A Thermal Analysis of a Hot-Wire Probe for Icing Applications

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Motivation

- Further understand behavior of hot-wire measurements in icing conditions
- Newer applications in ice crystals and mixed phase

SEA Multi-Element Probe

Video in IWC cloud

Particle impingement uniformity → Rigby et al. (Aviation 2014)
Temperature profile along wire → Thermal model (this paper)
Outline

• Describe Thermal Model
  – Governing equations
  – Model parameters

• Results (model vs. experiment)
  – Dry (air flow only, no water)
  – Wet (i.e. cloud on with LWC or IWC)

• Conclusions
Thermal Model Based on Energy Balance

\[ d\dot{q}_{gen} = d\dot{q}_{conv} + d\dot{q}_{cond} + d\dot{q}_{rad} + d\dot{q}_{wet} \]
Governing Equation

\[ \frac{d^2 T_s}{dx^2} + \frac{1}{k d T_s} \left[ \frac{dT_s}{dx} \right]^2 - \frac{P}{d_o k A_C} \frac{k_a}{Nu} [T_s - T_{a,0}] - \frac{\varepsilon \sigma P}{k A_C} [T_s^4 - T_{sur}^4] + \frac{i^2 \rho}{k A_C^2} - \frac{1}{k A_C} \frac{d \dot{q}_{wet}}{dx} = 0 \]

Conduction  Convection  Radiation  Heat Generation  Heat Sink (Water / Ice)

2\textsuperscript{nd} Order, Non-Linear ODE solved using MATLAB routine \textit{bvp4c}

\textit{Boundary Value Problem need two boundary conditions}
Model Parameters

- **Boundary Conditions**
  \[ \frac{dT}{dx} \bigg|_{x_L} = 0 \]
  \[ T \bigg|_R = 50^\circ C \]

- **Convection correlations**
  - (1) Sparrow et. al - 2004
  - (2) Generated from CFD

- **Probe operation**
  - Maintains const. avg. temp (e.g. resistance) ~140 °C
  - Temp (Res) ↓, Power ↑

\[ R = \sum_{j=1}^{n} dR(j) \]

Wire Power
\[ P = i^2 R \]
Results

• Experimental results from 2012 NRC RATFac Tests

• Dry Conditions ($P_0=13.5, 6.5$ psia; $U=85,100,135$ m/s)
  – Temperature profiles
  – Total power (experiment vs. model)
    • Effect of heat-transfer coefficient

• Wet results ($P_0 = 13.5$ psia, $U = 85$ m/s)
  – LWC Sweep 0-3 g/m$^3$
  – IWC Sweep 0-10 g/m$^3$
Results: *Temperature Profiles – Dry*
\[ dq/dx \text{ along Wire - Dry} \]

### Integrated Values (\( \text{Nu} = \text{Sparrow} \))

<table>
<thead>
<tr>
<th>Term</th>
<th>Dry</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_{\text{gen}} ) (W)</td>
<td>12.743</td>
</tr>
<tr>
<td>( q_{\text{conv}} ) (W)</td>
<td>-11.945</td>
</tr>
<tr>
<td>( q_{\text{cond}} ) (W)</td>
<td>-0.766</td>
</tr>
<tr>
<td>( q_{\text{rad}} ) (W)</td>
<td>-0.032</td>
</tr>
<tr>
<td>( q_{\text{wet}} ) (W)</td>
<td>0.000</td>
</tr>
<tr>
<td>Sum</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Experimental Data – Dry

083" - Wire
Tinf = -5°C

Total Power (W)

Tunnel Velocity (m/s)

P(psi) = 13.5
Exp.
6.5
TR
N/A
Model vs. Experiment – Dry

- 083" - Wire
- $T_{\text{inf}} = -5^\circ\text{C}$
- $Nu = \text{Sparrow 2004}$

Graph showing the relationship between tunnel velocity (m/s) and total power (W) with data points for $P(\text{psi}) = 13.5$ and $6.5$ compared to the model and Sparrow predictions.
Model vs. Experiment – Dry

083" - Wire
Tinf = -5°C

Total Power (W)

Tunnel Velocity (m/s)

P(ψ) = 13.5, 6.5
Exp. Model Sparrow CFD

www.nasa.gov
Heating & Evaporation Model

Data from Rigby et al. (2014)

