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Spacecraft Stabilization and Control for Capture of Non-Cooperative Space Objects

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Abstract

This paper addresses stabilization and control issues in autonomous capture and manipulation of non-cooperative space objects such as asteroids, space debris, and orbital spacecraft in need of servicing. Such objects are characterized by unknown mass-inertia properties, unknown rotational motion, and irregular shapes, which makes it a challenging control problem. The problem is further compounded by the presence of inherent nonlinearities, significant elastic modes with low damping, and parameter uncertainties in the spacecraft. Robust dissipativity-based control laws are presented and are shown to provide global asymptotic stability in spite of model uncertainties and nonlinearities. It is shown that robust stabilization can be accomplished via model-independent dissipativity-based controllers using thrusters alone, while stabilization with attitude and position control can be accomplished using thrusters and torque actuators.

1 Introduction

The proposed NASA asteroid redirect mission has created considerable excitement and interest in the public as well as in the worldwide science and engineering communities. A study by the Keck Institute [1] has concluded that it is feasible to autonomously capture and return an entire 7-m diameter, 500,000-kg near-Earth asteroid to a high lunar orbit. An alternate approach, consisting of picking a boulder off of a larger asteroid, is also being studied. Investigations are in progress at NASA and other organizations to conceive and develop alternative approaches and methods for capture, manipulation, and transport of asteroids.

Another important challenge that is technically similar to asteroid capture is orbital debris mitigation. The near-Earth as well as geostationary orbital debris population is continuously growing and its growth is expected to continue in the future due to ongoing space activities. On-orbit satellite explosions and collisions (accidental or intentional) create even larger numbers of debris items. Space debris poses a serious threat both to human-occupied vehicles and to commercial satellites. Some suggested approaches for orbital debris mitigation would involve docking a capture spacecraft with the debris or making a physical impact. This is a technologically challenging task as most debris are non-cooperative and possibly tumbling with unknown spin, precession, nutation, amplitude changes etc.

A third technically similar challenge is autonomous on-orbit capture and servicing of defunct or functioning satellites. For example, on-orbit refuelling or repairs can extend a satellite's life at a fraction of the cost of constructing and launching a new satellite. Also, with the rapid ongoing advancement of micro-component technologies, it may be desirable to replace components of older functioning satellites with advanced components and significantly increase the capabilities. (For refuelling or refurbishment of functioning satellites, the capture and manipulation should be somewhat simpler since they would likely be cooperative objects).

A related problem is asteroid strike threat mitigation, which would involve changing an asteroid's trajectory to avoid collision with Earth.

The problems described above, i.e., asteroid capture, space debris mitigation, on-orbit spacecraft servicing, and some methods for asteroid strike threat mitigation, involve capture and manipulation of non-cooperative space objects (NCSOs) and would require a common set of technologies.

The main technical problem in capture and manipulation of NCSOs is that they have unknown mass-inertia properties and their dynamic state (such as angular velocity vector- spin, precession, nutation) are unknown. Additionally, asteroids have irregular shapes and composition (e.g., ranging from solid monolithic bodies to collections of loose debris). Methods for capture and manipulation of non-cooperative space objects would likely require capture spacecraft having large solar arrays, multi-link manipulators, deployable bags, and/or other capture mechanisms. In addition to uncertainties in the non-cooperative space object's properties and state, the capture spacecraft will likely have significant lightly damped elastic modes with uncertain and varying modal parameters (frequencies and mode-shapes). Because of these problems, capture, stabilization, and post-capture control of non-cooperative space objects constitute a formidable technical challenge.

This paper addresses some basic issues in capturing a non-cooperative space object and stabilizing the combined spacecraft/object body during and after capture. The capture spacecraft is assumed to be a generic multi-link manipulator having a branched geometry configuration that includes a relatively large central body to which multiple articulated branches are attached through single degree-of-freedom (DOF) revolute joints. It is assumed that the object's spin/tumble rate is relatively small, so that pre-capture co-spin of the spacecraft (i.e., matching the object's angular velocity vector) is not performed. It is also assumed that orbital dynamics do not significantly impact the problem addressed in this paper, which will be the case when the time duration of the capture maneuver is small compared to the orbital period. For applications where orbital dynamics can significantly impact the system motion (e.g., in low Earth orbits) the orbital dynamics will also have to be considered. This paper presents in detail and extends the preliminary analyses and results presented by the authors in [2] for which there was no written version.

