A Thermal Analysis of a Hot-Wire Probe for Icing Applications

Peter Struk
NASA GRC

Dave Rigby
NASA GRC, Vantage

Krishna Venkataraman
University of Texas, Austin
Motivation

- Further understand behavior of hot-wire measurements in icing conditions
- Newer applications in ice crystals and mixed phase

Particle impingement uniformity → Rigby et al. (Aviation 2014)
Temperature profile along wire → Thermal model (this paper)
Outline

• **Describe Thermal Model**
  – Governing equations
  – Model parameters

• **Results (model vs. experiment)**
  – Dry (air flow only, no water)
  – Wet (i.e. cloud on with LWC or IWC)

• **Conclusions**
Thermal Model Based on Energy Balance

\[
dq_{gen} = dq_{conv} + dq_{cond} + dq_{rad} + dq_{wet}
\]
Governing Equation

\[
\frac{d^2 T_s}{dx^2} + \frac{1}{k} \frac{d}{dT_s} \left( \frac{dT_s}{dx} \right)^2 - \frac{P}{d_o kA_c} \frac{k_a}{kA_c} \text{Nu} [T_s - T_{a,0}] - \frac{\varepsilon \sigma P}{kA_c} [T_s^4 - T_{sur}^4] + \frac{i^2 \rho}{kA_c^2} - \frac{1}{kA_c} \frac{d \dot{q}_{wet}}{dx} = 0
\]

Conduction  Convection  Radiation  Heat Generation  Heat Sink (Water / Ice)

2\textsuperscript{nd} Order, Non-Linear ODE
solved using MATLAB routine \textit{bvp4c}

Boundary Value Problem
need two boundary conditions
Model Parameters

• **Boundary Conditions**
  \[ \frac{dT}{dx} \bigg|_{x_L} = 0 \]
  \[ T \bigg|_R = 50^\circ C \]

• **Convection correlations**
  – (1) Sparrow et. al - 2004
  – (2) Generated from CFD

• **Probe operation**
  – Maintains const. avg. temp (e.g. resistance) \(~140\, ^\circ C\)
  – Temp (Res) ↓, Power ↑

\[ R = \sum_{j=1}^{n} dR(j) \]

Wire Power
\[ P = i^2 R \]
Results

• Experimental results from 2012 NRC RATFac Tests

• Dry Conditions ($P_0=13.5, 6.5$ psia; $U=85, 100, 135$ m/s)
  – Temperature profiles
  – Total power (experiment vs. model)
    • Effect of heat-transfer coefficient

• Wet results ($P_0 = 13.5$ psia, $U = 85$ m/s)
  – LWC Sweep 0-3 g/m$^3$
  – IWC Sweep 0-10 g/m$^3$
Results: *Temperature Profiles – Dry*

![Diagram showing temperature profiles and labels for length (L) and position (x), with specific data points for element temperature and position.]

- **083, Dry**
  - $U_{in} = 85$ m/s
  - $P_{in} = 13.5$ psia
  - $T_{in} = -5 \degree C$
dq/dx along Wire - Dry

Integrated Values (Nu = Sparrow)

<table>
<thead>
<tr>
<th>Term</th>
<th>Dry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{q}_{gen}$ (W)</td>
<td>12.743</td>
</tr>
<tr>
<td>$\dot{q}_{conv}$ (W)</td>
<td>-11.945</td>
</tr>
<tr>
<td>$\dot{q}_{cond}$ (W)</td>
<td>-0.766</td>
</tr>
<tr>
<td>$\dot{q}_{rad}$ (W)</td>
<td>-0.032</td>
</tr>
<tr>
<td>$\dot{q}_{wet}$ (W)</td>
<td>0.000</td>
</tr>
<tr>
<td>Sum</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Experimental Data – Dry

083" - Wire
T_infinity = -5°C

Total Power (W)

Tunnel Velocity (m/s)

