



# Rapid State Space Modeling Tool for Rectangular Wing Aeroservoelastic Studies

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# Outline

- Overview & Motivation
- Aeroservoelastic tool
- Verification and Validation studies
- State Space Model Development and Results
- Conclusions





# Overview & Motivation

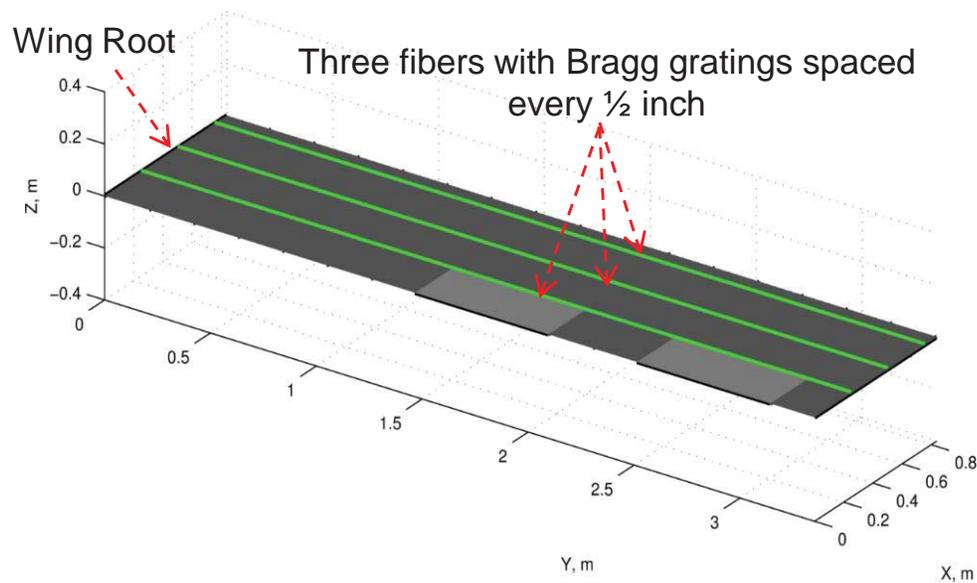
- Overview
  - Presentation of computational and experimental results from a recently developed rectangular wing aeroservoelastic modeling tool
- Motivation
  - Compare tool to independently published work<sup>2</sup>
  - To support rapid investigation of aeroservoelastic phenomena in a medium-fidelity tool
    - Also novel sensors such as fiber optics
  - Provide a rapid aeroservoelastic design platform which can serve students of aeroservoelasticity

<sup>2</sup>SConyers, H. J., Dowell, E. H., and Hall, K. C., "Aeroservoelastic Studies of a Rectangular Wing with a Hole: Correlation of Theory and Experiment," 2010 Aerospace Systems Conference

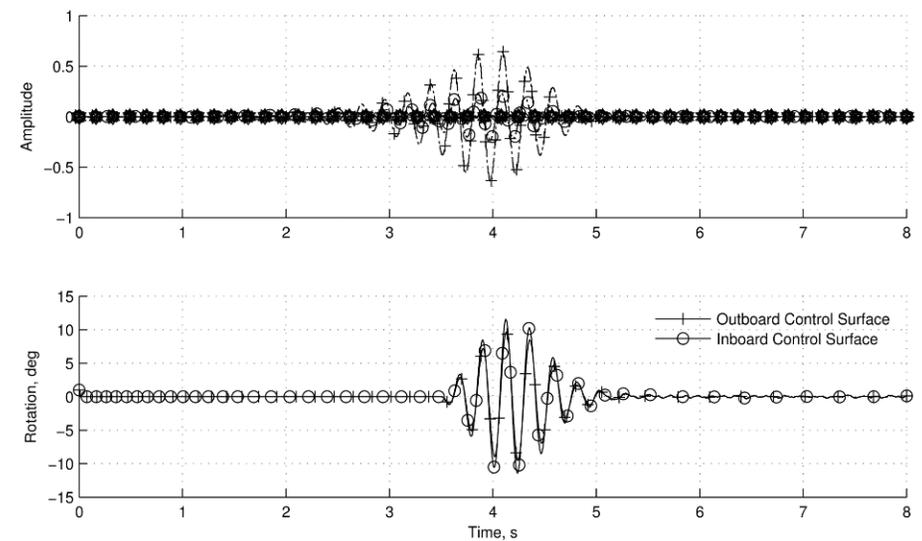
# Background

- In previous work<sup>1</sup>, tool used to model a clamped wing structure with two control surfaces and fiber optic sensor feedback used for flutter suppression

## Computational Wing Model



## Active Flutter Suppression



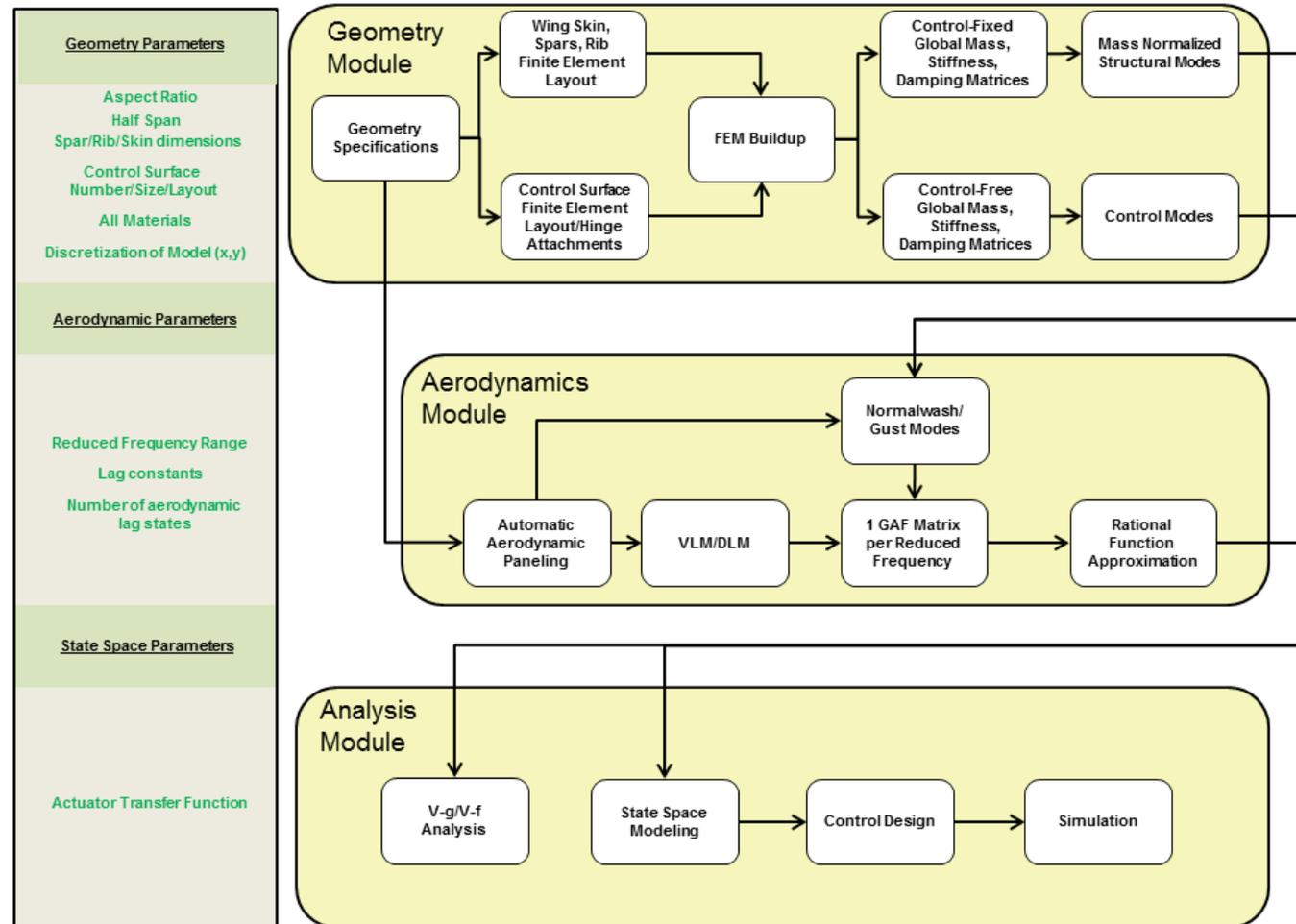
<sup>1</sup>Suh, P. M., and Mavis, D. N., Modal Filtering for Control of Flexible Aircraft, AIAA 2013-1741