1.1 Control Objectives

Prior to capture, it will be necessary to position the spacecraft at a desirable position relative to the NCSO, with a desired posture (defined by the base body attitude and the link joint angles) so that the end-effectors are appropriately positioned for capture. During capture, it will be necessary to stabilize the spacecraft/NCSO combination in the presence of unknown contact forces acting on the end effector. In the post-capture stage, it will be necessary to stabilize the combined body, i.e., to bring the rotational and translational motion as well as all elastic motion to rest. It is assumed that the primary actuators on the spacecraft would be thrusters, possibly augmented by torque actuators such as control-moment-gyros (CMGs). It is also assumed that each thruster can produce the commanded variable thrust (i.e., proportional thrust, e.g., see [3], [4]) rather than operating in an on/off mode. In addition, each link joint is assumed to have a torque actuator. In the pre-capture stage, the spacecraft dynamics are known to some extent (although uncertainties

will still exist in the mass inertia properties and elastic mode dynamics). In the mid-capture and post-capture stages, however, large unknowns and uncertainties will exist in all aspects of the combined spacecraft/NCSO dynamics. Hence, robust stabilization during and after capture is a major technical challenge. In the post-capture phase, the link joints would likely be locked, and the dynamics of the combined spacecraft/NCSO can be represented by a single body. During capture, the stabilizing control laws (which keep the velocities and elastic motion near zero) would be in effect, so that the stabilized spacecraft subjected to contact forces ('disturbances') would likely result in relatively small angular velocities that would be eventually brought down to zero after the capture operation is completed. While attitude and posture of the spacecraft (represented by the base body attitude and the link joint angles) are important in the pre-capture stage, the primary post-capture control objective is to stabilize the combined body, i.e., bring it to rest. A secondary objective is to achieve a desired attitude of the combined body in order to prepare for transporting it to another location (such as a Lunar Lagrange point in the case of asteroid capture).

1.2 Outline of the Paper

Section 2 addresses control of single-body spacecraft, such as a post-capture spacecraft/NCSO combination. Stabilization is addressed first, followed by attitude and position control. Both rigid and flexible spacecraft are considered. Section 3 addresses stabilization as well as position, attitude, and posture control of multibody/multilink spacecraft. Section 4 presents concluding remarks.

2 Single-Body Spacecraft Stabilization and Control

2.1 Stabilization of Rigid Spacecraft

In the mid- and post-capture stages, the objective is to stabilize the combined spacecraft/NCSO, i.e., to eliminate the inertial translational and rotational velocities. This can be accomplished by using thrusters alone, i.e., without the need for torque actuators. The next subsection addresses the case when only thrusters are available.

2.1.1 Stabilization Using Thrusters

Suppose there are m bi-directional thrusters at locations $r_1, r_2, \ldots r_m$, ($r_i \in \Re^3$) relative to the center of mass (c.m.) of the combined spacecraft/NCSO (if post-capture), expressed in a body-fixed coordinate system and the ith thruster produces a scalar force F_i in the direction d_i where $d_i \in \Re^3$ is a unit vector. Thus, the ith thruster produces the vector force $d_i F_i$ expressed in the body-fixed coordinate system. It is assumed that $r_1, r_2, \cdots, r_m$ are such that rank$[\tilde{r}_1 d_1, \tilde{r}_2 d_2, \ldots \tilde{r}_m d_m]$ is 3, where overhead $\sim$ denotes the cross-product matrix of a vector, i.e., if $\xi = [\xi_x, \xi_y, \xi_z]^T$,

$$\tilde{\xi} = \begin{bmatrix} 0 & -\xi_z & \xi_y \\ \xi_z & 0 & -\xi_x \\ -\xi_y & \xi_x & 0 \end{bmatrix} \tag{1}$$

The equations of motion are given by

$$\begin{bmatrix} \mathcal{M} & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \mathcal{M}(\widetilde{\omega} v) \\ \widetilde{\omega} J \omega \end{bmatrix} = \begin{bmatrix} d_1 & d_2 & \dots & d_m \\ \widetilde{r_1} d_1 & \widetilde{r_2} d_2 & \dots & \widetilde{r_m} d_m \end{bmatrix} F \tag{2}$$

where $\mathcal{M} = m_0 I_3$, m_0 being the mass (including the NCSO if post-capture), I_3 denotes the 3×3 identity matrix, J is the 3×3 moment-of-inertia matrix, and v, ω are the inertial translational velocity vector of the c.m. and the angular velocity vector, expressed in the body-fixed coordinate system; $F = [F_1, F_2, \dots F_m]^T$ is the $m \times 1$ thrust vector. It is assumed that $m \geq 6$ because at least 6 bi-directional thrusters are needed to simultaneously control rotational and translational motion.

Denote

$$X = (v^T, \omega^T)^T, \quad M = \begin{bmatrix} \mathcal{M} & 0 \\ 0 & J \end{bmatrix}, \quad \text{and } \Gamma = \begin{bmatrix} d_1 & d_2 & \dots & d_m \\ \widetilde{r_1} d_1 & \widetilde{r_2} d_2 & \dots & \widetilde{r_m} d_m \end{bmatrix} \tag{3}$$

In the post-capture stage, the c.m. of the combined spacecraft/NCSO is unknown, (as are $\mathcal{M}$ and J), therefore r_is are not known. It is assumed that Γ continues to have full rank ($= 6$) regardless of the unknown c.m. location of the combined spacecraft/NCSO. (This can be ensured by analyzing scenarios for a range of NCSO mass-inertia properties. It may also be desirable to include sufficient redundancy, i.e., deploy more than 6 thrusters as needed).

