Wing Shape Sensing from Measured Strain

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Overview

- What the technology does
- Previous technologies
- Technical features of new technology
- Computational Validation
  - Uniform 1g load
  - Wing tip torsion load
  - Aerodynamic load under 1° angle of attack at Mach 0.715
- Experimental Testing
  - Leading-edge load
  - Uniform load
- Conclusions
**What the technology does**

**Problem Statement**
- Improving fuel efficiency for an aircraft
  - Reducing weight or drag
  - Similar effect on fuel savings
  - Multidisciplinary design optimization (design phase) or active control (during flight)

- Real-time measurement of deflection, slope, and loads in flight are a valuable tool.

- Wing deflection and slope (complete degrees of freedom) are essential quantities for load computations during flight.
  - Loads can be computed from the following governing equations of motion.

\[ M\ddot q(t) + G\dot q(t) + Kq(t) = F_a(Mach, q(t)) \]

  - Internal Loads: using finite element structure model
    - \( M\ddot q(t), G\dot q(t), Kq(t) \) : Inertia, damping, and elastic loads
  - External Load: using unsteady aerodynamic model
    - \( F_a \) : Aerodynamic load

- Traditionally, strain over the wing are measured using strain gages.
  - Cabling would create weight and space limitation problems.
  - A new innovation is needed. Fiber optic strain sensor (FOSS) is an ideal choice for aerospace applications.

**Wing deflection & slope will be computed from measured strain.**
Previous technologies

Beam theory; Sectional bending moment and shear loads

  - NASA LRC; Application is limited for "beam".
  - University of Arizona and NASA LRC; using an inverse interpolation formulation.
  - KAIST; displacement-strain-transformation (DST) matrix. Use strain mode shape. Application was based on beam structure.
  - JAXA; using inverse analysis. “Beam” application only.
  - NASA AFRC; closed-form equations (based on beam theory)
  - NASA AFRC; "sectional" bending moment and shear force along the "beam".
  - NASA LRC; curve-fitting
  - Harvard University, Stanford University, and Howard Hughes Medical Institute; Uses beam theory.
  - Oregon State University; Aerodynamic loads are estimated from measured strain using virtual strain sensor technique.

Previous technologies are applied to a beam structure.
Technical features of new technology

Proposed solutions:

- The new method for obtaining the deflection over a flexible full 3D aircraft structure is based on the following two steps.
  - First Step: Compute wing deflection along fibers using measure strain data
    - Wing deflection will be computed along the fiber optic sensor line.
    - Strains at selected locations will be “fitted”.
    - These fitted strain will be integrated twice to have deflection information. (Relative deflection w.r.t. the reference point)
    - This is a finite element model independent method.
  - Second Step: Compute wing slope and deflection of entire structures
    - Slope computation will be based on a finite element model dependent technique.
    - Wing deflection and slope will be computed at all the finite element grid points.

A new two-step theory is investigated for predicting the deflection and slope of an entire structure using strain measurements at discrete locations.
Technical features of new technology (continued)

- First Step
  - Use piecewise least-squares method to minimize noise in the measured strain data (strain/offset)
  - Obtain cubic spline (Akima spline) function using re-generated strain data points:
    \[ \frac{d^2 \delta}{ds^2} = -\epsilon(s)/c(s) \]
  - Integrate fitted spline function to get slope data:
    \[ \frac{d\delta}{ds} = \theta(s) \]
  - Obtain cubic spline (Akima spline) function using computed slope data
  - Integrate fitted spline function to get deflection data:
    \[ \delta(s) \]

A measured strain is fitted using a piecewise least-squares curve fitting method together with the cubic spline technique.
Technical features of new technology (continued)