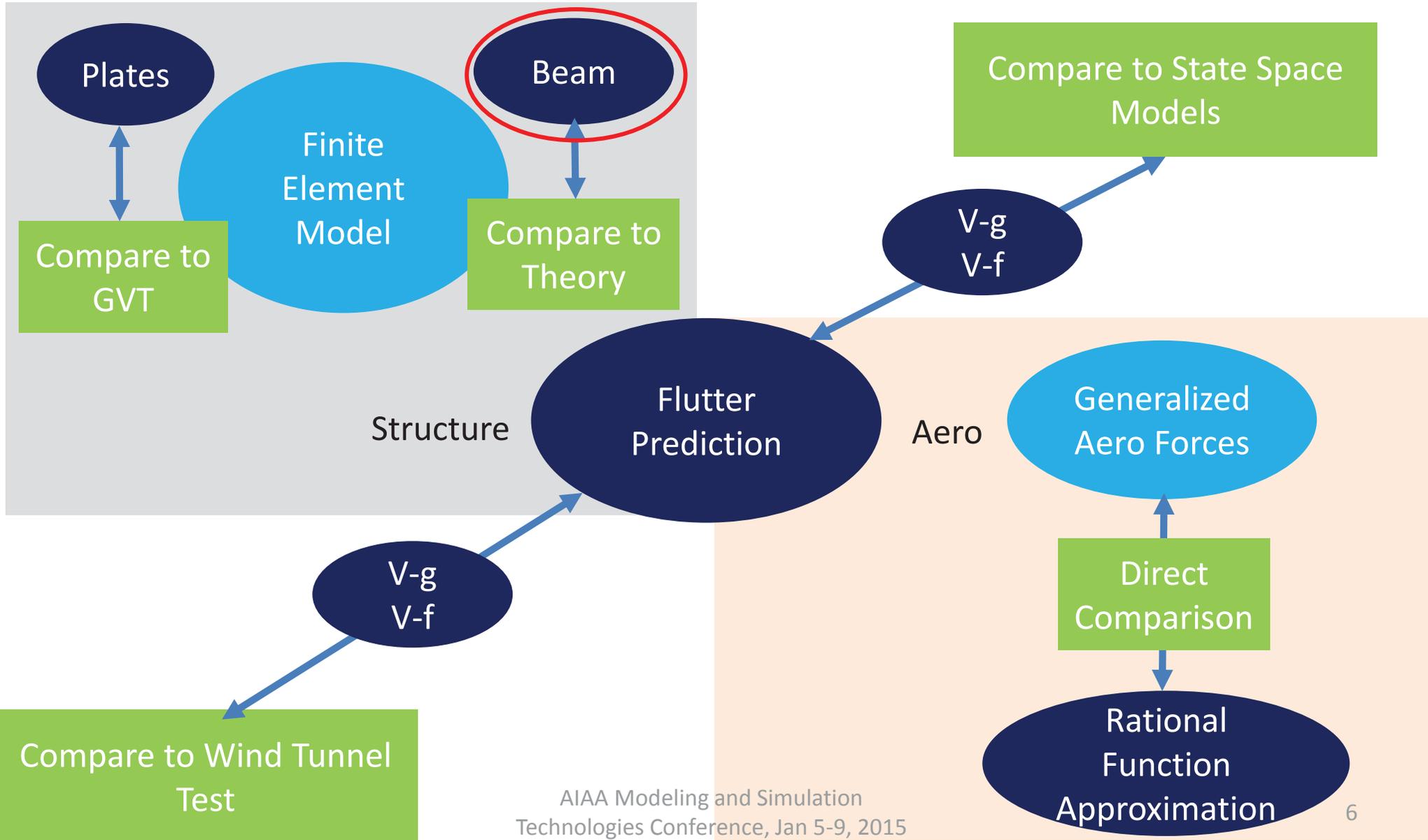
# Aeroservoelastic Tool Overview

## Tool for Rectangular Wing Aeroservoelastic Design in MATLAB

- Tool allows the user to quickly move from inputs like aspect ratio, control surface count, and half span to a linear time invariant state space model which can be used for control
  - A few seconds of real time computation
  - Most important structural and aerodynamic properties are parametric

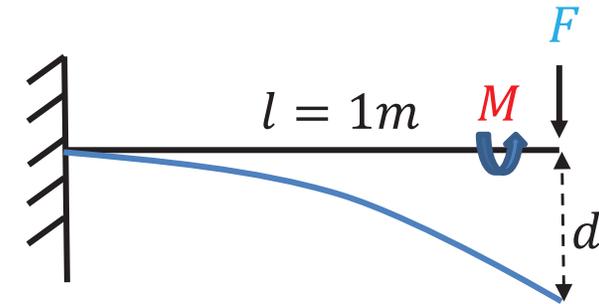


# Graphical Path of Verification and Validation of Tool

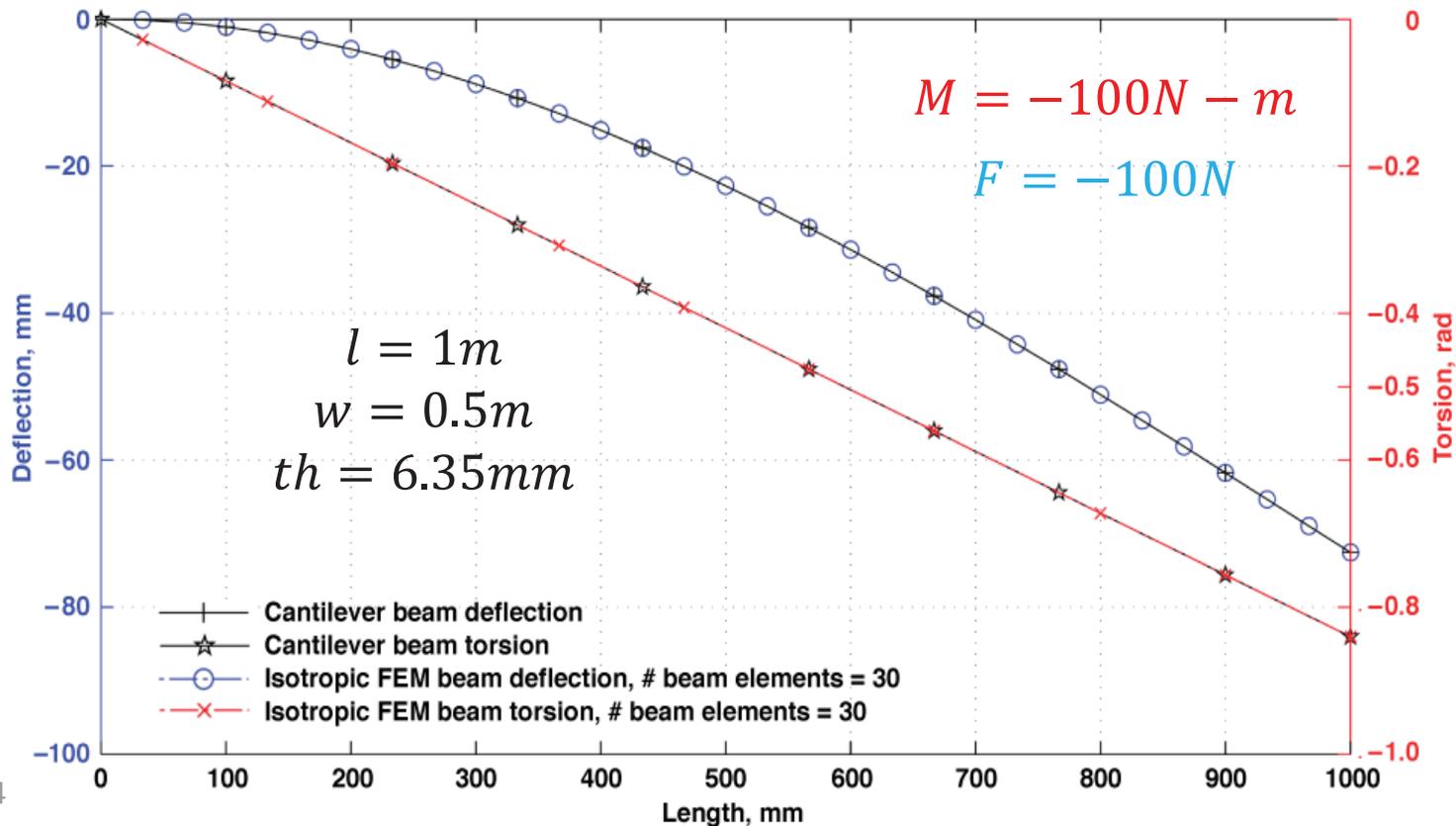


# Beam Model Verification

- Beams used to model wing structure
  - FEM with 30 elements compared to theory show good matches in **bending** and **torsion**

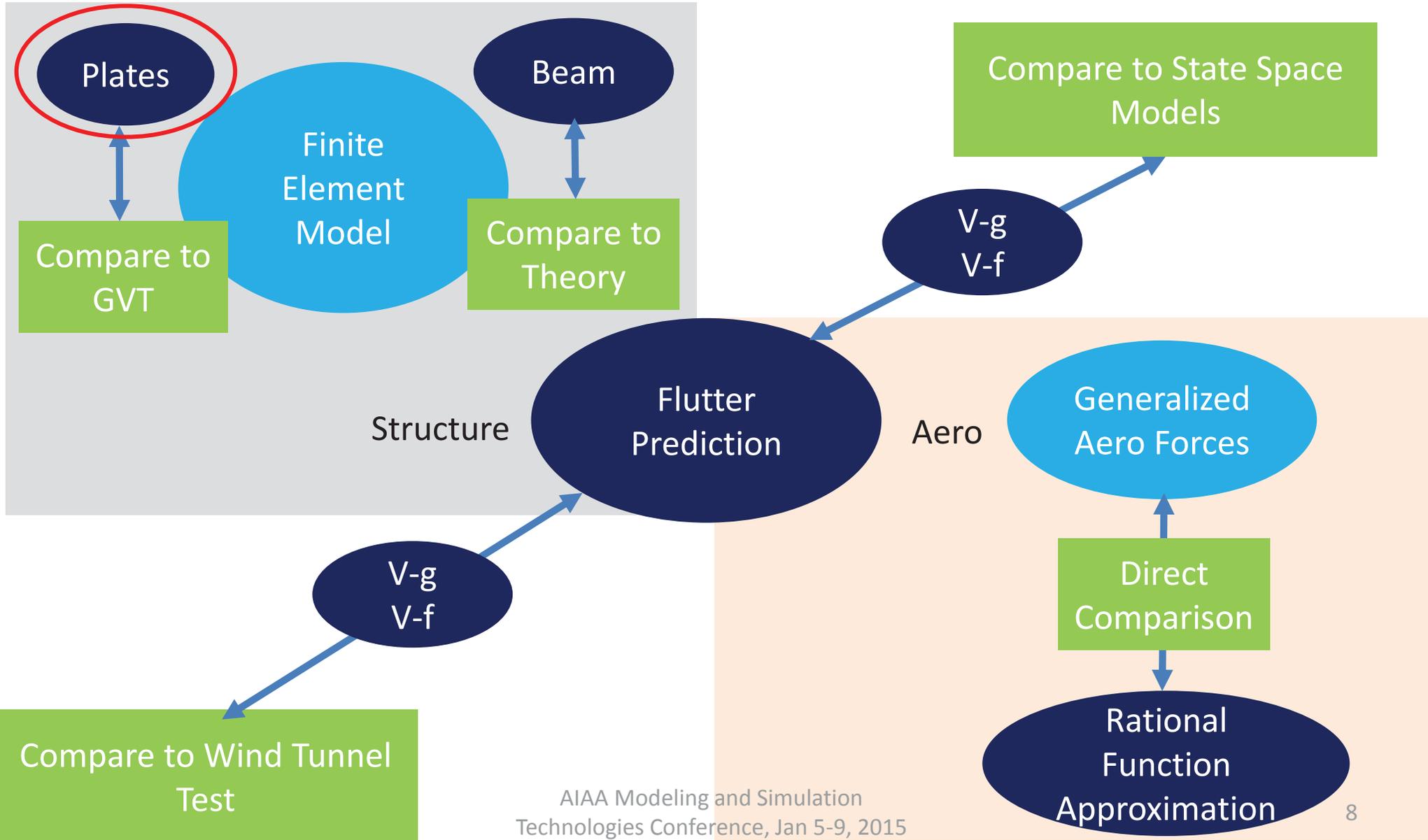


## Cantilever Theory versus FEM Beam Model: Deflection and Twist





# Graphical Path of Verification and Validation of Tool





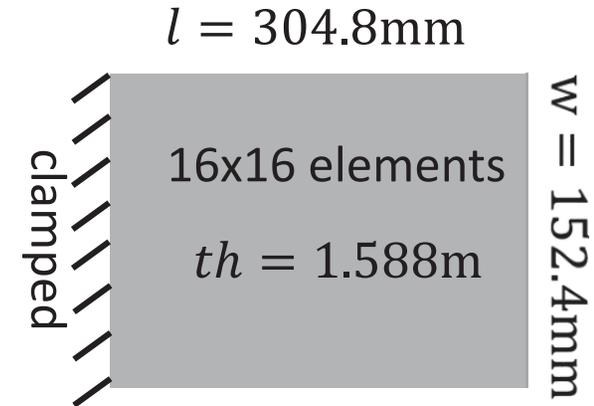
# Plate FEM Validation

- Ground Vibration Test (GVT) on a article used for validation of plate FEM
- Plate FEM Discretized with 16x16 12 DOF isotropic plate elements
- Experiment shows good correlation with ANSYS and tool