Suppose inertial velocity sensors are placed at the thruster locations. (From a practical viewpoint, placement of sensors near the thrusters would produce noisy measurements and it may be necessary to use noise-reduction filters. However, it would not affect closed-loop stability and would likely have a relatively small effect on the closed-loop motion). The velocity measurement v_{fi} in direction d_i at the location of thruster i (ignoring sensor noise) is given by

$$v_{fi} = d_i^T (v + \widetilde{\omega} r_i) = d_i^T (v + \widetilde{r}_i^T \omega) \tag{4}$$

Therefore the velocity measurement vector $y_r \in \Re^m$ is given by

$$y_r = v_f = \Gamma^T X \tag{5}$$

Consider the control law:

$$F = -G_r y_r; \quad \text{where } G_r = G_r^T > 0, \tag{6}$$

which yields the closed loop equation:

$$M\dot{X} + [(\mathcal{M}\widetilde{\omega} v)^T \ (\widetilde{\omega} J \omega)^T]^T + \Gamma G_r \Gamma^T X = 0 \tag{7}$$

It is shown below that this control law provides closed-loop global asymptotic stability. Furthermore, the stability is robust to $(0, \infty)$-sector actuator nonlinearities as well as first-order actuator dynamics.

Theorem 1-The closed-loop system consisting of (2), (5), and (6) is globally asymptotically stable.

Proof- Consider the Lyapunov function

$$V(X) = X^T M X \tag{8}$$

Because $M = M^T > 0$, V is positive definite. Differentiation with respect to (w.r.t.) time t and simplification utilizing properties of the cross product yields

$$\dot{V} = -2X^T \Gamma G_r \Gamma^T X \tag{9}$$

Because Γ has full rank, $\dot{V} < 0$ (negative definite), i.e., the closed-loop system is globally asymptotically stable $\square$

Note that, although the system comes to rest ($v, \omega \to 0$ as $t \to \infty$), the final inertial position and orientation (attitude) are arbitrary. An important feature of this control law is that *the knowledge of the parameters or the c.m. location* (m_0, J, r_i), *is not needed*, and therefore the stability is robust in spite of parameter errors or lack of knowledge of the parameters.

The robust stability property also holds in the presence of $(0, \infty)$ sector actuator nonlinearities, as well as first-order actuator dynamics, as shown next.

Suppose the actuators have $(0, \infty)$-sector nonlinearities, i.e., the actual actuator force is given by

$$F = \phi(-G_r y_r) \tag{10}$$

where $\phi(.) = [\phi_1(.), \phi_2(.), \ldots, \phi_m(.)]^T$, and

$$\sigma\phi_i(\sigma) > 0 \text{ for } \sigma \neq 0 \text{ and } \phi_i(0) = 0 \text{ for } i \in [1, \ldots, m] \tag{11}$$

$\phi_i(.)$ are assumed to be continuous functions.

Theorem 2- The control law in (10) provides global asymptotic stability of the closed-loop system in the presence of $(0, \infty)$-sector actuator nonlinearities provided G_r is a diagonal matrix.

Proof- Let

$$\psi_i(\sigma) = -\phi_i(-\sigma) \text{ for } i \in [1, 2, \ldots, m] \tag{12}$$

Then ψ_i also belong to the $(0, \infty)$-sector. Proceeding with the Lyapunov function in Theorem 1 yields

$$\dot{V} = -2X^T \Gamma \psi(G_r y_r) = -2 \sum_i^m y_{ri} G_{r_{ii}} \psi_i(y_{ri}) \leq 0 \tag{13}$$

Thus $\dot{V}$ is negative semidefinite. Also $\dot{V} \equiv 0$ implies $y_r \equiv 0$ (because of (11)), therefore $X \equiv 0$ (because Γ is full rank). Therefore the closed-loop system is globally asymptotically stable. $\square$

It can be shown that this control law also retains global asymptotic stability in the simultaneous presence of linear time-invariant (LTI) first-order actuator dynamics followed by $(0, \infty)$-sector actuator nonlinearities. The proof is along the lines of [5] (Theorem 16 in Chapter 2) and is omitted.

Remark- As stated in Section 1.1, it is assumed throughout this paper that the thrusters can produce proportional variable thrusts rather than operating in an on/off mode. If the thrusters are operated in an on/off mode (that usually includes a deadzone), it is expected that this control law would yield ultimate boundedness (e.g., a limit cycle), although further analysis would be needed to investigate that case.

