Acoustics of Jet Surface Interaction Scrubbing Noise

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Motivation

Interaction of jet exhaust with nearby solid surfaces:

- Hybrid Wing Body (HWB) concepts
- High aspect ratio rectangular exhaust with extended beveled surfaces
- Over the wing engine mount
- Nearby structural components could provide noise shielding
- They could also produce new sources of sound
Measurements*
Rectangular Jet (AR = 8)

\[ M_a = 0.90, \ h = 0, \ \theta = \pi/2 \]

\[ M_a = 0.50, \ h/D_e = 0.45, \ \theta = \pi/2 \]

\[ X_{TE}/D_e = 5.6, \ D_e = 2.14'' \]

*James Bridges, AIAA-2014-0876
Outline

- Governing Equations
- Propagation GF Applicable to High-AR Rectangular Jet
- A Parametric Study of the GF
  - Frequency
  - Temperature
  - Source Location
  - Directivity/ Flight Effect
  - Wall Impedance Effect
  - Reflected / Isolated Jet
**Scrubbing Noise**

- NS Equations \(\rightarrow\) (Mean Flow + Linear Eqs. for Fluctuations)
- Locally Parallel Mean Flow
- Compressible
- Constant Static Pressure
- Ideal Gas Law

**Variable density inhomogeneous PB eq.**

\[
L\pi' = \Gamma, \quad \pi' \approx \frac{p'(\bar{x},t)}{\gamma \bar{p}}
\]

\[
L \equiv D \left( D^2 - \frac{\partial}{\partial x_j} \left( c^2 \frac{\partial}{\partial x_j} \right) \right) + 2c^2 \frac{\partial U}{\partial x_j} \frac{\partial^2}{\partial x_1 \partial x_j}, \quad D \equiv \frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}
\]

- Source term \(\Gamma\) is defined according to the generalized Acoustic Analogy, Goldstein 2010)
Green’s Function Method

\[ \pi'(\bar{x}, t) = \int \int G(\bar{x}, \bar{y}, \tau) \Gamma(\bar{y}, \tau) d\tau d\bar{y} \]

\[ LG(\bar{x}, t; \bar{y}, \tau) = \delta(\bar{x} - \bar{y}) \delta(t - \tau) \]

- **Wetted side of the plate only** (scattered noise component discussed by Goldstein et al, 2013)

**Transform:** \((x_1, x_2, t) \rightarrow (k_1, k_2, \omega)\)

\[ G(\bar{x}, t; \bar{y}, \tau) \rightarrow \hat{G}(\vec{k}_t, x_3; y_3, \omega) \quad \vec{k}_t \equiv (k_1, k_2) \]

\[
\frac{\partial^2 \hat{G}}{\partial x_3^2} + \left( \frac{(c')^2}{c^2} - \frac{2k_1U'}{c^2} \right) \frac{\partial \hat{G}}{\partial x_3} + \left( \frac{(-\omega + k_1U)^2}{c^2} - k_1^2 - k_2^2 \right) \hat{G} = \frac{i}{(2\pi)^3} \frac{\delta(x_3 - y_3)}{c^2(-\omega + k_1U)}
\]

- **Far-field spectral density**

\[
\overline{p^2}(\bar{x}, \omega) = \int \int \int G^*(\bar{x}, \bar{y} - \bar{\xi}/2; \omega) G(\bar{x}, \bar{y} + \bar{\xi}/2; \omega) q(\bar{y}, \bar{\xi}, \tau) e^{i\omega\tau} d\tau d\bar{\xi} d\bar{y}
\]
The Eq is re-arranged into a self-adjoint 2\textsuperscript{nd} order ODE