GVT on Article (with a hole for a different test)



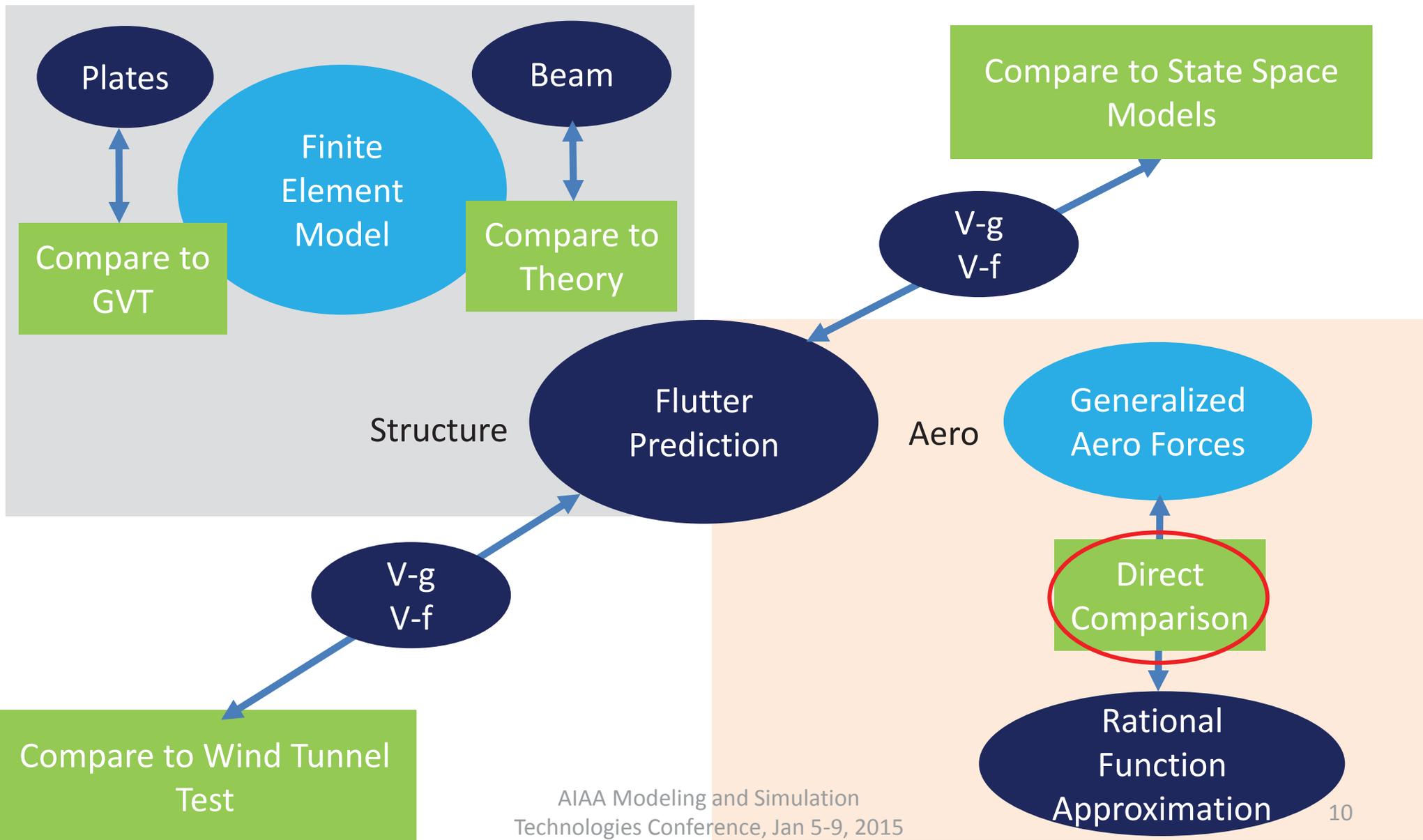
Computational Model



Experimental Data<sup>2</sup>

	ANSYS Frequencies, Hz	Tool FEM Frequencies, Hz	Conyers et al. GVT, Hz
Mode # 1	3.99	3.99	4.13
Mode # 2	16.96	16.97	17.24
Mode # 3	24.86	24.89	24.38
Mode # 4	55.33	55.40	54.25
Mode # 5	69.84	69.92	69.00

# Graphical Path of Verification and Validation of Tool





# RFA Verification

- Generalized aerodynamic forces (GAF) computed for plate
- Roger's rational function approximation (RFA) used to fit GAF coefficients
  - 4 lag states
- Least squares error for bending and twist coefficients

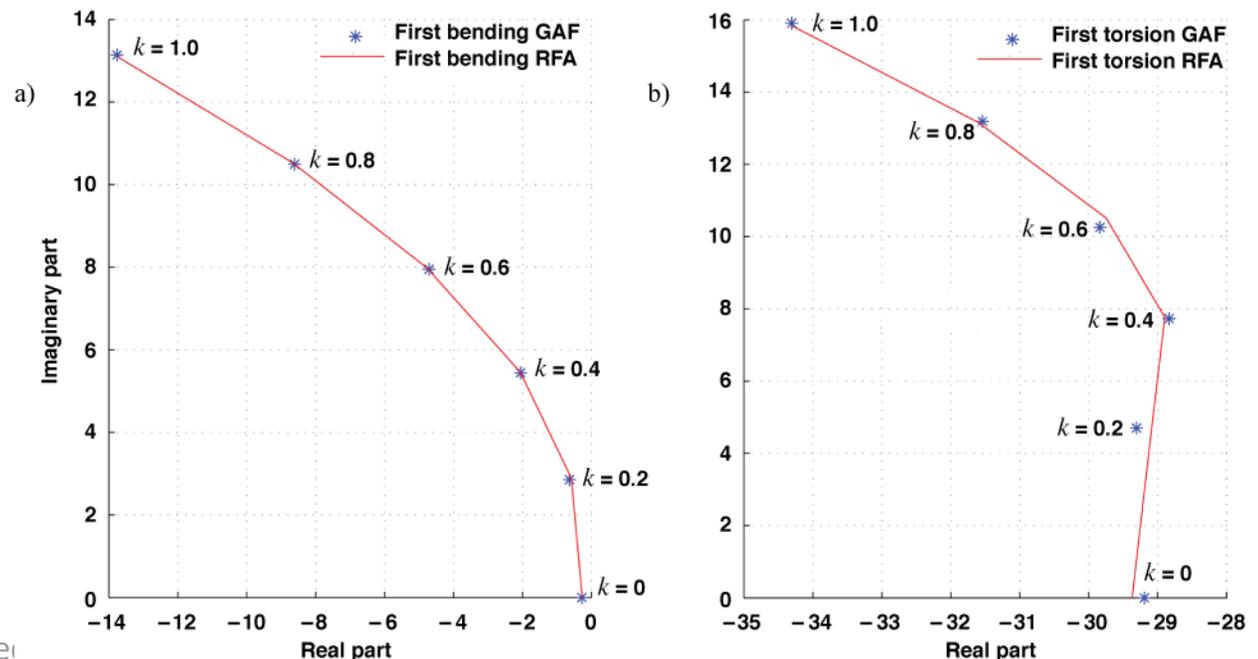
## Generalized Aerodynamic Force

$$Q(\hat{ik}) = Z_f^T D(\hat{ik})^{-1} A_p W_{c.p.}$$

## Rational Function Approximation of GAF

$$\hat{Q}(\bar{s}) = A_0 + \bar{s}A_1 + \bar{s}^2A_2 + \sum_{l=1}^L \frac{\bar{s}}{\bar{s} + \beta_l} A_{2+l}$$

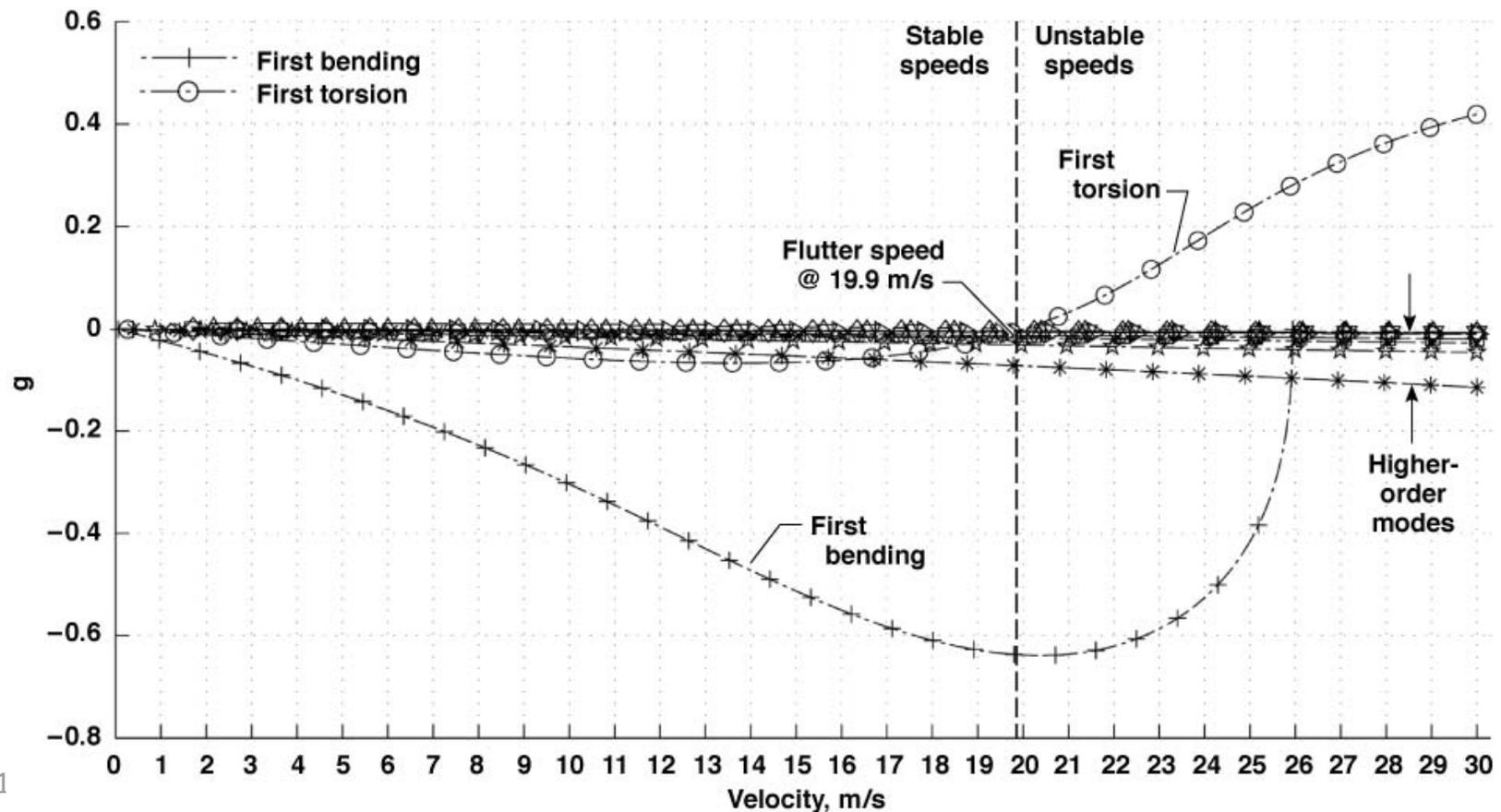
## Comparison of GAF and RFA Curve Fits



# V-g Analysis using RFA

- The test plate article flutter speed was predicted to be 19.9 m/s
  - traditional bending/torsion flutter mode

