spanned by the last m_k components of a basis for $\Gamma^{\varphi}(s)$. The elements of $\Gamma^{\varphi}_{m_k}(s)$ may be characterized similarly to those of $\Gamma^{\xi}_{m_k}(s)$ in Section 4.2, where $\mathcal{G}^{\varphi}_{m_k}(s)$ corresponds to the input-zero directions, and $\overline{\mathcal{G}}^{\varphi}_{m_k}(s)$ corresponds to the non-input-zero-directions.

Theorem 3. *The pair (C_φ, A_φ) is detectable (observable) if and only if all of the following are satisfied:*

(i) The pair (C^k_ξ, A^k_ξ) is detectable (observable),

(ii) The pair (C_η, A_η) is detectable (observable),

(iii) for $\lambda_{a,s} \in \Lambda_u(A_a) \cap \Lambda_u(A_s)$ ($\lambda_{a,s} \in \Lambda(A_a) \cap \Lambda(A_s)$) not an invariant zero of $\Sigma(s)$ with v and ψ eigenvectors of A_a and A_s associated with $\lambda_{a,s}$, respectively, when $l < m_k + q$ then either $C_a v \notin \overline{\mathcal{G}}^{\varphi}_{m_k}(\lambda_{a,s})$ or $C_2\zeta \neq \alpha C_s \psi$, for $\alpha \in \mathbb{C}$,

(iv) for $\lambda_{a,s} \in \Lambda_u(A_a) \cap \Lambda_u(A_s)$ ($\lambda_{a,s} \in \Lambda(A_a) \cap \Lambda(A_s)$) an invariant zero of $\Sigma(s)$, with v and ψ are eigenvectors of A_a and A_s associated with $\lambda_{a,s}$, respectively, either $C_a v \notin \mathcal{G}^{\varphi}_{m_k}(\lambda_{a,s})$ or $C_2\zeta \neq \alpha C_s \psi$, for $\alpha \in \mathbb{C}$.

Proof. The pair (C_φ, A_φ) is detectable (observable) if and only if the equation

$$\begin{bmatrix} sI - A & -\overline{B}^k C_a & 0_{n \times n_s} \\ 0_{n_a \times n} & sI - A_a & 0_{n_a \times n_s} \\ 0_{n_s \times n} & 0_{n_s \times n_a} & sI - A_s \\ C_1 & 0_{(l-q)\times n_a} & 0_{(l-q)\times n_s} \\ C_2 & 0_{q \times n_a} & C_s \end{bmatrix} \begin{bmatrix} \zeta \\ v \\ \psi \end{bmatrix} = 0, \qquad (47)$$

is satisfied only by the trivial solution for $s \in \Lambda_u(A) \cup \Lambda_u(A_a) \cup \Lambda_u(A_s)$ (for $s \in \Lambda(A) \cup \Lambda(A_a) \cup \Lambda(A_s)$). All of the possible solution cases to be considered are enumerated in Table 1 and treated subsequently. Observe

Case	$\zeta = 0$	$v = 0$	$\psi = 0$
(a)	T	T	T
(b)	T	T	F
(c)	T	F	T
(d)	T	F	F
(e)	F	T	T
(f)	F	T	F
(g)	F	F	T
(h)	F	F	F

TABLE 1: General form of possible solutions to Equation (47).

that Case (a) represents the trivial solution, and so is always a solution to Equation (47). Case (b) can be reduced to considering the equation

$$\begin{bmatrix} sI - A_s \\ C_s \end{bmatrix} \psi = 0. \tag{48}$$

A nontrivial solution to Equation (47) exists for Case (b) if and only if Equation (48) is also nontrivially satisfied. That is, if the pair (C_s, A_s) is not detectable (observable), which corresponds to Condition (i) of Theorem 2.

Case (c) can be reduced to considering the equation

$$\begin{bmatrix} sI - A_a \\ -\overline{B}^k C_a \end{bmatrix} v = 0. \tag{49}$$

A nontrivial solution to Equation (47) exists for Case (c) if and only if Equation (49) is also nontrivially satisfied. By Lemma 1, this is possible only in the case when the pair (C_a, A_a) is not detectable (observable), which corresponds to Condition (iii) of Theorem 1.