Two linearly independent solutions \( V_j(\kappa_t, x_3, \omega), \ j = 1, 2 \)

\[
V_j'' + f(\kappa_t, x_3, \omega)V_j = 0
\]

\[
\begin{align*}
IVP & \quad V_1(x_3) = 1 \\
\quad \frac{\partial V_1(x_3)}{\partial x_3} - \psi V_1(x_3) = 0, \\
\quad \psi(k_1, \omega, Z) = \left( \frac{i \kappa_o c^2}{Z} \frac{c'(0)}{c(0)} + \frac{k_1 U'(0)}{\omega} \right) \\
\quad x_3 = 0
\end{align*}
\]

\[
BVP & \quad V_2(x_3) = 1, \quad x_3 = 0 \\
\quad \frac{\partial V_2(x_3)}{\partial x_3} + i \chi \psi V_2 = 0, \quad x_3 \to \infty
\]

Radiation condition

\[
\chi^2 = (-\kappa_o + k_1 M_\infty)^2 - k_1^2 - k_2^2 > 0, \quad \kappa_o = \omega / c_o
\]
**GF Method (cont’d)**

\[ G(\bar{x}, \bar{y}; \omega) = \frac{i}{(2\pi)^3} \frac{1}{c(y_3)c(x_3)} \int \int_{k_1k_2} \frac{-\omega + k_1 U(x_3)}{(-\omega + k_1 U(y_3))^2} b_2 V_1(\bar{k}_1, y_3; \omega) W_o(\bar{k}_1, \omega, \bar{Z}) e^{i\Theta(\bar{k}_1, \bar{x}, \omega)} dk_1 dk_2 \]

\[ \Theta(\bar{k}_1, \bar{x}, \omega) = k_1(x_1 - y_1) + k_2(x_2 - y_2) - \chi_\infty x_3 \]

\[ V_2(\bar{k}_1, x_3, \omega) = b_2(\bar{k}_1, \omega) e^{-i\chi_\infty x_3} \quad x_3 \to \infty \]

- **Stationary Phase solution** \( (\kappa_o R \gg 1, \kappa_o = \omega / c_\infty) \)

\[ \bar{k}_1^s = \kappa_o (\sin \phi^s \cos \theta^s, \cos \phi^s) \]

\[ 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \]

\[ G(\bar{x}, \bar{y}; \omega) \sim -i \frac{e^{i\Theta(\bar{k}_1^s, \bar{x}, \omega)}}{(2\pi)^3 R} \frac{\sin \theta^s \sin^2 \phi^s}{c_\infty^2 c(y_3)} \frac{b_2(\bar{k}_1^s, \omega) V_1(\bar{k}_1^s, y_3; \omega)}{W_o(\bar{k}_1^s, \omega, \bar{Z})} \mathcal{Z} \left( 1 - \frac{U(y_3)}{c_\infty} \sin \phi^s \cos \theta^s \right)^2 \]
Stationary-Phase Point

- Stationary point angles are obtained from

\[
\tan \theta = -S(\theta^s, \phi^s, M_\infty) / \left( M_\infty + (1 - M_\infty^2) \cos \theta^s \sin \phi^s \right)
\]

\[
\sin \theta \tan \phi = -S(\theta^s, \phi^s, M_\infty) / \cos \phi^s
\]
Temperature & Velocity Profiles

$(M_\infty = 0, U_j / c_\infty = 0.90)$

- Analytical profiles are selected for mean velocity & static temperature

\[ \eta = y_3 / D_j \]

\[ T_R = 1.0 \text{ (Unheated)} \]
\[ T_R = 3.0 \text{ (heated)} \]
Numerical Results

\((\phi = \pi/2, \theta = \pi/4, U_j / c_\infty = 0.90, T_R = 3.0)\)

- Normalized GF \( G_N \equiv \frac{\pi c_\infty^3 G}{G_{FS}} \)

\[ S_t = \frac{\omega D_j}{2\pi U_j} \]

\[ G_N \]

\[ (1, 2, 3, 6) \]

\[ G_N \]

\[ St=0.25 \]

\[ 0.1 \quad 0.6 \quad 1.1 \quad 2 \]

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\[ \text{Abs} \quad \text{Arg} \]
Effect of Frequency
($\phi = \pi/2, \theta = \pi/4, U_j / c_\infty = 0.90, T_R = 3.0$)