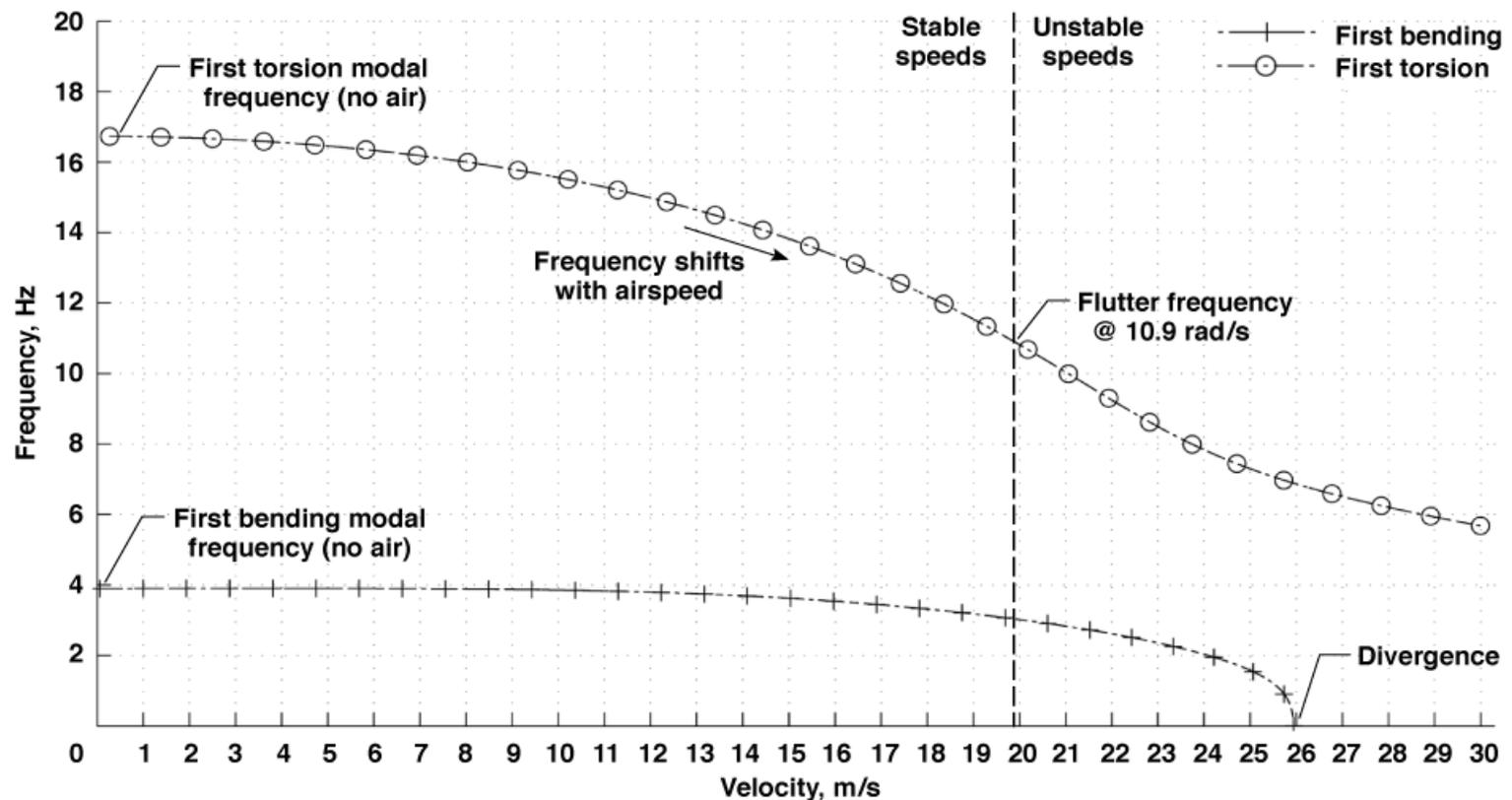
## V-g Analysis on Computational Plate Article



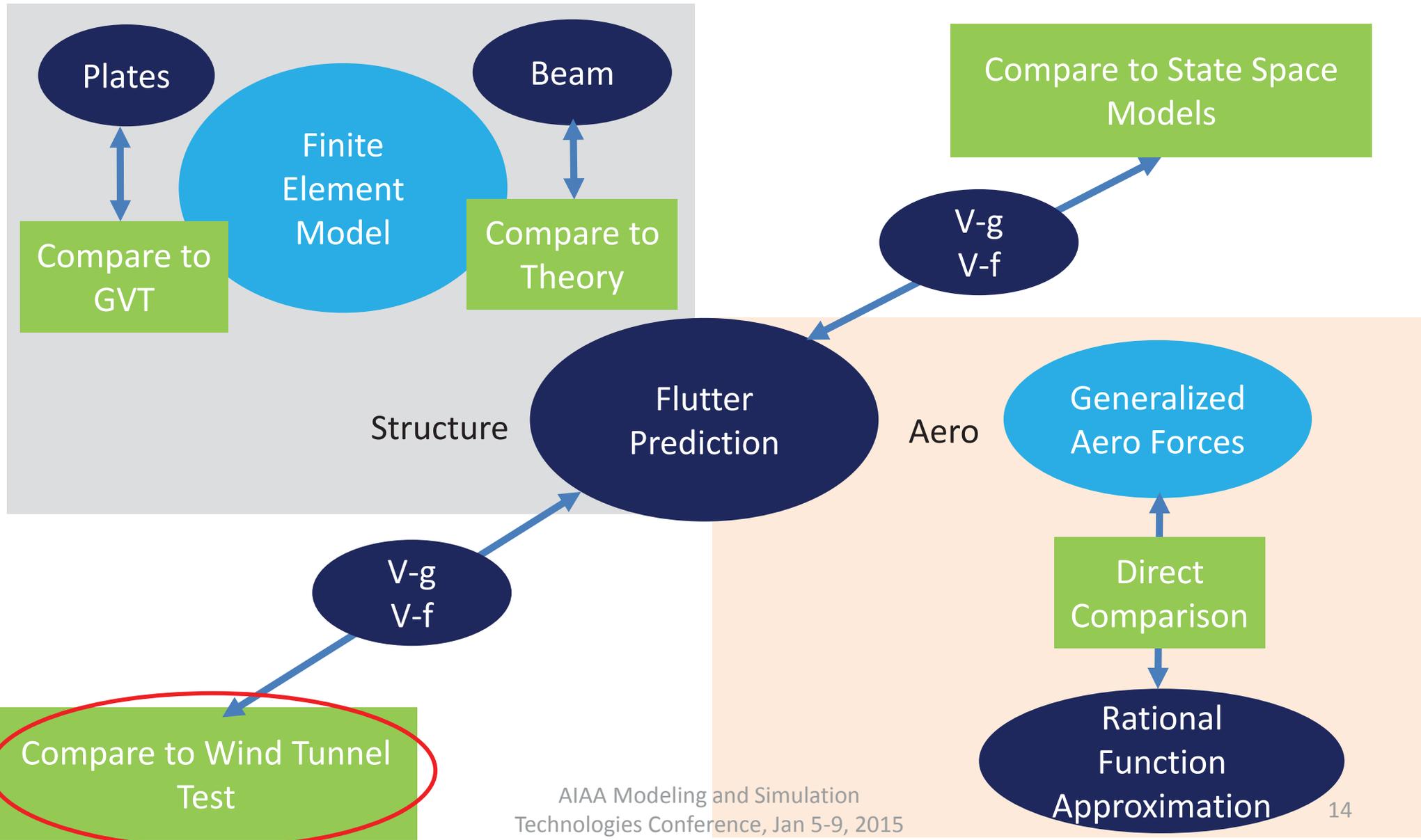
# V-f Analysis using RFA

- The test plate article flutter frequency was predicted to be 10.9 rad/s
  - Torsional mode shifts closer to bending mode
  - Characteristic of a one side clamped plate flutter mode

## V-f Analysis on Computational Plate Article



# Graphical Path of Verification and Validation of Tool

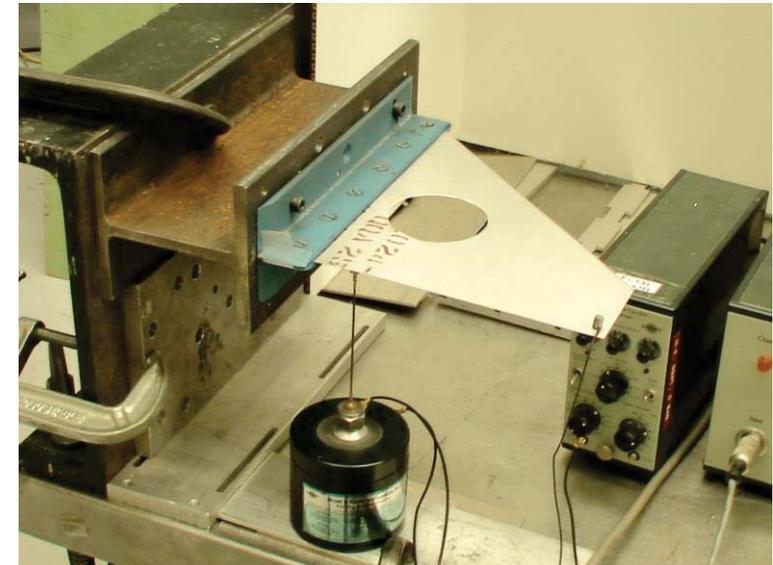




# Flutter Validation Experimental Study

- A wind tunnel investigation was completed at Duke University in previous work
  - Tool flutter speed shows good correlation with Conyers et al.'s flutter code
    - Differences may be due to use of more aero panels in the tool
  - Wind tunnel results were comparably close

