Wing Shape Sensing from Measured Strain

Prepared For:
AIAA SciTech 2015 (AIAA Infotech @ Aerospace)
January 5-9, Kissimmee, Florida

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Overview

- What the technology does
- Previous technologies
- Technical features of new technology
- Computational Validation
  - Uniform 1g load
  - Wing tip torsion load
  - Aerodynamic load under 1° angle of attack at Mach 0.715
- Experimental Testing
  - Leading-edge load
  - Uniform load
- Conclusions
What the technology does

**Problem Statement**

- Improving fuel efficiency for an aircraft
  - Reducing **weight** or **drag**
  - Similar effect on fuel savings
  - Multidisciplinary design optimization (design phase) or active control (during flight)

- Real-time measurement of deflection, slope, and loads in flight are a valuable tool.

- Wing deflection and slope (complete degrees of freedom) are essential quantities for load computations during flight.
  - Loads can be computed from the following governing equations of motion.
    \[
    \mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{G}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{F}_a(Mach, q(t))
    \]
    - Internal Loads: using finite element structure model
      - \(\mathbf{M}\ddot{\mathbf{q}}(t), \mathbf{G}\dot{\mathbf{q}}(t), \mathbf{K}\mathbf{q}(t)\): Inertia, damping, and elastic loads
    - External Load: using unsteady aerodynamic model
      - \(\mathbf{F}_a\): Aerodynamic load

- Traditionally, strain over the wing are measured using strain gages.
  - Cabling would create **weight and space limitation** problems.
  - A **new innovation** is needed. **Fiber optic strain sensor** (FOSS) is an ideal choice for **aerospace** applications.

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**Wing deflection & slope will be computed from measured strain.**
Previous technologies

Beam theory; Sectional bending moment and shear loads

- Carpenter, T.J. and Albertani, R., “Aerodynamic Load Estimation: Pressure Distribution from Virtual Strain Sensors for a Pliant Membrane Wing,” AIAA 2013-1917. Oregon State University; Aerodynamic loads are estimated from measured strain using virtual strain sensor technique.

Previous technologies are applied to a beam structure.
Technical features of new technology

Proposed solutions:

- The new method for obtaining the deflection over a flexible full 3D aircraft structure is based on the following two steps.
  - First Step: Compute wing deflection along fibers using measure strain data
    - Wing deflection will be computed along the fiber optic sensor line.
    - Strains at selected locations will be “fitted”.
    - These fitted strain will be integrated twice to have deflection information. (Relative deflection w.r.t. the reference point)
    - This is a finite element model independent method.
  - Second Step: Compute wing slope and deflection of entire structures
    - Slope computation will be based on a finite element model dependent technique.
    - Wing deflection and slope will be computed at all the finite element grid points.

A new two-step theory is investigated for predicting the deflection and slope of an entire structure using strain measurements at discrete locations.
First Step
- Use piecewise least-squares method to minimize noise in the measured strain data (strain/offset)
- Obtain cubic spline (Akima spline) function using re-generated strain data points:
  \[
  \frac{d^2\delta}{ds^2} = -\epsilon(s)/c(s)
  \]
- Integrate fitted spline function to get slope data:
  \[
  \frac{d\delta}{ds} = \theta(s)
  \]
- Obtain cubic spline (Akima spline) function using computed slope data
- Integrate fitted spline function to get deflection data: \(\delta(s)\)

A measured strain is fitted using a piecewise least-squares curve fitting method together with the cubic spline technique.
Technical features of new technology (continued)

- **Second Step: Based on General Transformation**
  - For all model reduction/expansion techniques, there is a relationship between the **master (measured or tested)** degrees of freedom and the **slave (deleted or omitted)** degrees of freedom which can be written in general terms as
    \[
    \{q\} = \begin{bmatrix} q_M \\ q_S \end{bmatrix} = [T]\{\tilde{q}_M\}; \quad \{q\} = \text{general displacement vector}
    \]
  - Where, an eigen-matrix is defined as \(\begin{bmatrix} q_M \\ q_S \end{bmatrix} = [\Phi_M \Phi_S] \{\eta\}; \quad \{\eta\} = \text{orthogonal displacement vector}
  - Transformation matrix \([T]\) can be one of the followings:
    - Guyan (or static) condensation, dynamic condensation, improved reduced system (IRS), or system equivalent reduction expansion process (SEREP)