2.1.2 Stabilization Using Thrusters and Torque Actuators

When torque actuators are available in addition to thrusters, the equations of motion can be written as:

$$\begin{bmatrix} \mathcal{M} & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \mathcal{M}\widetilde{\omega}v \\ \widetilde{\omega}J\omega \end{bmatrix} = \begin{bmatrix} \Gamma_{6\times m} & \begin{pmatrix} 0_{3\times 3} \\ I_3 \end{pmatrix} \end{bmatrix} \begin{bmatrix} F \\ \tau_a \end{bmatrix} \tag{14}$$

wherein it is assumed for simplicity that a single 3-axis torque actuator is used to produce control torque $\tau_a \in \Re^3$. In addition to velocity sensors, suppose angular velocity sensors are available. Then the sensor output vector is given by

$$y_r = [v_f^T \; \omega^T]^T = \overline{\Gamma}^T X \tag{15}$$

where

$$\overline{\Gamma} = \begin{bmatrix} \Gamma_{6\times m} & \begin{pmatrix} 0_{3\times 3} \\ I_3 \end{pmatrix} \end{bmatrix} \tag{16}$$

Using the control law

$$u = -G_r y_r \text{ where } G_r = G_r^T > 0 \tag{17}$$

and following the same procedure as in Sec. 2.1.1, it can be shown that the closed-loop system is globally asymptotically stable, and that the global asymptotic stability is preserved (if G_r is a diagonal matrix) in the presence of $(0,\infty)$-sector actuator nonlinearities as well as simultaneous $(0,\infty)$-sector actuator nonlinearities and first-order actuator dynamics.

2.2 Attitude and Position Control of Rigid Spacecraft

Assuming both thrusters and torque actuators are available, the equations of motion using quaternions to represent attitude are:

$$\begin{aligned} \mathcal{M}\ddot{R} &= T_{IB}^T \Sigma_{i=1}^m \overline{F}_i \\ J\dot{\omega} + \omega \times J\omega &= \tau \\ \dot{\overline{\alpha}} &= \frac{1}{2}\left[\omega \times \overline{\alpha} + (\beta + 1)\,\omega\right] \\ \dot{\beta} &= -\frac{1}{2}\omega^T \overline{\alpha} \end{aligned} \tag{18}$$

where R denotes the c.m. position in an inertial coordinate system, T_{IB} denotes the transformation matrix from inertial to body-fixed coordinate system, $\overline{F}_i = d_i F_i$ denotes the ith vector thrust expressed in the body-fixed coordinate system, and the quaternion parameters satisfy

$$\alpha = [\overline{\alpha}^T, \alpha_4]^T; \;\; \beta = \alpha_4 - 1; \tag{19}$$

$$\overline{\alpha} = [\alpha_1, \alpha_2, \alpha_3]^T; \;\; \alpha^T \alpha = 1. \tag{20}$$

The control torque τ applied to the spacecraft consists of the moments generated by the thrusters and the torque τ_a provided by the torque actuators such as control moment gyros (CMGs).

$$\tau = \Sigma_{i=1}^m (r_i \times \overline{F}_i) + \tau_a \tag{21}$$

Consider the nonlinear attitude control law:

$$\begin{aligned}\tau &= -\frac{1}{2}[(\tilde{\overline{\alpha}} + \alpha_4 I)G_p + \nu(1-\alpha_4)I]\overline{\alpha} - G_r\omega \\ &= -\frac{1}{2}[(\tilde{\overline{\alpha}} + (\beta+1)I)G_p - \nu\beta I]\overline{\alpha} - G_r\omega \end{aligned} \tag{22}$$

where $G_p,\ G_r$ are symmetric positive definite (3×3) matrices, and ν is a positive scalar. The control law for translational motion may be designed using

$$T_{IB}^T[I_3, I_3, \ldots, I_3]\overline{F} = T_{IB}^T[d_1, d_2, \ldots, d_m]F = -G_p^{trans}R - G_r^{trans}\dot{R} \tag{23}$$

where $G_p^{trans},\ G_r^{trans}$ are 3×3 positive definite matrices.

It was shown in [6] that the rotational part of the closed-loop system has exactly two equilibrium solutions: $[\overline{\alpha} = \omega = 0,\ \beta = 0]$ and $[\overline{\alpha} = \omega = 0, \beta = -2]$, which correspond to the same equilibrium point in the physical space, i.e., the equilibrium state is unique, defined as: $R = 0,\ \dot{R} = 0,\ \overline{\alpha} = 0,\ \omega = 0,$ and $\beta = 0$. By following the procedure in [6] with a minor modification (i.e., addition of the term $(\dot{R}^T\mathcal{M}\dot{R} + R^T G_p^{trans} R)$ to the Lyapunov function), it can be shown that this control law globally asymptotically stabilizes the physical equilibrium (the origin of the state-space). The proof is based on Lyapunov analysis and LaSalle's invariance principle.