Case (d) can be reduced to considering the equation

$$\begin{bmatrix} -\bar{B}^k C_a & 0 \\ sI - A_a & 0 \\ 0 & sI - A_s \\ 0 & C_s \end{bmatrix} \begin{bmatrix} v \\ \psi \end{bmatrix} = 0. \tag{50}$$

However, Equation (50) can be further represented as two separate equations, which must be simultaneously satisfied (i.e., Equations (48) and (49)), thus reestablishing the conditions covered under Cases (b) and (c), respectively.

Case (e) represents a nontrivial solution to Equation (47) if and only if the equation

$$\begin{bmatrix} sI - A \\ C \end{bmatrix} \xi = 0, \tag{51}$$

has a nontrivial solution. Such a solution exists for Case (e) if and only if the pair (C, A) is not detectable (observable). This condition is addressed in Conditions (ii) and (i) of Theorems 1 and 2, respectively.

Case (f) can be reduced to considering the following equation.

$$\begin{bmatrix} sI - A & 0 \\ 0 & sI - A_s \\ C_1 & 0 \\ C_2 & C_s \end{bmatrix} \begin{bmatrix} \zeta \\ \psi \end{bmatrix} = 0. \tag{52}$$

By inspection of Equation (52), a nontrivial solution exists if either or both of the pairs (C, A) and (C_s, A_s) are not detectable (observable). These cases are covered through of Condition (ii) of Theorem 1 and Conditions (i) and (ii) of Theorem 2. Furthermore, a nontrivial solution to Equation (52) (where both ζ and ψ are nonzero) still exists if: 1) $s = \lambda_s \in \Lambda(A_s)$, where ψ is a corresponding a eigenvector, 2) λ_s corresponds to an undetectable (unobservable) eigenvalue of the pair (C_1, A), where ζ is a corresponding eigenvector, and 3) $C_2\zeta \in \alpha C_s \psi$, where $\alpha \in \mathbb{C}$. The situation of 1) through 3) for Case (f) is addressed in Condition (iii) of Theorem 2.

Case (g) can be reduced to considering the following equation.

$$\begin{bmatrix} sI - A & -\bar{B}^k C_a \\ 0 & sI - A_a \\ C & 0 \end{bmatrix} \begin{bmatrix} \zeta \\ v \end{bmatrix} = 0. \tag{53}$$

Inspection of Equation (53) reveals a nontrivial solution if either pair or both pairs, (C, A) and (C_a, A_a), are not detectable (observable). Conditions (ii) and (iii) of Theorem 1 and Condition (i) of Theorem 2 together address such a solution. Now, a nontrivial solution to Equation (53) still exists if: 1) $s = \lambda_a \in \Lambda(A_a)$ and 2) either Condition (i) or (iv) of Theorem 1 are violated.

Cases (a)-(g) are addressed entirely by Theorems 1 and 2, where it is furthermore the case that all conditions have been applied (i.e., no conditions remain unused). Thus, Conditions (i) and (iii) of Theorem 3 are established.

The only remaining case to consider is the situation in Case (h), where ζ, v, and ψ are all nonzero vectors. Considering a nontrivial solution to Equation (47), it is necessary that $s = \lambda_{a,s} \in \Lambda(A_a) \cap \Lambda(A_s)$, where v and ψ are corresponding eigenvectors. Under these assumptions,

Equation (47) can be reduced and expressed as

$$\left[\begin{array}{cc:c} \lambda_{a,s}I - A & -\overline{B}^k & 0 \\ C_1 & 0 & 0 \\ \hdashline C_2 & 0 & I_q \end{array}\right] \left[\begin{array}{c} \zeta \\ C_a v \\ \hdashline C_s\psi \end{array}\right] = 0. \tag{54}$$

Equation (54) has a nontrivial solution if and only if either:

(h.1) for $\lambda_{a,s} \in \Lambda_u(A_a) \cap \Lambda_u(A_s)$ ($\lambda_{a,s} \in \Lambda(A_a) \cap \Lambda(A_s)$) not an invariant zero of $\Sigma_{(C_1,A,\overline{B}^k)}(s)$, when $l < m_k + q$ then both $C_a v \in \overline{\mathcal{G}}^{\varphi}_{m_k}(\lambda_{a,s})$ and $C_2\zeta = \alpha C_s \psi$, where $\zeta \in \mathbb{C}^n$ and $\alpha \in \mathbb{C}$, or

(h.2) for $\lambda_{a,s} \in \Lambda_u(A_a) \cap \Lambda_u(A_s)$ ($\lambda_{a,s} \in \Lambda(A_a) \cap \Lambda(A_s)$) an invariant zero of $\Sigma_{(C_1,A,\overline{B}^k)}(s)$, both $C_a v \in \mathcal{G}^{\varphi}_{m_k}(\lambda_{a,s})$ and $C_2\zeta = \alpha C_s\psi$, where $\zeta \in \mathbb{C}^n$ and $\alpha \in \mathbb{C}$.

Thus, Conditions (iii) and (iv) of Theorem 3 are established, and Conditions (i)-(iv) are together a necessary and sufficient condition for detectability (observability) of the pair $(C^{\varphi}, A^{\varphi})$. $\square$

Corollary 5. *For the special case when all of the simultaneous faults are constant biases, i.e., step faults, Conditions (i)-(iv) of Theorem 3 reduce as follows.*

(i) the pair (C, A) is observable,

(ii) $l \geq m_k + q$, and

(iii) $\Sigma(s)$ has no invariant zeros at the origin.

Proof. Observe that when all the actuator and sensor faults are step faults, Equation (47) can be expressed as

$$\begin{bmatrix} sI - A & -\overline{B}^k & 0 \\ 0 & sI_{m_k} & 0 \\ 0 & 0 & sI_q \\ C_1 & 0 & 0 \\ C_2 & 0 & I_q \end{bmatrix} \begin{bmatrix} \zeta \\ v \\ \psi \end{bmatrix} = 0. \tag{55}$$

First, observe the requirement that the pair (C, A) be observable (since $s = 0$ does not lie strictly in the open, left-half plane). This requirement ensures that the first n columns are linearly independent. Letting $s = 0$, Equation 55 can be expressed in three equations as

$$-A\zeta - \overline{B}^k v = 0 \tag{56}$$

$$C_1\zeta = 0 \tag{57}$$

$$C_2\zeta + I_q\psi = 0 \tag{58}$$

By inspection, Equation 58 is satisfied for any ζ. Thus, it is required that the remaining two equations be satisfied only by the trivial solution. The remaining two equations can be expressed as

$$\begin{bmatrix} -A & -\overline{B}^k \\ C_1 & 0 \end{bmatrix} \begin{bmatrix} \zeta \\ v \end{bmatrix} = 0. \tag{59}$$

Now, by appeal to Corollary 2 and the symmetry of the problem, the conditions that $l \geq m_k + q$ and $\Sigma(s)$ has no invariant zeros at the origin follows. $\square$

Remark 2. Conditions (i)-(iii) of Corollary 5 are identical to those presented in Theorem 3 of [22] for the simultaneous actuator and sensor step fault case.

Example 3. Consider the 4th-order, linearized vertical-plane dynamics of a vertical takeoff and landing (VTOL) aircraft, flying in the airspeed range of 60-170 knots, given in [21] and [30] given by

$$A = \begin{bmatrix} -0.0336 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.7070 & 1.3229 \\ 0 & 0 & 1 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.0447 & -7.5922 \\ -5.5200 & 4.9900 \\ 0 & 0 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The state vector is comprised of the horizontal velocity, vertical velocity, pitch rate, and pitch angle, that is, $x(t) = \begin{bmatrix} v & w & q & \theta \end{bmatrix}^T$. The control vector is comprised of the collective pitch angle and longitudinal cyclic pitch angle. The collective pitch angle input controls the vertical motion, and the longitudinal cyclic pitch angle input controls the horizontal velocity [21]. The output vector is comprised of the horizontal velocity and vertical velocity, that is, $y(t) = \begin{bmatrix} v & w \end{bmatrix}^T$. It can be verified that $\Lambda(A) = \{2.8174 \cdot 10^{-1} \pm i9.7701 \cdot 10^{-2}, -3.3318 \cdot 10^{-1}, -1.9809\}$, and that $\{A, B, C, 0\}$ is a minimal realization, that is, both controllable and observable.