- **Expansion of displacement using SEREP: kinds of least-squares method; most accurate reduction-expansion technique**
  - \(\{\tilde{q}_M\}\): master DOF; deflection along the fiber **computed from the first step**
  - \(\{q_S\} = [\Phi_S]\left(([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\right)\{\tilde{q}_M\}\): deflection and slope all over the structure
  - \(\{q_M\} = [\Phi_M]\left(([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\right)\{\tilde{q}_M\}\): smoothed master DOF
    \[
    [T] = \begin{bmatrix}
    \Phi_M ([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T \\
    \Phi_S ([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T
    \end{bmatrix}
    \]

**Computed deflection along the fibers are combined with a finite element model of the structure in order to interpolate and extrapolate the deflection and slope of the entire structure through the use of the System Equivalent Reduction and Expansion Process.**
Computational Validation

Cantilevered rectangular wing model
Cantilevered Rectangular Wing Model

- Wind tunnel test wing (thickness = 0.065 in.)
  - Uniform 1g load
  - Wing tip torsion (1 lbf at leading-edge and -1 lbf at trailing-edge of wing tip section)
  - Aerodynamic load under 1° angle of attack at Mach 0.715

- MSC/NASTRAN
  - Compute strain
  - Compute deflection (target)

- ZAERO
  - Compute aerodynamic load

- Two-step approach
  - Compute deflection from computed strain
  - Compare computed deflection with respect to target value

Wing with 22 FOSS

Fiber optic strain sensors: 11(upper) + 11(lower)
Cantilevered Rectangular Wing Model: Uniform 1g

- **Uniform 1g load**

- **Curvature distribution (step 1 results)**
  - Positive curvature distribution along the fiber direction.

- **Wing deflection over FE model (step 2 results with 10 modes)**
  - Undeformed shape
  - Deformed shape

- **Wing deflection (step 2 results with 10 modes)**

- **Wing span-wise slope (step 2 results with 10 modes)**

Cantilevered Rectangular Wing Model: Wing tip torsion

- Wing tip torsion
- Negative
- Positive

Wing deflection over FE model (step 2 results with 10 modes)

Markers: NASTRAN Result
Lines: Two-step Approach

Fiber 1
Fiber 3
Fiber 5
Fiber 7
Fiber 9
Fiber 11

Markers: NASTRAN Result
Lines: Two-step Approach

Fiber 1
Fiber 3
Fiber 5
Fiber 7
Fiber 9
Fiber 11

Markers: NASTRAN Result
Lines: Two-step Approach

Fiber 1
Fiber 3
Fiber 5
Fiber 7
Fiber 9
Fiber 11

Markers: NASTRAN Result
Lines: Two-step Approach

Fiber 1
Fiber 3
Fiber 5
Fiber 7
Fiber 9
Fiber 11

(a) Strain distribution (step 1 results)

(b) Step 2 results with 50 Modes

(c) Step 2 results with 50 Modes

(d) Step 2 results with 50 Modes
Cantilevered Rectangular Wing Model: Aerodynamic load

- Aerodynamic load under 1° angle of attack at Mach 0.715

Markers: NASTRAN Result
Lines: two-step approach

(a) Curvature distribution (step 1 results)

Markers: NASTRAN Result
Lines: two-step approach

Fiber 1
Fiber 5
Fiber 9
Fiber 13
Fiber 17
Fiber 21

Extrapolation
Extrapolation

Along the fiber direction, in.

Curvature, in.

0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35

0 1 2 3 4 5 6 7 8 9 10 11

(b) Splined load vectors

(a) Pressure distribution
### Cantilevered Rectangular Wing Model: Aerodynamic load (continued)

- Aerodynamic load under 1° angle of attack at Mach 0.715 (continued)
- Wing tip deflections