The required control thrusts and torque can be obtained by using

$$\begin{bmatrix} d_1 & \cdots & d_m & 0_{3\times 3} \\ \tilde{r}_1 d_1 & \cdots & \tilde{r}_m d_m & I_3 \end{bmatrix} \begin{bmatrix} F \\ \tau_a \end{bmatrix} = \overline{\Gamma} \begin{bmatrix} F \\ \tau_a \end{bmatrix} = \begin{bmatrix} -T_{IB}[G_p^{trans}R + G_r^{trans}\dot{R}] \\ -\frac{1}{2}[(\tilde{\overline{\alpha}} + (\beta+1)I)G_p - \nu\beta I]\alpha - G_r\omega \end{bmatrix} \tag{24}$$

Γ is assumed to have full rank, therefore $\overline{\Gamma}$ also has full rank, and the minimum-norm solution u is obtained as

$$u = \begin{bmatrix} F \\ \tau_a \end{bmatrix} = \overline{\Gamma}^T(\overline{\Gamma\Gamma}^T)^{-1} \begin{bmatrix} -T_{IB}[G_p^{trans}R + G_r^{trans}\dot{R}] \\ -\frac{1}{2}[(\tilde{\overline{\alpha}} + (\beta+1)I)G_p - \nu\beta I]\alpha - G_r\omega \end{bmatrix} \tag{25}$$

Suppose the inertial position R_P and velocity v_P, expressed along the body axes, are measured at a point P that is fixed to the spacecraft at a location denoted by the vector r_P in the body-fixed coordinate system centered at the c.m. Then R and $\dot{R}$ can be obtained as

$$R = T_{IB}^T(R_P - r_P); \quad \dot{R} = T_{IB}^T[v_P - \widetilde{\omega} r_P] \tag{26}$$

for implementation in the right-hand side of (25).

If there are no torque actuators, τ must be produced by thrusters alone, i.e., it is necessary to solve the following equation for F

$$\Gamma F = \begin{bmatrix} -T_{IB}[G_p^{trans}R + G_r^{trans}\dot{R}] \\ -\frac{1}{2}[(\tilde{\overline{\alpha}} + (\beta+1)I)G_p - \nu\beta I]\alpha - G_r\omega \end{bmatrix} \tag{27}$$

Γ is assumed to have full rank, therefore the minimum-norm solution F can be obtained similar to (25). *Note that $r_1, \cdots, r_m$, and r_P (i.e., the c.m. location) must be known for computing u in (25) or F from (27).* This is possible in the pre-capture stage because the spacecraft c.m. location is known reasonably accurately.

In the post-capture stage, the c.m. location is not known, therefore it is usually not possible to implement this control law. However, while attitude and position control are important in the pre-capture phase, the primary objective in the post-capture phase is stabilization; therefore attitude and position control may not be necessary post capture. If torque actuators are available, they can be used for post-capture attitude control (without using the thrusters) for globally stable attitude maneuvering using the nonlinear control law (22).

2.3 Stabilization of Flexible Spacecraft

The capture spacecraft would likely have significant elastic mode dynamics because of long booms, solar arrays, and manipulator links. The problem of stabilization is first considered i.e., the translational and angular velocities as well as elastic motion of the post-capture spacecraft/NCSO need to be brought to zero.

2.3.1 Stabilization Using Thrusters

The equations of motion including elastic motion can be written as:

$$\begin{bmatrix} \mathcal{M} & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & I_{n_q} \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} \mathcal{M}\widetilde{\omega}v \\ \widetilde{\omega}J\omega \\ D\dot{q} + \Lambda q \end{bmatrix} = \begin{bmatrix} I_3 & I_3 & \dots & I_3 \\ \widetilde{\rho_1} & \widetilde{\rho_2} & \dots & \widetilde{\rho_m} \\ \Phi_1^T & \Phi_2^T & \dots & \Phi_m^T \end{bmatrix} \bar{F}_{3m\times 1} \tag{28}$$

where $q(t) \in \Re^{n_q}$ is the modal amplitude; $\Lambda = diag(\omega_1^2, \omega_2^2, \dots, \omega_{nq}^2)$ where ω_i denotes the natural frequency of the i^{th} elastic mode; and $D = D^T > 0$ ($D \in \Re^{n_q \times n_q}$) is the damping matrix. $\rho_i \in \Re^3$ represents the location of the ith thruster (including elastic displacement) expressed in the body-fixed coordinate system (fixed to the nominal rigid body), given by

$$\rho_i = r_i + \Phi_i q \tag{29}$$

where $\Phi_i \in \Re^{3\times n_q}$ is the mode shape matrix at thruster i. (This formulation assumes relatively small elastic deformations. For large elastic deformations, a fully coupled rigid-elastic model would be appropriate). Velocity measurements at thruster locations in the thruster direction d_i, measured in the local coordinate system (that includes local elastic deflections) are:

$$y_{ri} = d_i^T \Theta_i(\Psi_i q)(v + \widetilde{\omega}\rho_i + \Phi_i \dot{q}) \tag{30}$$

where $\Psi_i \in \Re^{3\times n_q}$ is the mode-slope matrix at the ith sensor location. $\Psi_i q \in \Re^3$ represents the local relative attitude (with respect to the body-fixed coordinate system) due to elastic modes, and $\Theta_i(\Psi_i q) \in \Re^{3\times 3}$ is the corresponding transformation matrix. Scalar thrusts (denoted by $\mathcal{F}_i$) are generated in the local coordinate systems. The ith thruster force, expressed as a 3×1 vector in the body-fixed system, is given by