- Second Step: Based on General Transformation
  - For all model reduction/expansion techniques, there is a relationship between the master (measured or tested) degrees of freedom and the slave (deleted or omitted) degrees of freedom which can be written in general terms as:

    \[
    \{q\} = \begin{bmatrix} q_M \\ q_S \end{bmatrix} = [T] \{\tilde{q}_M\}; \quad \{\tilde{q}_M\} = \text{general displacement vector}
    \]

    \[
    \{q\} = \begin{bmatrix} q_M \\ q_S \end{bmatrix} = [T] \{\tilde{q}_M\}; \quad \{\tilde{q}_M\} = \text{general displacement vector}
    \]

    \[
    \{q\} = \begin{bmatrix} q_M \\ q_S \end{bmatrix} = [T] \{\tilde{q}_M\}; \quad \{\tilde{q}_M\} = \text{orthogonal displacement vector}
    \]

    - Transformation matrix \([T]\) can be one of the followings:
      - Guyan (or static) condensation, dynamic condensation, improved reduced system (IRS), or system equivalent reduction expansion process (SEREP)

- Expansion of displacement using SEREP: kinds of least-squares method; most accurate reduction-expansion technique
  - \{\tilde{q}_M\}: master DOF; deflection along the fiber “computed from the first step”
  - \{\tilde{q}_M\} = [\Phi_S] ([\Phi_M]^{-1} [\Phi_M]^T \{q_M\})^{-1} [\Phi_M]^{-1} [\Phi_M]^T \{q_M\}; deflection and slope all over the structure
  - \{q_M\} = [\Phi_M] ([\Phi_M]^{-1} [\Phi_M]^T \{q_S\})^{-1} [\Phi_M]^{-1} [\Phi_M]^T \{q_S\}; smooth master DOF

\[
[T] = \begin{bmatrix} \Phi_M ([\Phi_M]^T [\Phi_M])^{-1} [\Phi_M]^T \\ \Phi_S ([\Phi_M]^T [\Phi_M])^{-1} [\Phi_M]^T \end{bmatrix}
\]

Computed deflection along the fibers are combined with a finite element model of the structure in order to interpolate and extrapolate the deflection and slope of the entire structure through the use of the System Equivalent Reduction and Expansion Process.
Computational Validation

Cantilevered rectangular wing model
Cantilevered Rectangular Wing Model

- Wind tunnel test wing (thickness = 0.065 in.)
  - Uniform 1g load
  - Wing tip torsion (1 lbf at leading-edge and -1 lbf at trailing-edge of wing tip section)
  - Aerodynamic load under 1° angle of attack at Mach 0.715

- MSC/NASTRAN
  - Compute strain
  - Compute deflection (target)

- ZAERO
  - Compute aerodynamic load

- Two-step approach
  - Compute deflection from computed strain
  - Compare computed deflection with respect to target value

Wing with 22 FOSS
Fiber optic strain sensors: 11(upper) + 11(lower)

Uniform 1g loading
Positive

Wing tip torsion
(1 lbf at leading-edge and -1 lbf at trailing-edge of wing tip section)
Positive
Negative

Aerodynamic load under 1° angle of attack at Mach 0.715
Negative
Cantilevered Rectangular Wing Model: Uniform 1g

- Uniform 1g load

- Wing deflection over FE model (step 2 results with 10 modes)

- Curvature distribution (step 1 results)

- Wing deflection (step 2 results with 10 modes)

- Wing span-wise slope (step 2 results with 10 modes)
Cantilevered Rectangular Wing Model: Wing tip torsion

- Wing tip torsion

Wing deflection over FE model (step 2 results with 10 modes)

Deformed shape

Undeformed shape

Markers: NASTRAN Result
Lines: Two-step Approach

(a) Strain distribution (step 1 results)

Markers: NASTRAN Result
Lines: Two-step Approach

(d) Step 2 results with 50 Modes

Markers: NASTRAN Result
Lines: Two-step Approach

Markers: NASTRAN Result
Lines: Two-step Approach
Cantilevered Rectangular Wing Model: Aerodynamic load

- Aerodynamic load under 1° angle of attack at Mach 0.715

Markers: NASTRAN Result
Lines: two-step approach

Fiber 1
Fiber 5
Fiber 9
Fiber 13
Fiber 17
Fiber 21

Extrapolation
Extrapolation

(a) Curvature distribution (step 1 results)

(b) Splined load vectors
Cantilevered Rectangular Wing Model: Aerodynamic load (continued)