$LWC$

Total Collection Efficiency

\[ dm_{LWC,\infty} = E_m \cdot \dot{m}_{LWC,\infty} \cdot d_o \cdot dx \]

\[ dq_{wet} = \begin{cases} 
  dm_{LWC,\infty} \left( C_{P,LWC} \left[ T_s - T_{LWC,\infty} \right] + \eta_w L_v \right), & T_s < T_b \\
  dm_{LWC,\infty} \left( C_{P,LWC} \left[ T_b - T_{LWC,\infty} \right] + \eta_w L_v \right), & T_s \geq T_b 
\end{cases} \]

“Evaporation Efficiency” (next slide)
Evaporation Efficiency

• Needed a way to estimate fraction of water that does not evaporate

• Defined a parameter called evaporation efficiency, $\eta$

$$\eta = \frac{\text{evaporation potential}}{\text{incoming mass flux}} \quad 0 \leq \eta \leq 1$$

• Use analogy of heat & mass transfer
  – Mass flux related to heat flux

• Evaporation area:
  – For 083,021 entire circumference
  – For HP, forward face of element

• Does not include bounce / splash
Effect of LWC on Temperature Profiles

85 m/s, 13.5 psia, CFD-based Nu

LWC (g/m³)
0.0
1.0
2.0
3.0

Temperature (°C)

x, position (mm)

LWC (g/m³)
1.0
2.0
3.0

η_w

x, position (mm)
Wet Power - Effect of Conduction Losses

Traditional wet power calculation:

\[ P_{\text{wet}} = P_{\text{total}} - P_{\text{dry}} \]

However, energy needed to evaporate LWC was 7.41W (1.9% diff) Under measurement is due to conduction loss differences

<table>
<thead>
<tr>
<th>Term</th>
<th>Dry</th>
<th>Wet</th>
</tr>
</thead>
<tbody>
<tr>
<td>LWC (g/m³)</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>( \dot{q}_{\text{gen}} ) (W)</td>
<td>6.34</td>
<td>13.61</td>
</tr>
<tr>
<td>( \dot{q}_{\text{conv}} ) (W)</td>
<td>-6.05</td>
<td>-6.05</td>
</tr>
<tr>
<td>( \dot{q}_{\text{cond}} ) (W)</td>
<td>-0.28</td>
<td>-0.14</td>
</tr>
<tr>
<td>( \dot{q}_{\text{rad}} ) (W)</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>( \dot{q}_{\text{wet}} ) (W)</td>
<td>0</td>
<td>-7.41</td>
</tr>
</tbody>
</table>
IWC

\[ \eta = \frac{\text{evaporation potential}}{\text{incoming mass flux}} \]

IWC (g/m³)
- 0
- 4
- 7
- 10

Temperature (°C)

Position (mm)
Wet Power vs. IWC

![Graph showing Wet Power vs. IWC]
Conclusions (1 of 2)

• **Developed steady-state hot-wire thermal model**
  – Includes resistive heating, convection, axial conduction, radiation, and water/ice evaporation

• **Examined:**
  – Temperature & power variation along the wire
  – Steady-state power

• **Model compared to SEA multi-wire probe data**

• **For dry conditions:**
  – Matched experiment to within:
    • 5.5% for 021
    • 9.2% for 083
    • 14% for HP
  – Max. conduction loss ~ 4% of total power for conditions examined
Conclusions (2 of 2)

• **Wet conditions:**
  – Introduced “evaporation potential” to estimate % water evaporated
    • Needs validation
  – **LWC:**
    • Affected temperature profile of 021 most significantly;
    • In all cases, high evaporation potential, effect minimal
    • Conduction losses can be different dry vs. wet
      – For 021 at 3 g/m³ → 1.9% difference in P_wet measured vs. actual
  – **IWC (HP only)**
    • Model suggests a non-linear behavior of wet power and IWC_t
      – Below 4 g/m³, linear relationship
      – Above 4 g/m³, non linear due to incomplete evaporation everywhere along wire
    • Limited available experimental data to see if trend is correct
    • Bouncing or splashing-type loss present in experiment complicate interpretation

• **Further examination & development of model planned**
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