P(ψ) = 13.5  6.5  T_R
Exp.  N/A
Model vs. Experiment – Dry

083” - Wire
T\textsubscript{inf} = -5\textdegree C
Nu = Sparrow 2004
Model vs. Experiment – Dry

083" - Wire
Tinf = -5°C

P(psi)=

Exp. 13.5 6.5 Nu
Model
Sparrow
CFD
Heating & Evaporation Model

Data from Rigby et al. (2014)

LWC

Total Collection Efficiency

\[ d\dot{m}_{\text{LWC},\infty} = E_m \cdot \dot{m}_{\text{LWC},\infty} \cdot d_0 \cdot dx \]

\[ d\dot{q}_{\text{wet}} = \begin{cases} 
    d\dot{m}_{\text{LWC},\infty} \left( C_{P,\text{LWC}} \left[ T_s - T_{\text{LWC},\infty} \right] + \eta_w L_v \right), & \text{if } T_s < T_b \\
    d\dot{m}_{\text{LWC},\infty} \left( C_{P,\text{LWC}} \left[ T_b - T_{\text{LWC},\infty} \right] + \eta_w L_v \right), & \text{if } T_s \geq T_b 
\end{cases} \]

“Evaporation Efficiency” (next slide)
Evaporation Efficiency

- Needed a way to estimate fraction of water that does not evaporate
- Defined a parameter called evaporation efficiency, $\eta$
  \[ \eta = \frac{\text{evaporation potential}}{\text{incoming mass flux}} \quad 0 \leq \eta \leq 1 \]
- Use analogy of heat & mass transfer
  - Mass flux related to heat flux
- Evaporation area:
  - For 083,021 entire circumference
  - For HP, forward face of element
- Does not include bounce / splash
Effect of LWC on Temperature Profiles

85 m/s, 13.5 psia, CFD-based Nu
Wet Power - Effect of Conduction Losses

\[ P_{wet} = P_{total} - P_{dry} \]

- Traditional wet power calculation:

\[ P_{wet} = 13.61\,W - 6.34\,W = 7.27\,W \]

However, energy needed to evaporate LWC was 7.41W (1.9% diff)

Under measurement is due to conduction loss differences
\( \eta = \frac{\text{evaporation potential}}{\text{incoming mass flux}} \)
Wet Power vs. IWC

![Graph showing the relationship between wet power and IWC](image_url)

- **Exp.**:
  - \( E_m = 0.981 \)
  - \( P_{\text{max}} = 0.981 \)

- **Model**

- **IWC\(_i\) (g/m\(^3\))**
- **Wet Power (W)**
Conclusions (1 of 2)

• **Developed steady-state hot-wire thermal model**
  – Includes resistive heating, convection, axial conduction, radiation, and water/ice evaporation

• **Examined:**
  – Temperature & power variation along the wire
  – Steady-state power

• **Model compared to SEA multi-wire probe data**

• **For dry conditions:**
  – Matched experiment to within:
    • 5.5% for 021
    • 9.2% for 083
    • 14% for HP
  – Max. conduction loss ~ 4% of total power for conditions examined
Conclusions (2 of 2)

• **Wet conditions:**
  – Introduced “evaporation potential” to estimate % water evaporated
    • Needs validation
  
  – **LWC:**
    • Affected temperature profile of 021 most significantly;
    • In all cases, high evaporation potential, effect minimal
    • Conduction losses can be different dry vs. wet
      – For 021 at 3 g/m³ → 1.9% difference in \( P_{wet} \) measured vs. actual
  
  – **IWC (HP only)**
    • Model suggests a non-linear behavior of wet power and \( IWC_t \)
      – Below 4 g/m³, linear relationship
      – Above 4 g/m³, non-linear due to incomplete evaporation everywhere along wire
    • Limited available experimental data to see if trend is correct
    • Bouncing or splashing-type loss present in experiment complicate interpretation

• **Further examination & development of model planned**
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