Markers: NASTRAN Result    Lines: Two-step Approach

X deflection (step 2 results with 10 modes)

Y deflection (step 2 results with 10 modes)

Z deflection (step 2 results with 10 modes)

Wing slope in roll direction (step 2 results with 10 modes)

Wing slope in pitch direction (step 2 results with 10 modes)

Wing slope in yaw direction (step 2 results with 10 modes)
Cantilevered Rectangular Wing Model: Aerodynamic load (continued)

- Aerodynamic load under 1° angle of attack at Mach 0.715 (continued)
- Wing tip deflections

**Deflection in X, Y, & Z direction**

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target (inch)</th>
<th>Computed X deflection (inch)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>0.0006927</td>
<td>N/A</td>
<td>0.0006703</td>
</tr>
<tr>
<td>5</td>
<td>0.0006975</td>
<td>N/A</td>
<td>0.0006776</td>
</tr>
<tr>
<td>9</td>
<td>0.0007029</td>
<td>N/A</td>
<td>0.0006936</td>
</tr>
<tr>
<td>13</td>
<td>0.0007047</td>
<td>N/A</td>
<td>0.0007081</td>
</tr>
<tr>
<td>17</td>
<td>0.0007035</td>
<td>N/A</td>
<td>0.0007144</td>
</tr>
<tr>
<td>21</td>
<td>0.0007023</td>
<td>N/A</td>
<td>0.0007143</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target (inch)</th>
<th>Computed Y deflection (inch)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>-0.003280</td>
<td>N/A</td>
<td>-0.003282</td>
</tr>
<tr>
<td>5</td>
<td>-0.003272</td>
<td>N/A</td>
<td>-0.003278</td>
</tr>
<tr>
<td>9</td>
<td>-0.003260</td>
<td>N/A</td>
<td>-0.003268</td>
</tr>
<tr>
<td>13</td>
<td>-0.003248</td>
<td>N/A</td>
<td>-0.003257</td>
</tr>
<tr>
<td>17</td>
<td>-0.003239</td>
<td>N/A</td>
<td>-0.003246</td>
</tr>
<tr>
<td>21</td>
<td>-0.003235</td>
<td>N/A</td>
<td>-0.003242</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target (inch)</th>
<th>Computed Z deflection (inch)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>0.9280</td>
<td>0.9269</td>
<td>0.9275</td>
</tr>
<tr>
<td>5</td>
<td>0.9085</td>
<td>0.9091</td>
<td>0.9087</td>
</tr>
<tr>
<td>9</td>
<td>0.8889</td>
<td>0.8894</td>
<td>0.8895</td>
</tr>
<tr>
<td>13</td>
<td>0.8691</td>
<td>0.8696</td>
<td>0.8699</td>
</tr>
<tr>
<td>17</td>
<td>0.8493</td>
<td>0.8497</td>
<td>0.8498</td>
</tr>
<tr>
<td>21</td>
<td>0.8296</td>
<td>0.8300</td>
<td>0.8297</td>
</tr>
</tbody>
</table>

- \( q_M \): master DOF
- \( q_S \): slave DOF
- Input to Step 2
- Boundary
- Deformed shape
- Undeformed shape
- Wing deflection over FE model (step 2 results with 10 modes)
- Smoothing effect
Cantilevered Rectangular Wing Model: Aerodynamic load (continued)

- Aerodynamic load under 1° angle of attack at Mach 0.715 (continued)
- Wing tip slopes

**Slope in roll, pitch, & yaw direction**

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target</th>
<th>Computed roll slope</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1*</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>0.10090</td>
<td>0.1010</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>0.10070</td>
<td>0.1016</td>
<td>0.96</td>
</tr>
<tr>
<td>9</td>
<td>0.10030</td>
<td>0.1012</td>
<td>0.94</td>
</tr>
<tr>
<td>13</td>
<td>0.09993</td>
<td>0.1009</td>
<td>0.93</td>
</tr>
<tr>
<td>17</td>
<td>0.09966</td>
<td>0.1006</td>
<td>0.92</td>
</tr>
<tr>
<td>21</td>
<td>0.09954</td>
<td>0.1004</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Note: Roll slope without effect of X and Y deflections (These slopes are not used during step 2 computation.)