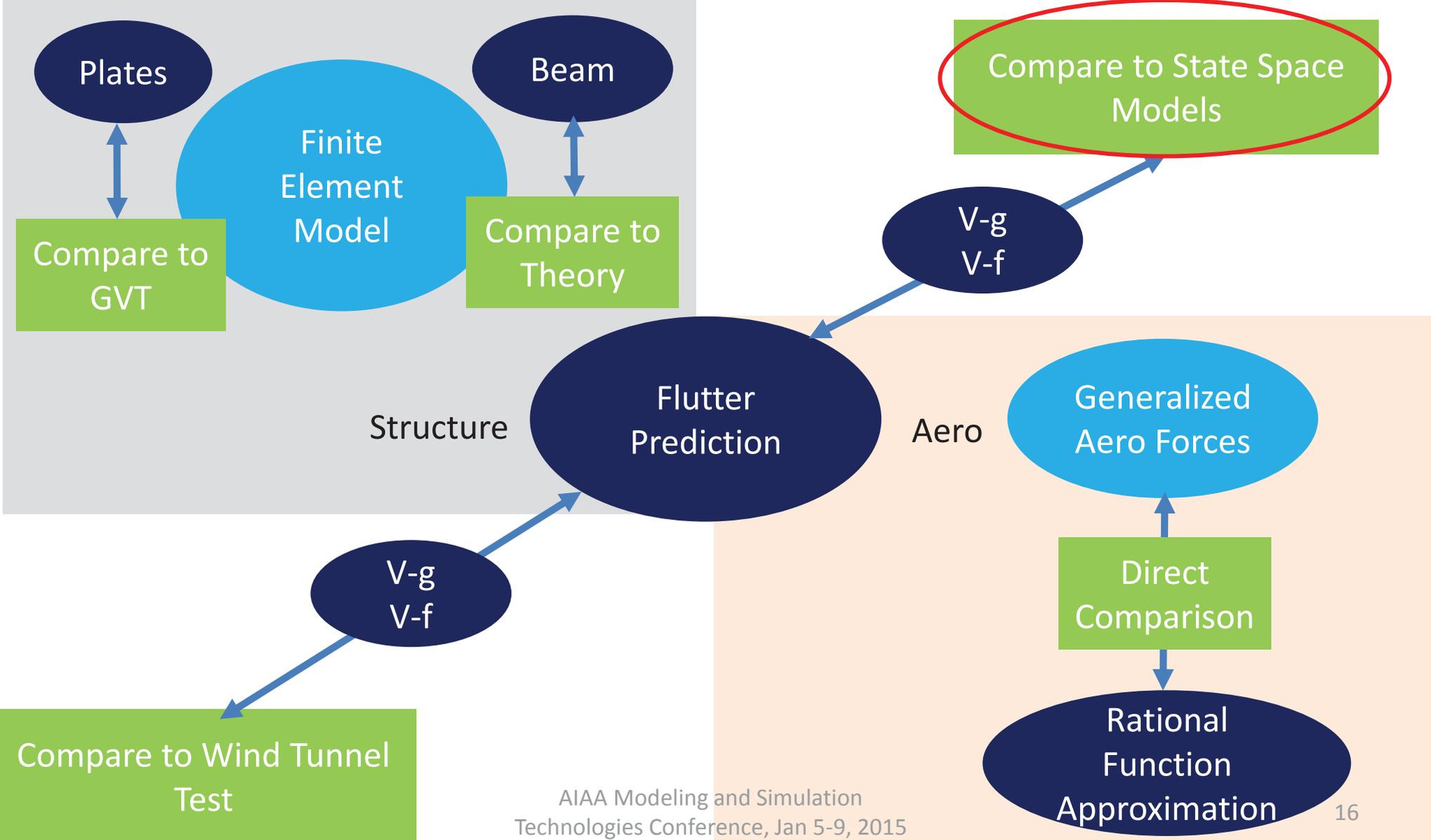
Configuration for Wind Tunnel Test at Duke  
(different article shown than flat plate)



## Experimental Data<sup>2</sup>

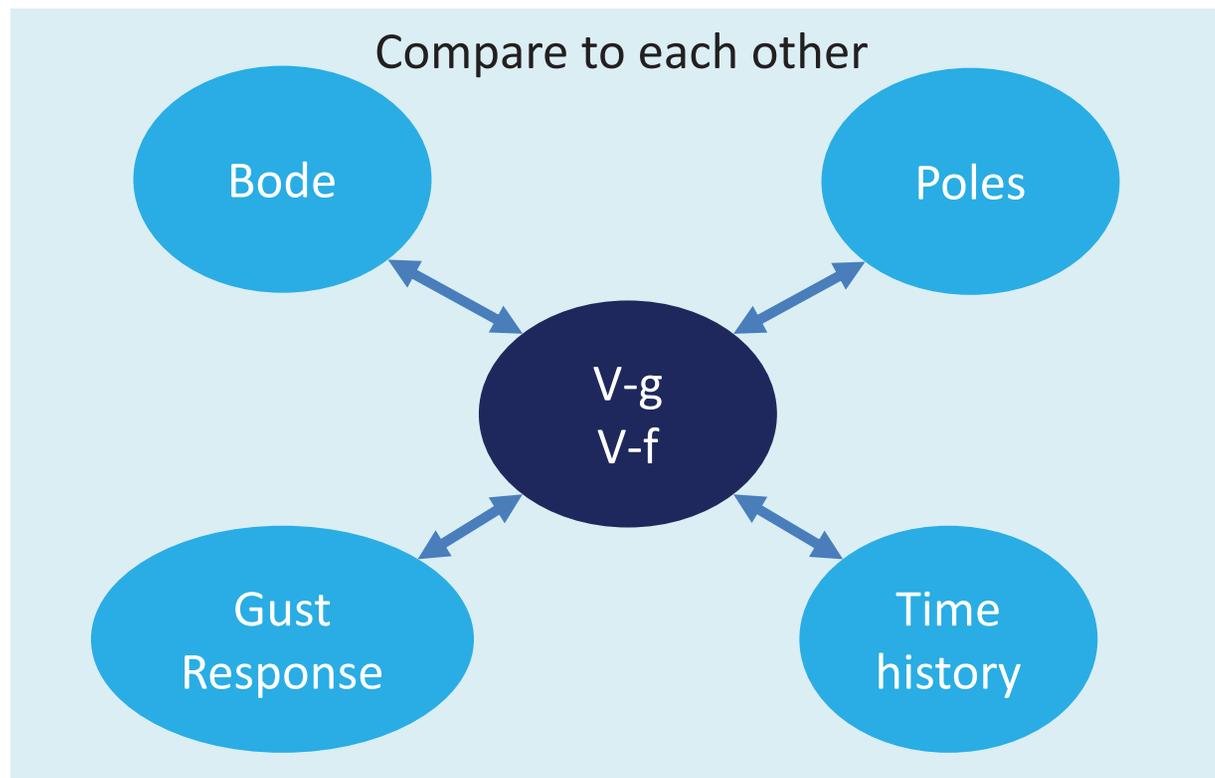
	Conyers et al. Flutter Code <sup>3</sup>	Tool Flutter Code	Conyers et al. Wind Tunnel Results <sup>3</sup>
Flutter speed , m/s	20.8	19.9	20.05
Flutter frequency, Hz	10.3	10.9	11.50

# Graphical Path of Verification and Validation of Tool



# State Space Model Verification

- We verify that the state space models correlate with what was predicted from the V-g and V-f analyses





# State Space Model Architecture

- Components of state space models
  - FEM mass, stiffness, damping and modal matrices
  - Rational function approximation coefficients
  - Actuator dynamic models
  - Flight condition

## State Space Model Architecture

$$\dot{x} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_6 \end{Bmatrix} = \begin{bmatrix} 0 & I & 0 & \dots & 0 \\ -\hat{M}^{-1}(\hat{K} & \hat{C} & \bar{q}I & \dots & \bar{q}I) \\ 0 & A_3 & -\beta_1(\frac{2V_\infty}{\bar{c}}) & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & A_6 & 0 & 0 & -\beta_4(\frac{2V_\infty}{\bar{c}}) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_6 \end{Bmatrix}$$

Modal displacement (points to  $x_1$ )  
Modal velocity (points to  $x_2$ )  
Aero lag states (bracketed around  $x_3$  to  $x_6$ )