#### Deflection in X, Y, & Z direction

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target (inch)</th>
<th>Computed X deflection (inch)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>0.0006927</td>
<td>N/A</td>
<td>0.0006703</td>
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<td>5</td>
<td>0.0006975</td>
<td>N/A</td>
<td>0.0006776</td>
</tr>
<tr>
<td>9</td>
<td>0.0007029</td>
<td>N/A</td>
<td>0.0006936</td>
</tr>
<tr>
<td>13</td>
<td>0.0007047</td>
<td>N/A</td>
<td>0.0007081</td>
</tr>
<tr>
<td>17</td>
<td>0.0007035</td>
<td>N/A</td>
<td>0.0007144</td>
</tr>
<tr>
<td>21</td>
<td>0.0007023</td>
<td>N/A</td>
<td>0.0007143</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target (inch)</th>
<th>Computed Y deflection (inch)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>-0.003280</td>
<td>N/A</td>
<td>-0.003282</td>
</tr>
<tr>
<td>5</td>
<td>-0.003272</td>
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<td>-0.003278</td>
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<tr>
<td>9</td>
<td>-0.003260</td>
<td>N/A</td>
<td>-0.003268</td>
</tr>
<tr>
<td>13</td>
<td>-0.003248</td>
<td>N/A</td>
<td>-0.003257</td>
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<tr>
<td>17</td>
<td>-0.003239</td>
<td>N/A</td>
<td>-0.003246</td>
</tr>
<tr>
<td>21</td>
<td>-0.003235</td>
<td>N/A</td>
<td>-0.003242</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target (inch)</th>
<th>Computed Z deflection (inch)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>0.9280</td>
<td>0.9269</td>
<td>0.9275</td>
</tr>
<tr>
<td>5</td>
<td>0.9085</td>
<td>0.9091</td>
<td>0.9087</td>
</tr>
<tr>
<td>9</td>
<td>0.8889</td>
<td>0.8894</td>
<td>0.8895</td>
</tr>
<tr>
<td>13</td>
<td>0.8691</td>
<td>0.8696</td>
<td>0.8699</td>
</tr>
<tr>
<td>17</td>
<td>0.8493</td>
<td>0.8497</td>
<td>0.8498</td>
</tr>
<tr>
<td>21</td>
<td>0.8296</td>
<td>0.8300</td>
<td>0.8297</td>
</tr>
</tbody>
</table>

**Boundary Deformed shape**

**Undeformed shape**

**Wing deflection over FE model (step 2 results with 10 modes)**

**Input to Step 2\(q_M\): master DOF \(q_S\): slave DOF**

**Smoothing effect**
Cantilevered Rectangular Wing Model: Aerodynamic load (continued)

- Aerodynamic load under 1° angle of attack at Mach 0.715 (continued)
- Wing tip slopes
  **Slope in roll, pitch, & yaw direction**

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target</th>
<th>Computed roll slope</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1*</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>0.10090</td>
<td>0.1010</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>0.10070</td>
<td>0.1016</td>
<td>0.0909</td>
</tr>
<tr>
<td>9</td>
<td>0.10030</td>
<td>0.1012</td>
<td>0.1060</td>
</tr>
<tr>
<td>13</td>
<td>0.09993</td>
<td>0.1009</td>
<td>0.1020</td>
</tr>
<tr>
<td>17</td>
<td>0.09966</td>
<td>0.1006</td>
<td>0.09989</td>
</tr>
<tr>
<td>21</td>
<td>0.09954</td>
<td>0.1004</td>
<td>0.09974</td>
</tr>
</tbody>
</table>

*Step 1* Roll slope without effect of X and Y deflections (These slopes are not used during step 2 computation.)

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target</th>
<th>Computed pitch slope</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>0.02131</td>
<td>N/A</td>
<td>0.02063</td>
</tr>
<tr>
<td>5</td>
<td>0.02146</td>
<td>N/A</td>
<td>0.02085</td>
</tr>
<tr>
<td>9</td>
<td>0.02163</td>
<td>N/A</td>
<td>0.02134</td>
</tr>
<tr>
<td>13</td>
<td>0.0216</td>
<td>N/A</td>
<td>0.02179</td>
</tr>
<tr>
<td>17</td>
<td>0.02165</td>
<td>N/A</td>
<td>0.02198</td>
</tr>
<tr>
<td>21</td>
<td>0.02161</td>
<td>N/A</td>
<td>0.02198</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiber number</th>
<th>Target</th>
<th>Computed yaw slope</th>
<th>Absolute error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Step 1</td>
<td>Step 2 with 10 modes</td>
</tr>
<tr>
<td>1</td>
<td>2.2e-31</td>
<td>N/A</td>
<td>7.9e-18</td>
</tr>
<tr>
<td>5</td>
<td>1.9e-31</td>
<td>N/A</td>
<td>6.6e-18</td>
</tr>
<tr>
<td>9</td>
<td>1.7e-31</td>
<td>N/A</td>
<td>6.1e-18</td>
</tr>
<tr>
<td>13</td>
<td>1.7e-31</td>
<td>N/A</td>
<td>5.7e-18</td>
</tr>
<tr>
<td>17</td>
<td>1.4e-31</td>
<td>N/A</td>
<td>5.9e-18</td>
</tr>
<tr>
<td>21</td>
<td>1.2e-31</td>
<td>N/A</td>
<td>3.3e-18</td>
</tr>
</tbody>
</table>