- GF amplitude decreases with increasing frequency
- In a uniform mean flow $G_N \sim 1/\omega$
Effect of Temperature
\( \phi = \pi / 2, \theta = \pi / 4, St_o = 0.25, U_j / c_\infty = 0.90 \)

- GF amplitude decreases with increasing temperature
Effect of Surface Impedance

\( \phi = \pi/2, \theta = \pi/4, St_o = 0.25, \frac{U_j}{c_\infty} = 0.90, T_R = 1.0 \)

- Phase factor depends on source location for non-rigid surface
Flight Effect on Directivity
($\phi = \pi/2, St_o = 0.25, U_j / c_\infty = 0.90, T_R = 1.0$)

- GF peaks at smaller down-stream angles as flight Mach number is increased
Isolated Rectangular Jet

\[ V_j(x_3) = 1, \quad x_3 \to -\infty, \quad j = 1, 2 \]

\[ \frac{\partial V_j}{\partial x_3} - (-1)^j i \chi_\infty V_j = 0, \quad x_3 \to \begin{cases} -\infty, & j = 1 \\ +\infty, & j = 2 \end{cases} \]

Jet Profile – Unheated

\[ \eta = \frac{y_3}{D_j} \]

Observer at \( x_3 \to +\infty \)

\[ (\theta = \pi / 4, \ St_o = 0.25, \ U_j / c_\infty = 0.90, \ T_R = 1.0) \]
Isolated vs. Reflected Jet

( $M_a = 0.90, \, T_R = 1.0$ )

GF Directivity ($St = 0.25$)

Measured SPL ($St = 0.28, \, AR = 8$)

James Bridges, 2014

$X_{TE} / D_e = 5.6, \, D_e = 2.14''$

- A reflecting surface enhances the GF (5-6 dB) relative to an isolated jet (at polar angles larger than peak directivity angle)
Summary

Within the region of nonzero sources:

- GF magnitude decreases with increasing frequency
- GF magnitude decreases with increasing temperature
- At a fixed polar angle GF magnitude varies with source location, generally increases at smaller downstream angle
- Phase factor varies with source location for non-rigid surface impedance
- GF peaks at smaller down-stream angles as flight Mach number is increased
- Presence of a reflecting surface enhances the GF magnitude (5-6 dB) relative to an isolated jet at polar angles larger than peak directivity angle.
QUESTIONS ?
Measurements (JSIT Tests*)

SMC000, SP 7, xTE = 4, h = 0
Q = 90°

SMC000, SP 7, xTE = 16, h = 0
Q = 90°

*Cliff Brown (GRC), ASME paper GT2012
*Gary Podboy (GRC), ASME paper GT2012
Mean Flow – Analytical Profiles

• Axial Velocity \( \eta = y_3 / D_j \)

\[
U(\eta) = \begin{cases} 
\tanh\left(\frac{D_j \eta}{d_1}\right), & \eta < 1.05 \\
\frac{1}{2} \left(1 + \frac{U_\infty}{U_j}\right) + \frac{1}{2} \left(1 - \frac{U_\infty}{U_j}\right) \tanh \frac{1}{d_2} \left(\frac{1}{\eta - 1} - \frac{1 - \eta - 1}{1/2}\right), & \eta \geq 1.05 
\end{cases}
\]

• Temperature

\[
T = T_1 + T_2
\]

\[
T_1(y_3) = \frac{d_1}{d_3} \left(1 + \frac{1}{2} \tanh \frac{1}{d_4} \left(\frac{1}{D_j \eta} - D_j \eta\right)\right)
\]

Crocco-Busemann Law

Frictional heat near the wall

\[
D_j = 2'' \quad \delta_o / D_j = 1.32 d_1 \quad (d_1, d_2, d_3, d_4) = (0.10, 2, 4, 3)
\]
Effect of Observer Angle

\( \phi = \pi/2, \quad St_o = 0.25, \quad U_j / c_\infty = 0.90, \quad T_R = 3.0 \)

- GF amplitude varies with source location, generally increases at smaller downstream angles