For the case when the actuators are subject to ramp faults, and the sensors are subject to step faults, there are a total of nine possible fault configurations having simultaneous faults. A check of the conditions in Theorem 3 show that

1. Conditions (i) and (ii) are satisfied,

2. for $\lambda_{a,s} \in \Lambda(A_a) \cap \Lambda(A_s) = \{0\}$ (where $\lambda_{a,s}$ is not an invariant zero of $(C_1, A, \overline{B}^k)$), there are five cases when $l < m_k + q$, i.e., the one case when all actuators and senors are faulty, the two cases when both actuators are faulty and either of the sensors are faulty, and the two cases when both sensors are faulty and either of the actuators are faulty. In all five cases, $C_a v \in \overline{\mathcal{G}}_{m_k}^{\varphi}(0)$. For each of these cases, since the geometric multiplicity of the zero eigenvalue of A_s is $n_s = q$, it is always the case that $C_2 \zeta = \alpha C_s \psi$, where $\alpha \in \mathbb{C}$. Thus, Condition (iii) of Theorem 3 is not satisfied for these five cases, and the faults are not identifiable,

3. $l \geq m$, therefore Condition (iv) of Theorem 3 is satisfied, and

4. for the four cases when $\Sigma(s)$ has invariant zeros, that is when only one or the other measurement is biased in conjunction with only one or the other input being faulty, none of the invariant zeros are at the origin. Thus for all four such fault cases, Condition (iv) of Theorem 3 is satisfied.

Thus, for the case when it is of interest to identify ramp faults in the actuators together with constant bias in the sensors, there are five fault configurations which are theoretically not possible to identify using state augmentation. In such cases, other identification methods need to be considered.

5 Conclusions and Future Research

The fundamental problem addressed in this paper was the determination of a theoretical set of necessary and sufficient conditions for identifiability of arbitrary combinations of a class of additive, time-varying actuator and sensor faults using state augmentation. The provided theorems give the conditions which must be satisfied in order for a practical state-augmentation-based solution to exist. One recommendation for future work on this topic is to investigate identifiability of multiplicative faults in the actuators and sensors. Such faults may manifest themselves as a loss-of-effectiveness in the actuators and sensors. One relevant example of such faults is a reduction in thrust provided by a propulsion system.

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14. ABSTRACT
Recent work has provided a set of necessary and sufficient conditions for identifiability of additive step faults (e.g., lock-in-place actuator faults, constant bias in the sensors) using state augmentation. This paper extends these results to an important class of faults which may affect linear, time-invariant systems. In particular, the faults under consideration are those which vary with time and affect the system dynamics additively. Such faults may manifest themselves in aircraft as, for example, control surface oscilations, control surface runaway, and sensor drift. The set of necessary and sufficient conditions presented in this paper are general, and apply when a class of time-varying faults affects arbitrary combinations of actuators and sensors. The results in the main theorems are illustrated by two case studies, which provide some insight into how the conditions may be used to check the theoretical identifiability of fault configurations of interest for a given system. It is shown that while state augmentation can be used to identify certain fault configurations, other fault configurations are theoretically impossible to identify using state augmentation, giving practicioners valuable insight into such situations. That is, the limitations of state augmentation for a given system and configuration of faults are made explicit. Another limitation of model-based methods is that there can be large numbers of fault configurations, thus making identification of all possible configurations impractical. However, the theoretical identifiability of known, credible fault configurations can be tested using the theorems presented in this paper, which can then assist the efforts of fault identification practicioners.

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