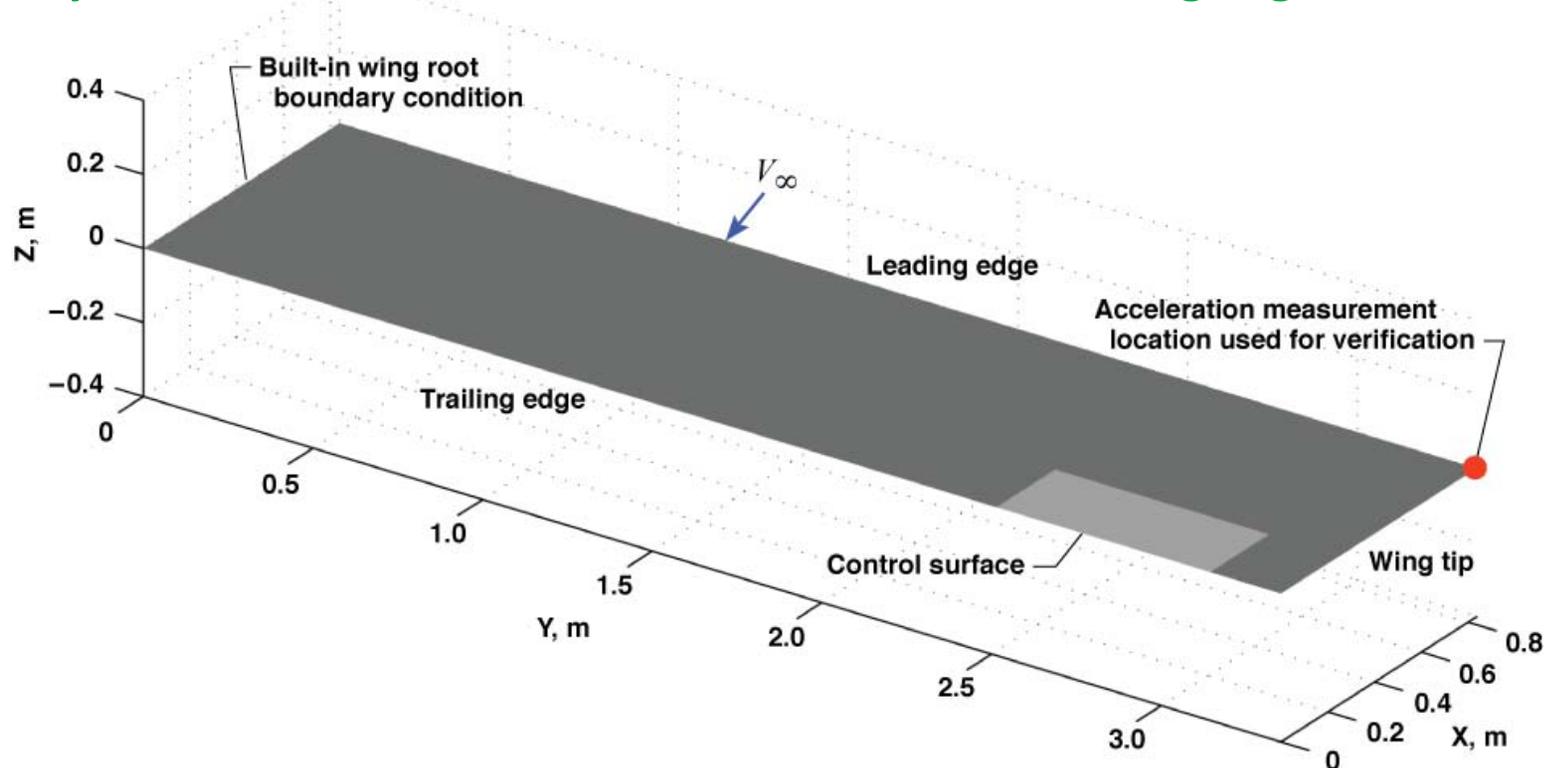
$$+ \begin{bmatrix} 0 & 0 & 0 \\ -\hat{M}^{-1}(\hat{K}_c & \hat{C}_c & \hat{M}_c) \\ 0 & A_{c1} & 0 \\ \vdots & \vdots & \vdots \\ 0 & A_{c4} & 0 \end{bmatrix} \begin{Bmatrix} u_c \\ \dot{u}_c \\ \ddot{u}_c \end{Bmatrix} + \frac{1}{V_\infty} \begin{bmatrix} 0 & 0 \\ -\hat{M}^{-1}(\hat{K}_g & \hat{C}_g) \\ 0 & A_{g1} \\ \vdots & \vdots \\ 0 & A_{g4} \end{bmatrix} \begin{Bmatrix} w_g \\ \dot{w}_g \end{Bmatrix}$$

Control states (bracketed around  $u_c, \dot{u}_c, \ddot{u}_c$ )  
Gust states (bracketed around  $w_g, \dot{w}_g$ )

# Analytical Model with Control Surfaces

- Verification of state space models is completed for a wing model with
  - internal aluminum beam spar and rib structure
  - aluminum skin
  - a control surface and a leading edge accelerometer

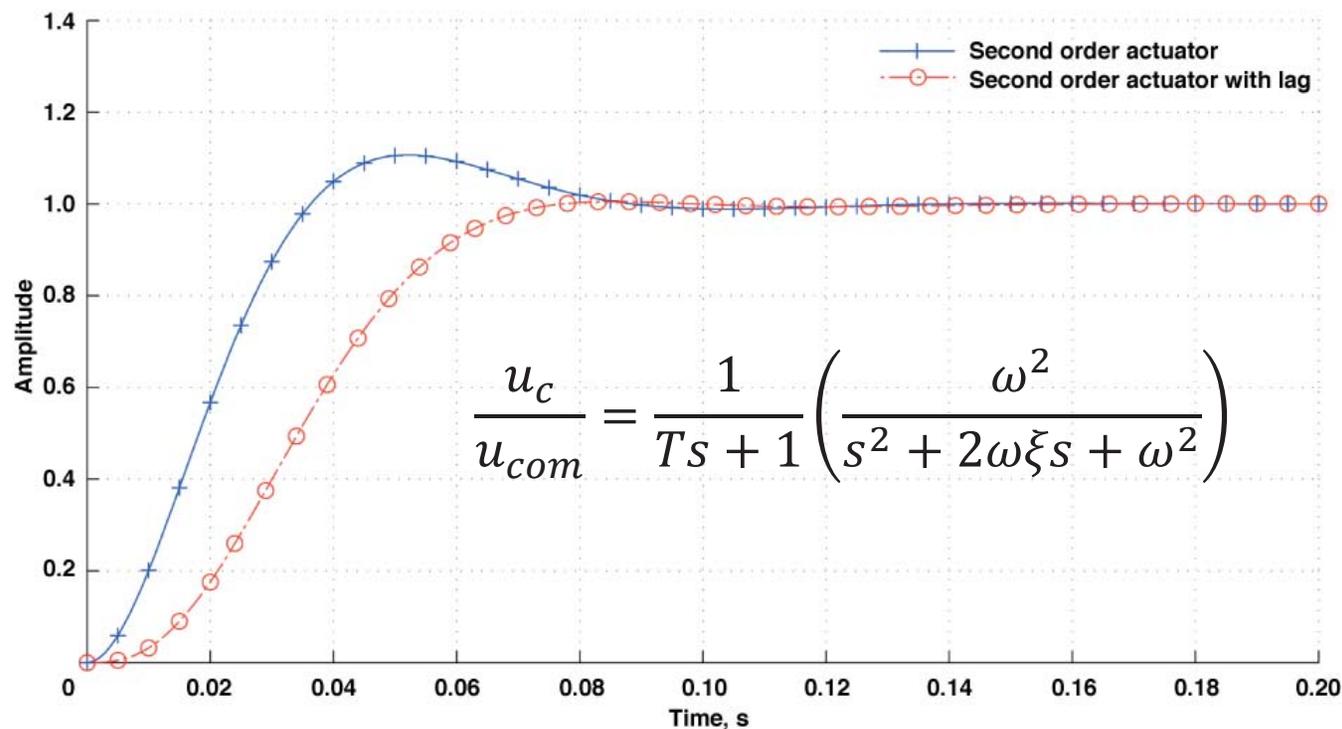
## Analytical Model with One Control Surface and a Leading Edge Accelerometer



# Actuator Dynamics

- Actuators are modeled as 3<sup>rd</sup> order transfer functions
  - 1<sup>st</sup> order command lag
  - 2<sup>nd</sup> order actuator dynamics

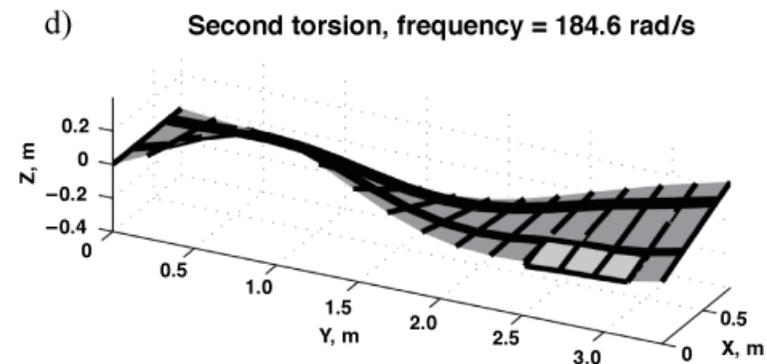
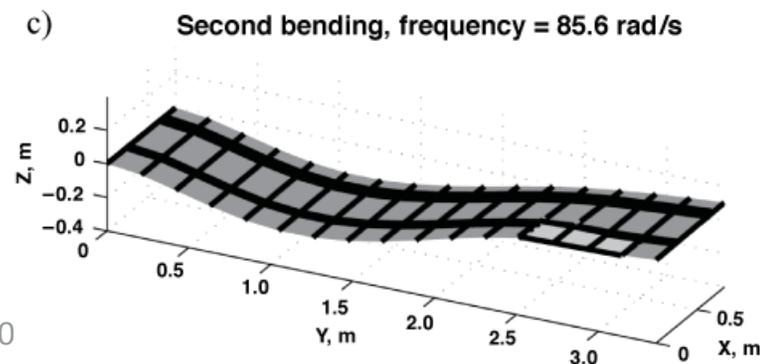
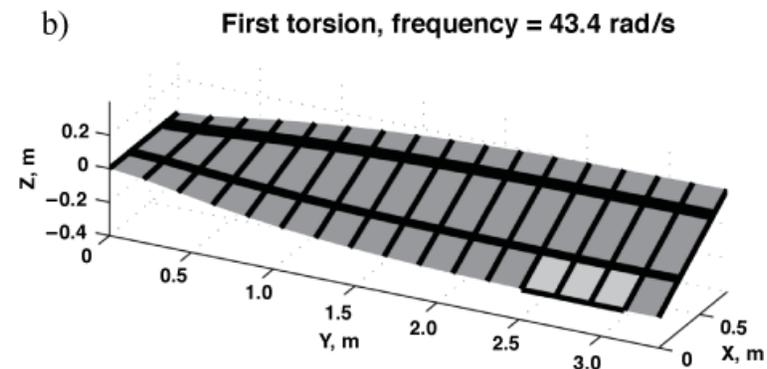
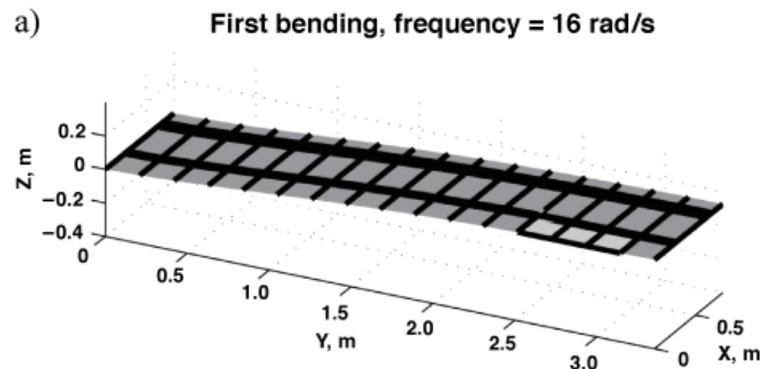
## Actuator Model with and without command lag