| Fiber number | Target   | Computed pitch slope  | Relative error (%) | |
|--------------|----------|------------------------|--------------------|
|              |          | Step 1                 | Step 2 with 10 modes | |
| 1            | 0.02131  | N/A                    | 0.02063            | -3.2               |
| 5            | 0.02146  | N/A                    | 0.02085            | -2.9               |
| 9            | 0.02163  | N/A                    | 0.02134            | -1.3               |
| 13           | 0.02168  | N/A                    | 0.02179            | 0.5                |
| 17           | 0.02165  | N/A                    | 0.02198            | 1.5                |
| 21           | 0.02161  | N/A                    | 0.02198            | 1.7                |

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target</th>
<th>Computed yaw slope</th>
<th>Absolute error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>2.2e-31</td>
<td>N/A</td>
<td>7.9e-18</td>
</tr>
<tr>
<td>5</td>
<td>1.9e-31</td>
<td>N/A</td>
<td>6.6e-18</td>
</tr>
<tr>
<td>9</td>
<td>1.7e-31</td>
<td>N/A</td>
<td>6.1e-18</td>
</tr>
<tr>
<td>13</td>
<td>1.7e-31</td>
<td>N/A</td>
<td>5.7e-18</td>
</tr>
<tr>
<td>17</td>
<td>1.4e-31</td>
<td>N/A</td>
<td>5.9e-18</td>
</tr>
<tr>
<td>21</td>
<td>1.2e-31</td>
<td>N/A</td>
<td>3.3e-18</td>
</tr>
</tbody>
</table>

- Slope in roll, pitch, & yaw direction
- Not usable for Step 2 computations

**Relative error (%)**

| Fiber number | Target   | Computed roll slope       | Relative error (%) | |
|--------------|----------|---------------------------|--------------------|
|              |          | Step 1*                   | Step 2 with 10 modes | |
| 1            | 0.10090  | 0.1010                    | 0.12               | 0.08               |
| 5            | 0.10070  | 0.1016                    | 0.96               | 0.18               |
| 9            | 0.10030  | 0.1012                    | 0.94               | 0.26               |
| 13           | 0.09993  | 0.1009                    | 0.93               | 0.28               |
| 17           | 0.09966  | 0.1006                    | 0.92               | 0.23               |
| 21           | 0.09954  | 0.1004                    | 0.88               | 0.20               |

| Fiber number | Target   | Computed pitch slope  | Relative error (%) | |
|--------------|----------|------------------------|--------------------|
|              |          | Step 1                 | Step 2 with 10 modes | |
| 1            | 0.02131  | N/A                    | 0.02063            | -3.2               |
| 5            | 0.02146  | N/A                    | 0.02085            | -2.9               |
| 9            | 0.02163  | N/A                    | 0.02134            | -1.3               |
| 13           | 0.02168  | N/A                    | 0.02179            | 0.5                |
| 17           | 0.02165  | N/A                    | 0.02198            | 1.5                |
| 21           | 0.02161  | N/A                    | 0.02198            | 1.7                |

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target</th>
<th>Computed yaw slope</th>
<th>Absolute error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>2.2e-31</td>
<td>N/A</td>
<td>7.9e-18</td>
</tr>
<tr>
<td>5</td>
<td>1.9e-31</td>
<td>N/A</td>
<td>6.6e-18</td>
</tr>
<tr>
<td>9</td>
<td>1.7e-31</td>
<td>N/A</td>
<td>6.1e-18</td>
</tr>
<tr>
<td>13</td>
<td>1.7e-31</td>
<td>N/A</td>
<td>5.7e-18</td>
</tr>
<tr>
<td>17</td>
<td>1.4e-31</td>
<td>N/A</td>
<td>5.9e-18</td>
</tr>
<tr>
<td>21</td>
<td>1.2e-31</td>
<td>N/A</td>
<td>3.3e-18</td>
</tr>
</tbody>
</table>
Experimental Testing