- Not usable for Step 2 computations
- Y deflection
- X deflection

{ qs: slave DOF}
Experimental Testing

Swept test plate
Swept Test Plate

- Tested at NASA AFRC

**Thickness = 0.19 in**
- **Swept angle = 45°**

- **Leading-edge fiber**
- **Trailing-edge fiber**
- **Mid-chord fiber**

**a) Leading-edge load**
- 3 lb
- 3 lb
- 2 lb
- 2 lb
- 2 lb
- 2 lb
- 2 lb

**b) Uniform load**
- 6 lb
- 6 lb
- 3 lb
- 3 lb
- 3 lb
- 3 lb
- 2 lb

Photogrammetry target
- Strain rosette
- Fiber optical strain sensing (under sealant)
Averaging the curvatures calculated by using each fiber individually eliminates the effect of the axial load.

This computation is performed after curve-fitting each set of data individually to minimize noise.

(a) Leading-edge load

(b) Uniform load
Swept Test Plate (continued)

- Under leading-edge load

Step 1 results

- Leading-edge fiber (step 1 result)
- Mid-chord fiber (step 1 result)
- Trailing-edge fiber (step 1 result)

Step 2 results

- Leading-edge fiber (step 2 result with 10 modes)
- Mid-chord fiber (step 2 result with 10 modes)
- Trailing-edge fiber (step 2 result with 10 modes)

larger error
Swept Test Plate (continued)

Under uniform load

- **Leading-edge**
  - Step 1 results
  - Step 2 results

- **Mid-chord**
  - Larger error

- **Trailing-edge**
  - Larger error

- **Step 1 results**
  - Leading-edge fiber
  - Mid-chord fiber
  - Trailing-edge fiber

- **Step 2 results**
  - Leading-edge fiber (step 2 result with 10 modes)
  - Mid-chord fiber (step 2 result with 10 modes)
  - Trailing-edge fiber (step 2 result with 10 modes)

Legend:
- : Photogrammetry data
- : Computed deflection
- : Bakalyar/Jutte deflection

Diagram notes:
- Average deflection, in.
- Along the fiber direction, in.
## Swept Test Plate (continued)

- **Deformed wing shape (step 2 results with 10 modes)**

<table>
<thead>
<tr>
<th></th>
<th>Measured (inch)</th>
<th>Computed (inch)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bakalyar and Jutte*</td>
<td>Step 1</td>
</tr>
<tr>
<td><strong>Leading-edge load</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leading edge fiber</td>
<td>-4.525</td>
<td>-4.500</td>
<td>-4.542</td>
</tr>
<tr>
<td>Middle fiber</td>
<td>-4.912</td>
<td>-4.952</td>
<td>-4.880</td>
</tr>
<tr>
<td>Trailing edge fiber</td>
<td>-5.300</td>
<td>-5.067</td>
<td>-5.091</td>
</tr>
<tr>
<td><strong>Uniform load</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leading edge fiber</td>
<td>-6.541</td>
<td>-6.546</td>
<td>-6.630</td>
</tr>
<tr>
<td>Middle fiber</td>
<td>-7.256</td>
<td>-7.408</td>
<td>-7.313</td>
</tr>
<tr>
<td>Trailing edge fiber</td>
<td>-7.971</td>
<td>-7.667</td>
<td>-7.750</td>
</tr>
</tbody>
</table>

*: extrapolated result

### Notes:
- In general larger error than current approach
- Smoothing effect

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**a) Leading-edge load.**

**b) Uniform load.**

**Stress concentration**

**Boundary**

**Undeformed shape**

**Deformed shape**

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Structural Dynamics Group
Conclusions

- The **two-step approach** for computing all the degrees of freedom in a structural FE model from measured strain along the FOSS is successfully applied to a cantilevered rectangular wing model and a test plate.

  - The first experiment investigates the accuracy of the theory by applying it to a cantilevered rectangular wing model analyzed using the MSC/NASTRAN and ZAERO codes.
    - 1g uniform load case
    - Wing tip torsion load
    - Aerodynamic loading
    - All six computed DOFs have excellent matching with target values.

  - The second experiment applies the theory to experimental data collected from a test plate fabricated and tested at the NASA AFRC.
    - The deflections calculated from the experimental model are extremely accurate.
Questions?