# Analytical Wing Mode Shapes

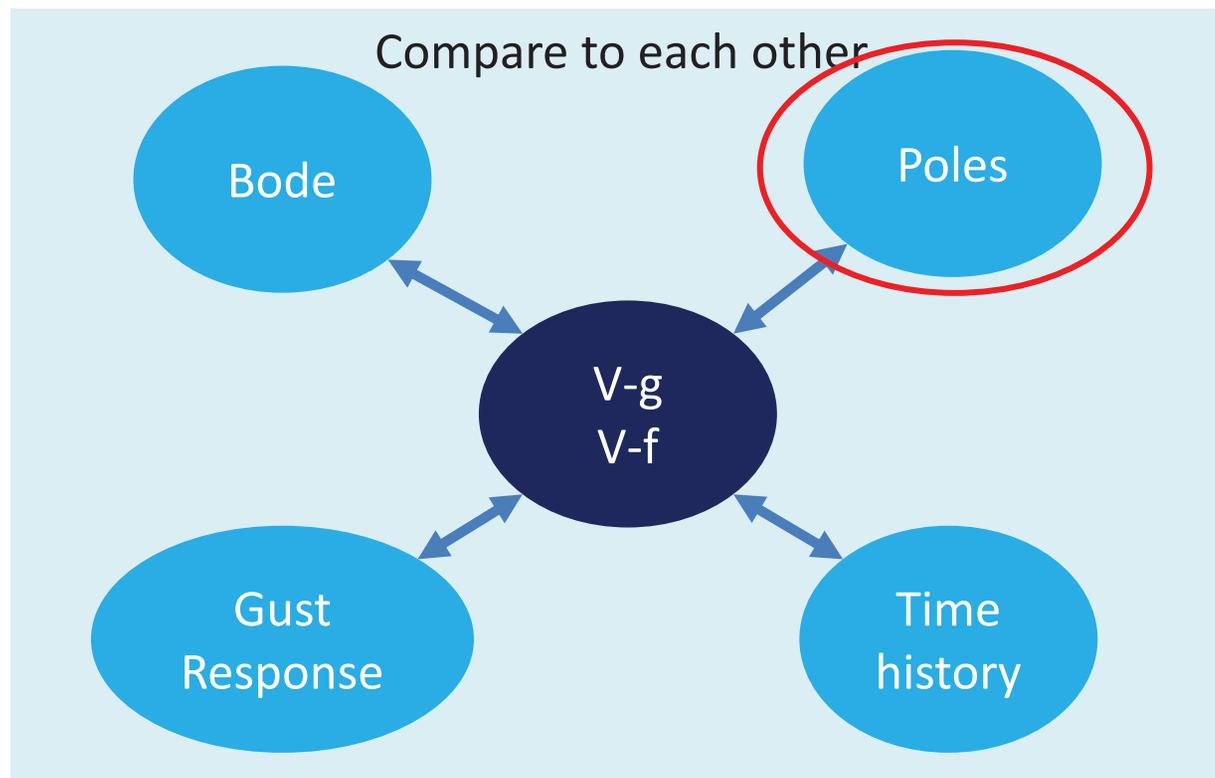
- Mass normalized mode shapes are computed with high torsional spring stiffness in connected control surfaces
- Control modes are computed with low torsional spring stiffness and a prescribed 1 deg. rotation boundary condition

## Analytical Wing Modal Analysis



# State Space Model Verification

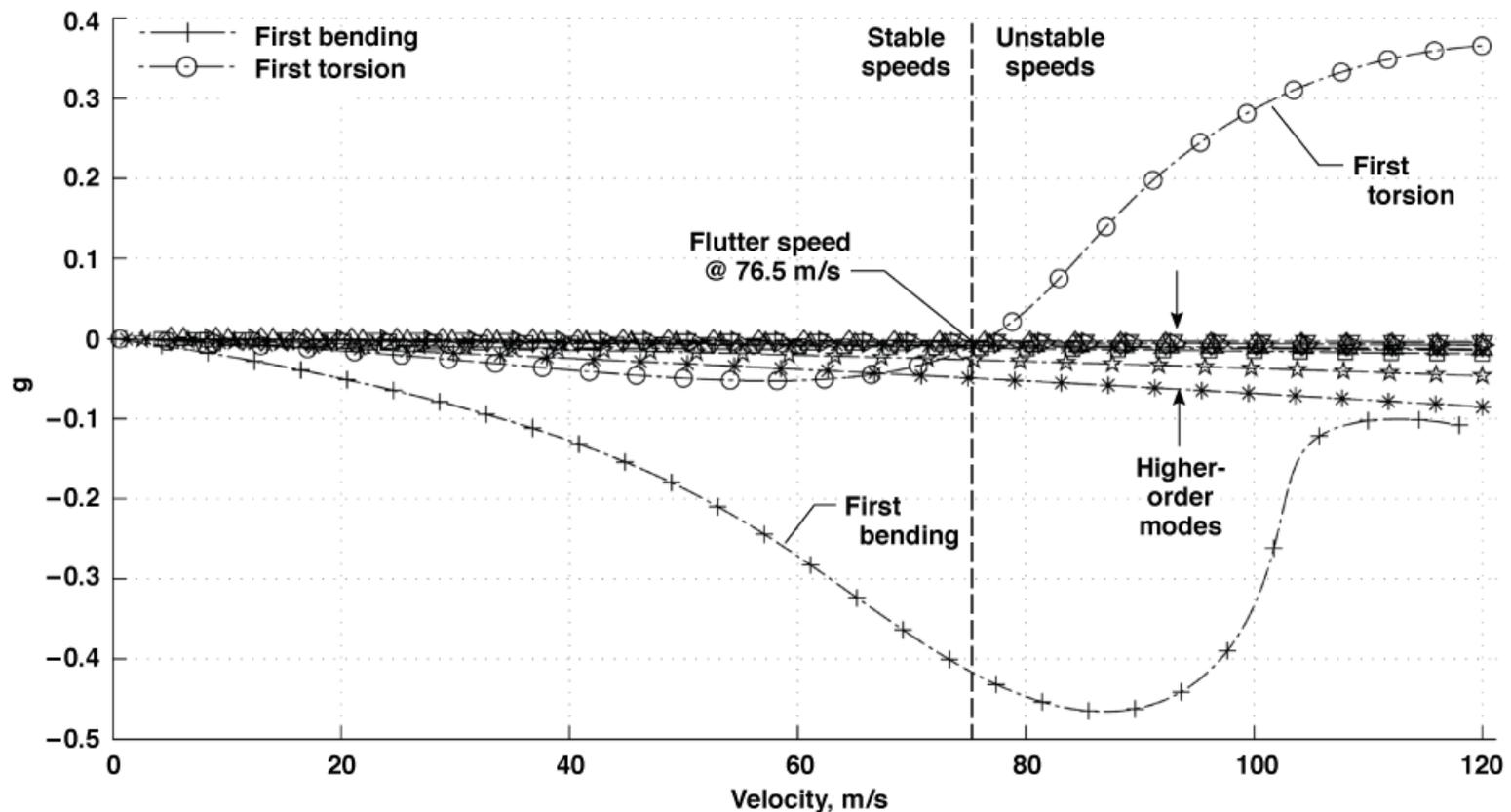
- We verify that the state space models correlate with what was predicted from the V-g and V-f analyses



# V-g Analysis with RFA

- V-g analysis of wing shows a traditional bending/torsion flutter mode appearing at 76.5 m/s