Swept test plate
Swept Test Plate

Tested at NASA AFRC

Thickness = 0.19 in
Swept angle = 45°

- Leading-edge fiber
- Mid-chord fiber
- Trailing-edge fiber

- 3 lb
- 2 lb
- 3 lb
- 2 lb
- 3 lb
- 2 lb
- 6 lb
- 2 lb
- 6 lb

- 12 in.
- 50 in.

a) Leading-edge load
b) Uniform load

- Photogrammetry target
- Strain rosette
- Fiber optical strain sensing (under sealant)
Averaging the curvatures calculated by using each fiber individually eliminates the effect of the axial load.

This computation is performed after curve-fitting each set of data individually to minimize noise.
Swept Test Plate (continued)

Under leading-edge load

a) Leading-edge fiber (step 1 result)

b) Leading-edge fiber (step 2 result with 10 modes)

c) Mid-chord fiber (step 1 result)

d) Mid-chord fiber (step 2 result with 10 modes)

e) Trailing-edge fiber (step 1 result)

f) Trailing-edge fiber (step 2 result with 10 modes)

Step 1 results

Step 2 results

larger error
Swept Test Plate (continued)

Under uniform load

- a) Leading-edge fiber (step 1 result)
- b) Leading-edge fiber (step 2 result with 10 modes)
- c) Mid-chord fiber (step 1 result)
- d) Mid-chord fiber (step 2 result with 10 modes)
- e) Trailing-edge fiber (step 1 result)
- f) Trailing-edge fiber (step 2 result with 10 modes)

- Step 1 results
- Step 2 results

Comparison of deflection results:
- Leading-edge: larger error
- Mid-chord: larger error
- Trailing-edge: larger error

Legend:
- Photogrammetry data
- Computed deflection
- Bakalyar/Jutte deflection
## Swept Test Plate (continued)

- **Deformed wing shape (step 2 results with 10 modes)**

<table>
<thead>
<tr>
<th></th>
<th>Measured (inch)</th>
<th>Computed (inch)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bakalyar and Jutte*</td>
<td>Bakalyar and Jutte</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with 10 modes</td>
<td></td>
</tr>
<tr>
<td><strong>Leading-edge load</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leading edge fiber</td>
<td>-4.525</td>
<td>-4.500</td>
<td>-4.542</td>
</tr>
<tr>
<td>Middle fiber</td>
<td>-4.912</td>
<td>-4.952</td>
<td>-4.880</td>
</tr>
<tr>
<td>Trailing edge fiber</td>
<td>-5.300</td>
<td>-5.067</td>
<td>-5.091</td>
</tr>
<tr>
<td><strong>Uniform load</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leading edge fiber</td>
<td>-6.541</td>
<td>-6.546</td>
<td>-6.630</td>
</tr>
<tr>
<td>Middle fiber</td>
<td>-7.256</td>
<td>-7.408</td>
<td>-7.313</td>
</tr>
<tr>
<td>Trailing edge fiber</td>
<td>-7.971</td>
<td>-7.667</td>
<td>-7.750</td>
</tr>
</tbody>
</table>

*: extrapolated result

In general larger error than current approach

The table shows the comparison of measured and computed deformed shapes for step 1 and step 2 results with 10 modes. The relative error is calculated as a percentage difference between the measured and computed values, indicating the accuracy of the computed results. The smoothing effect is observed in the deformed shapes, particularly around the boundary areas.
Conclusions

The **two-step approach** for computing all the degrees of freedom in a structural FE model from measured strain along the FOSS is successfully applied to a cantilevered rectangular wing model and a test plate.

- The first experiment investigates the accuracy of the theory by applying it to a cantilevered rectangular wing model analyzed using the MSC/NASTRAN and ZAERO codes.
  - 1g uniform load case
  - Wing tip torsion load
  - Aerodynamic loading
    - All six computed DOFs have excellent matching with target values.

- The second experiment applies the theory to experimental data collected from a test plate fabricated and tested at the NASA AFRC.
  - The deflections calculated from the experimental model are extremely accurate.
Questions?