$$\bar{F}_i = \Theta_i^T(\Psi_i q) d_i \mathcal{F}_i \tag{31}$$

where $\mathcal{F}_i \in \Re$ is the thrust in the local coordinate system. Equation (28) can then be written as

$$\begin{aligned} \overline{M} \begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} \mathcal{M}\widetilde{\omega}v \\ \widetilde{\omega}J\omega \\ D\dot{q} + \Lambda q \end{bmatrix} &= \begin{bmatrix} I_3 & I_3 & \cdots & I_3 \\ \widetilde{\rho}_1 & \widetilde{\rho}_2 & \dots & \widetilde{\rho_m} \\ \Phi_1^T & \Phi_2^T & \dots & \Phi_m^T \end{bmatrix} \overline{\Theta}^T \begin{bmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \\ \vdots \\ \mathcal{F}_m \end{bmatrix} \\ &:= \begin{bmatrix} \widehat{\Gamma} \\ \widehat{\Phi}^T \end{bmatrix} \mathcal{F} := \mathcal{B}\mathcal{F} \end{aligned} \tag{32}$$

where

$$\begin{aligned} \overline{\Theta}^T &= diag[\Theta_1^T(\Psi_1 q)d_1,\ \Theta_2^T(\Psi_2 q)d_2,\ \cdots, \Theta_m^T(\Psi_m q)d_m]_{3m\times m} \\ \widehat{\Gamma} &= \begin{bmatrix} I_3, & I_3, \cdots I_3 \\ \widetilde{\rho}_1, & \widetilde{\rho}_2, \cdots \widetilde{\rho}_m \end{bmatrix} \overline{\Theta}^T, \quad \widehat{\Phi}^T = \left[\Phi_1^T, \Phi_2^T, \cdots \Phi_m^T\right] \overline{\Theta}^T \end{aligned} \tag{33}$$

$\overline{M} = \text{diag}(\mathcal{M}, J, I_{n_q}) \in \Re^{(n_q+6)\times(n_q+6)}$, $\overline{\Theta} \in \Re^{m\times 3m}$, $\widehat{\Gamma} \in \Re^{6\times m}$, $\widehat{\Phi} \in \Re^{m\times n_q}$. As in the rigid spacecraft case, it is assumed that $m \geq 6$. It can be shown that the velocities at thruster locations in the local coordinate systems are given by

$$y_r = \mathcal{B}^T X = [\widehat{\Gamma}^T\ \widehat{\Phi}]X \tag{34}$$

where $X = (v^T, \omega^T, \dot{q}^T)^T$. Note that this system is highly nonlinear and time-varying because of cross-product terms and transformation matrices that are functions of the modal amplitude. The system is also unknown and uncertain because $\mathcal{M}$, J are unknown, modal amplitudes $q(t)$ cannot be measured and the modal parameters (natural frequencies, mode shapes, damping) are never accurately known. In addition, the number of modes (infinite in theory) can be very large, resulting in unmodeled modes. In spite of these problems, the closed-loop system can be shown to be globally asymptotically stable. It is assumed that $\widehat{\Gamma}$ has full row rank ($= 6$) for all t. Consider the control law:

$$\mathcal{F} = -G_r y_r;\ G_r = G_r^T > 0 \tag{35}$$

The closed-loop system is given by

$$\overline{M}\dot{X} + \begin{bmatrix} \mathcal{M}\widetilde{\omega}v \\ \widetilde{\omega}J\omega \\ D\dot{q} + \Lambda q \end{bmatrix} + \mathcal{B}G_r\mathcal{B}^T X = 0. \tag{36}$$

Theorem 3- The closed-loop system consisting of (32), (34), (35) is globally asymptotically stable, i.e., $v(t), \omega(t),$ and $q(t) \to 0$ as $t \to \infty$.
Proof- Consider the Lyapunov function

$$V(X) = X^T\overline{M}X + q^T\Lambda q \tag{37}$$

Differentiation w.r.t. t and simplification yields

$$\dot{V} = -2X^T\mathcal{B}G_r\mathcal{B}^T X - 2\dot{q}^T D\dot{q} \tag{38}$$

Thus, $\dot{V}$ is negative semidefinite. Also, $\dot{V} \equiv 0$ only when $\dot{q} = 0$ and $y_r = 0$, i.e., $v = \omega = 0$ (because $\widehat{\Gamma}$ has full row rank), $\mathcal{F} = 0$, and $q = 0$ (from (32)). Therefore the system is globally asymptotically stable. □

The significance of this result is that robust stabilization can be accomplished using thrusters in spite of *time-variation, unmodeled elastic mode dynamics, nonlinearities, and parametric uncertainties.*

Similar to the rigid spacecraft case, it can be shown that the global asymptotic stability is preserved in the presence of $(0, \infty)$-sector actuator nonlinearities. However, additional analysis will be needed to investigate stability in the simultaneous presence of actuator nonlinearities and actuator dynamics.

2.3.2 Stabilization Using Thrusters and Torque Actuators

When torque actuators are available in addition to thrusters, (32) is modified as

$$\overline{M}\dot{X} + \begin{bmatrix} \mathcal{M}\widetilde{\omega}v \\ \widetilde{\omega}J\omega \\ D\dot{q} + \Lambda q \end{bmatrix} = \begin{bmatrix} \mathcal{B} & \begin{pmatrix} 0_{3\times 3} \\ \Theta_t^T(\Psi_t q) \\ \Psi_t^T \end{pmatrix} \end{bmatrix} \begin{bmatrix} \mathcal{F} \\ \tau_a \end{bmatrix} := \overline{\mathcal{B}}u \tag{39}$$

wherein $u = [\mathcal{F}^T, \tau_a^T]^T$ and it is assumed that three axis torque $\tau_a \in \Re^3$ is produced by a torque actuator; $\Psi_t \in \Re^{3\times n_q}$ is the mode-slope matrix at the torque actuator location, and $\Theta_t \in \Re^{3\times 3}$ is the transformation matrix from body-fixed coordinates to local coordinates at the torque actuator location.

Assuming collocated sensors and actuators, the sensor output y_r consists of inertial velocities $v_f \in \Re^m$ at the thruster locations and the angular velocity $\omega_t \in \Re^3$ at the torque actuator location, all in local coordinates. It can be shown that

$$y_r = \begin{bmatrix} v_f \\ \omega_t \end{bmatrix} = \overline{\mathcal{B}}^T X \tag{40}$$

Using the control law

$$u = -G_r y_r \tag{41}$$

where G_r is positive definite and symmetric, and proceeding as in the previous subsection, it can be shown that the closed-loop system is globally asymptotically stable, and that this stability property is preserved in the presence of $(0, \infty)$-sector actuator nonlinearities when G_r is a diagonal matrix.

2.4 Attitude and Position Control of Flexible Spacecraft

When it is required to control attitude and position, it was seen in Section 2.2 for rigid spacecraft that the knowledge of the thruster locations w.r.t. the c.m. was needed. This is generally not possible in flexible spacecraft because of elastic motion unless additional instrumentation is used. One approach to position and attitude control would be to first achieve the desired translation (c.m. position) using thrusters (while ensuring no net moment, i.e., $\sum \rho_i \times \bar{F}_i \approx 0$), and then use the torque actuators to achieve the desired attitude. If non-zero moment generated due to uncertainties causes significant non-zero angular velocity, it will be necessary

to stabilize the spacecraft first (Sections 2.3.1 and 2.3.2) and then perform attitude control using torque actuators. Attitude control (using a 3-axis torque actuator and collocated attitude and rate sensors) is considered in this section.

Following [7], the rotational motion can be modeled as

$$M(p)\ddot{p} + C(p, \dot{p})\dot{p} + D\dot{p} + Kp = B^T \tau_a \tag{42}$$

where $\dot{p} = (\omega^T, q^T)^T$, $M(p) \in \Re^{(n_q+3)\times(n_q+3)}$ is a symmetric positive definite matrix, $C(p, \dot{p})$, $D \in \Re^{(n_q+3)\times(n_q+3)}$ corresponds to Coriolis and centrifugal forces and damping terms respectively.

$$K = \begin{bmatrix} 0_{3\times3} & 0_{3\times n_q} \\ 0_{n_q\times3} & \widehat{K}_{n_q\times n_q} \end{bmatrix}, \quad D = \begin{bmatrix} 0_{3\times3} & 0_3 \times n_q \\ 0_{n_q\times3} & \widehat{D}_{n_q\times n_q} \end{bmatrix} \tag{43}$$

From [7], $\widehat{K}$, $\widehat{D}$ are symmetric positive definite matrices, and

$$S(p, \dot{p}) = \dot{M}(p) - 2C(p, \dot{p}) \tag{44}$$

is skew-symmetric. The attitude is measured at the torque actuator location and is expressed in the quaternion form as in (18). Using the control law as in (22),

$$\tau_a = -\frac{1}{2}[(\tilde{\overline{\alpha}} + (\beta + 1)I)G_p - \nu\beta I]\overline{\alpha} - G_r\omega \tag{45}$$

where G_p, G_r are symmetric positive definite and $0 < \lambda_M(G_p) \leq 2\nu$ (where $\lambda_M(.)$ denotes the largest eigenvalue), it can be shown (as in [7]) that there are two closed-loop equilibrium solutions that are physically identical, i.e., the equilibrium is unique. The closed-loop global asymptotic stability can be established by using a procedure similar to [7] and the Lyapunov function

$$V = \dot{p}^T M(p)\dot{p} + q^T \widehat{K} q + \overline{\alpha}^T G_p \overline{\alpha} + \nu\beta^2 \tag{46}$$

3 Multi-body Spacecraft Stabilization and Control

In multi-body spacecraft such as space-based manipulators with multiple articulated joints, stabilization can be performed by locking the joints first, which essentially results in a single-body spacecraft. The methods in Sections 2.1.1, 2.1.2 and 2.3.1, 2.3.2 can then be used for rigid spacecraft and flexible spacecraft. For position and attitude control of multi-body spacecraft, one approach would be to first achieve the desired translation (c.m. position) with joints locked, using the thrusters while ensuring minimum net moment, and subsequently use the torque actuators and the joint torques to achieve the desired attitude and posture. If the translation maneuver results in significant angular velocity, stabilization can be performed prior to the attitude/posture control. Therefore only attitude/posture control results for flexible multi-body spacecraft are presented below.

Following [7], the flexible multi-body system consists of a central body having multiple articulated appendages connected through n_θ revolute joints. All bodies are assumed to be flexible. The dynamics are described by (42) where $\dot{p} = (\omega^T, \dot{\theta}^T, \dot{q}^T)^T$;

$\theta \in \Re^{n_\theta}$ is the joint angle vector; $M(p) \in \Re^{(n_q+n_\theta+3)\times(n_q+n_\theta+3)}$ is symmetric positive definite; $C(p,\dot{p}),\ D \in \Re^{(n_q+n_\theta+3)}$ corresponding to Coriolis and centrifugal forces and damping terms respectively, are similar to (43). The input vector is

$$u = \begin{bmatrix} \tau_a \\ \tau_\theta \end{bmatrix} \tag{47}$$

where $\tau_a \in \Re^3$ is the attitude control torque and $\tau_\theta \in \Re^{n_\theta}$ consists of the articulated joint torques. As in the previous cases, the attitude and angular velocity are measured at the attitude control torque actuator locations. Define

$$y_p = \begin{bmatrix} \overline{\alpha} \\ \theta \end{bmatrix}; \quad y_r = \begin{bmatrix} \omega \\ \dot{\theta} \end{bmatrix} \tag{48}$$

In [7], the following control law was presented:

$$u = -\begin{bmatrix} \frac{1}{2}\{(\tilde{\overline{\alpha}} + I_3(\beta+1)G_{p1} + \nu\beta I_3\} & 0 \\ 0 & G_{p2} \end{bmatrix} y_p - G_r y_r \tag{49}$$

where $G_{p1} \in \Re^{3\times 3},\ G_{p2} \in \Re^{n_\theta \times n_\theta}, G_r \in \Re^{(n_\theta+3)\times(n_\theta+3)}$ are symmetric positive definite. It was shown in [7] that, if $0 < \lambda_M(G_{p1}) \leq 2\nu$, there exists a unique equilibrium in the physical space, and that this equilibrium is globally asymptotically stable. Thus, the control law provides globally asymptotically stable large-angle maneuvering and posture control which can be used prior to capture to attain the desired end-effector positioning.

4 Concluding Remarks

Stabilization and control issues in capture and control of non-cooperative space objects were addressed. The capture spacecraft was assumed to consist of a multi-link manipulator characterized by a base body with articulated appendages. The main problem of post-capture stabilization of the combined spacecraft/captured object was considered using proportional (variable-thrust) thrusters and combinations of thrusters and torque actuators. Control laws that asymptotically eliminate all rotational and translational velocities as well as all elastic motion, were presented. The control laws provide global asymptotic stability in spite of unknown mass-inertia properties of the captured object, as well as nonlinearities, unmodeled elastic modes, and uncertainties. The stability was shown to be preserved in the presence of $(0,\infty)$-sector actuator nonlinearities (such as saturation). Combined attitude, position, and posture control was also considered using nonlinear quaternion-based feedback control laws, which were shown to provide global asymptotic stability. An important issue that needs to be addressed is control during the capture stage when unknown contact forces are exerted on the capture spacecraft. Because the stabilizing control laws would be in effect during capture, the contact forces would act as bounded disturbances on the closed-loop system and would likely result in bounded rotational and translational velocities of the spacecraft, which would eventually reduce to zero

after capture is complete. However, this issue needs to be investigated further using concepts such as input-to-state stability. It is also desirable to investigate the use of on/off (rather than proportional) thrusters, which would be expected yield ultimate boundedness (e.g., a limit cycle) rather than asymptotic stability. Further research is also needed on control gain design and optimization, estimation methods for the object's state, contact forces, and possibly mass-inertia properties, control methods for safe transportation of the object to another location, as well as detailed simulation studies and laboratory experiments.

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14. ABSTRACT
This paper addresses stabilization and control issues in autonomous capture and manipulation of non-cooperative space objects such as asteroids, space debris, and orbital spacecraft in need of servicing. Such objects are characterized by unknown mass-inertia properties, unknown rotational motion, and irregular shapes, which makes it a challenging control problem. The problem is further compounded by the presence of inherent nonlinearities, significant elastic modes with low damping, and parameter uncertainties in the spacecraft. Robust dissipativity-based control laws are presented and are shown to provide global asymptotic stability in spite of model uncertainties and nonlinearities. It is shown that robust stabilization can be accomplished via model-independent dissipativity-based controllers using thrusters alone, while stabilization with attitude and position control can be accomplished using thrusters and torque actuators.

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