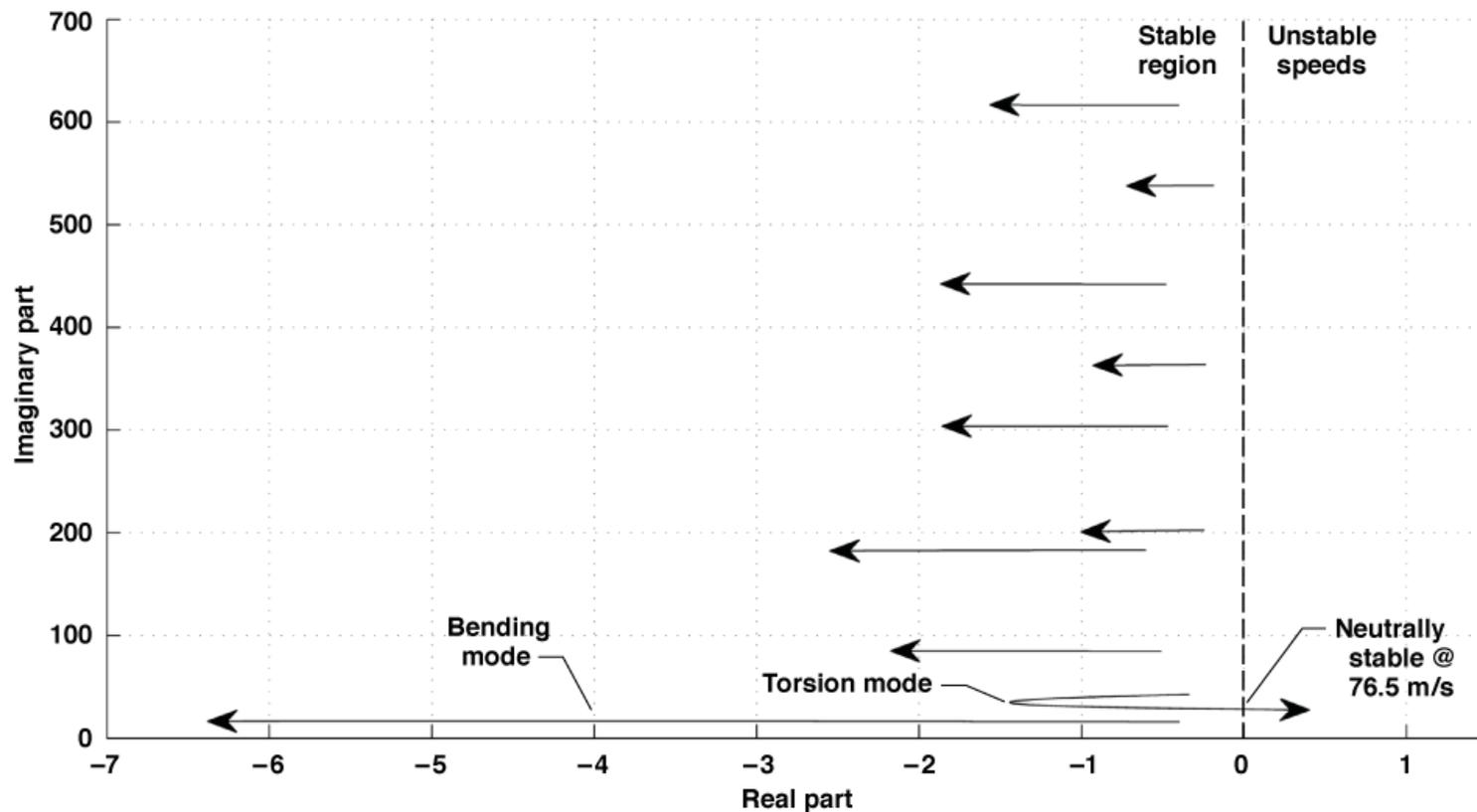
V-g Analysis of Analytical Wing Model



# Wing Model Pole Migration

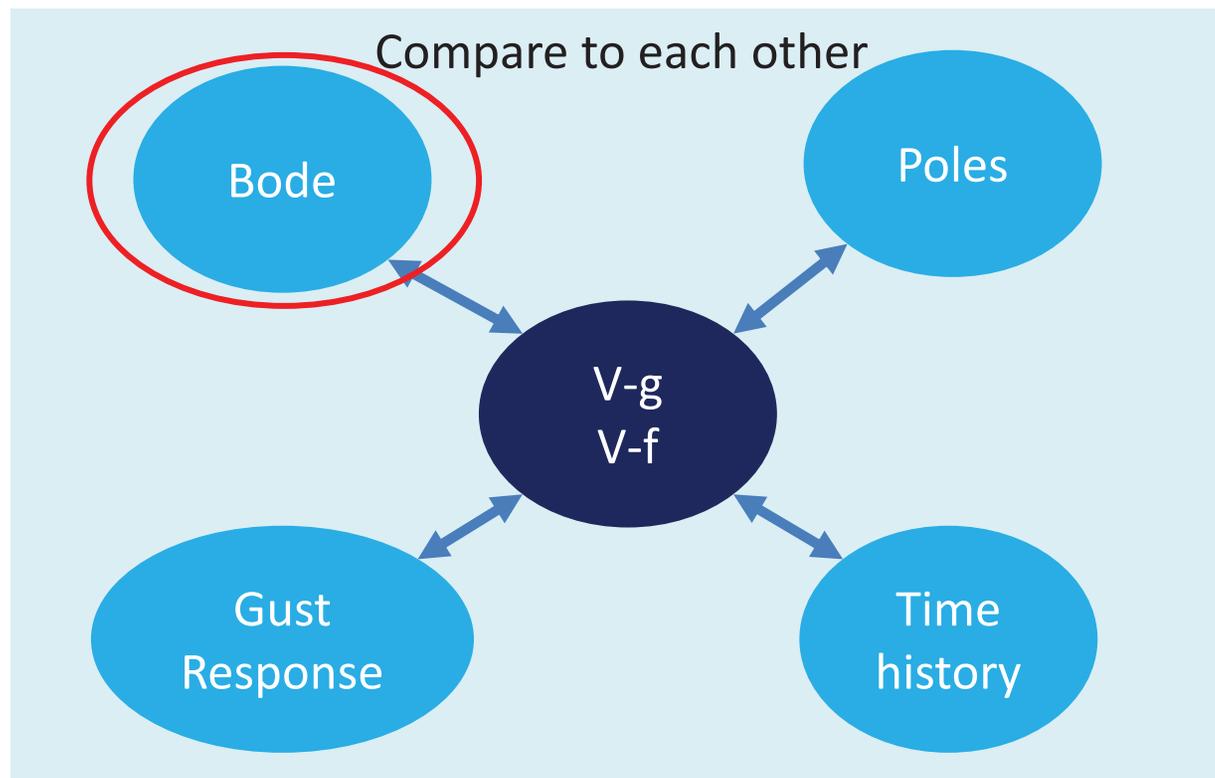
- The bending mode becomes more stable
- The torsion mode becomes neutrally stable at 76.5 m/s
- Flutter speed is the same as predicted in the V-g analysis

## Pole Migration of State Space Model from 20 – 78 m/s



# State Space Model Verification

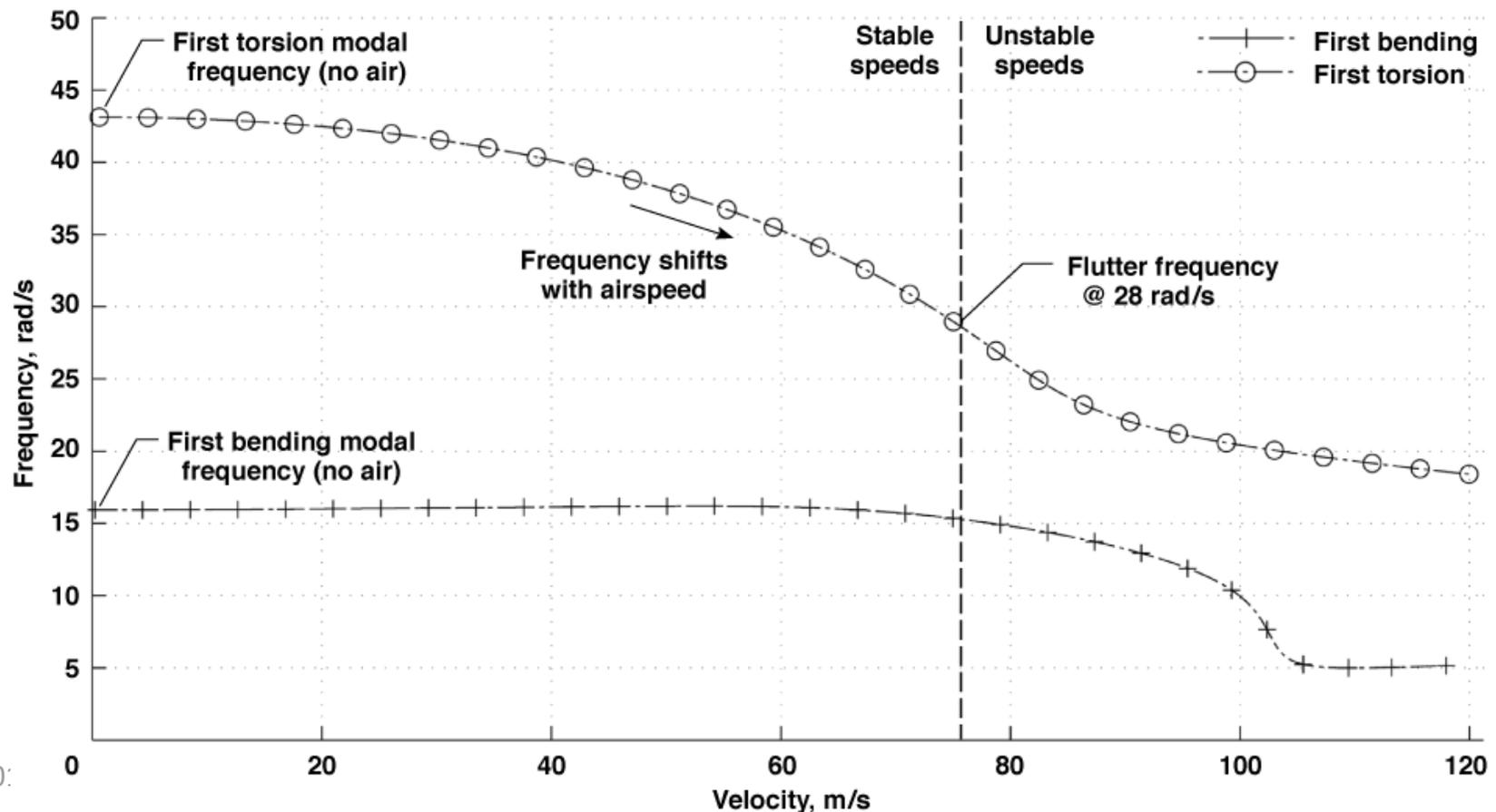
- We verify that the state space models correlate with what was predicted from the V-g and V-f analyses



# V-f Analysis

- Frequency analysis shows the flutter frequency at 28 rad/s

## V-f Analysis of Analytical Wing Model

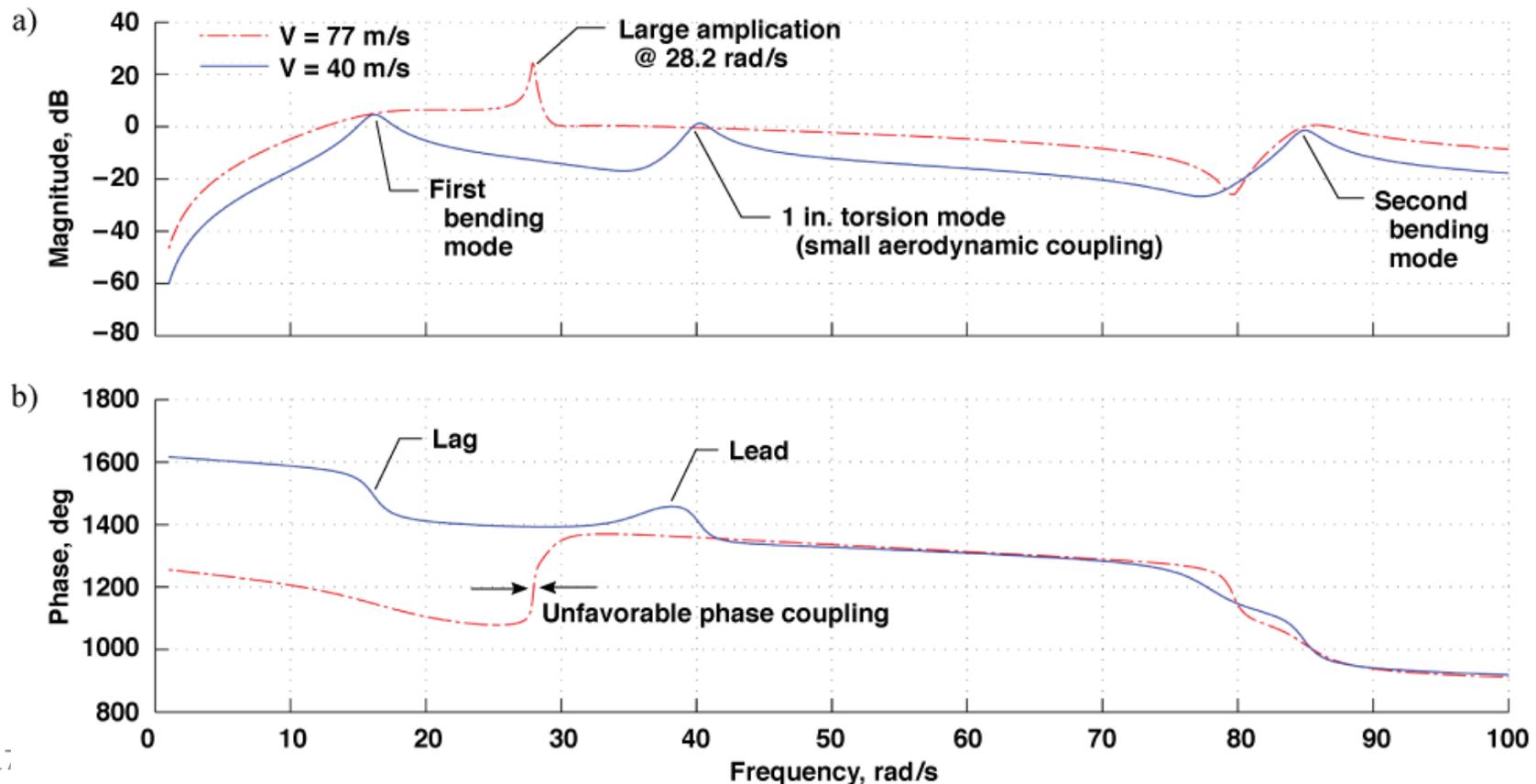




# Bode Plot of State Space Model

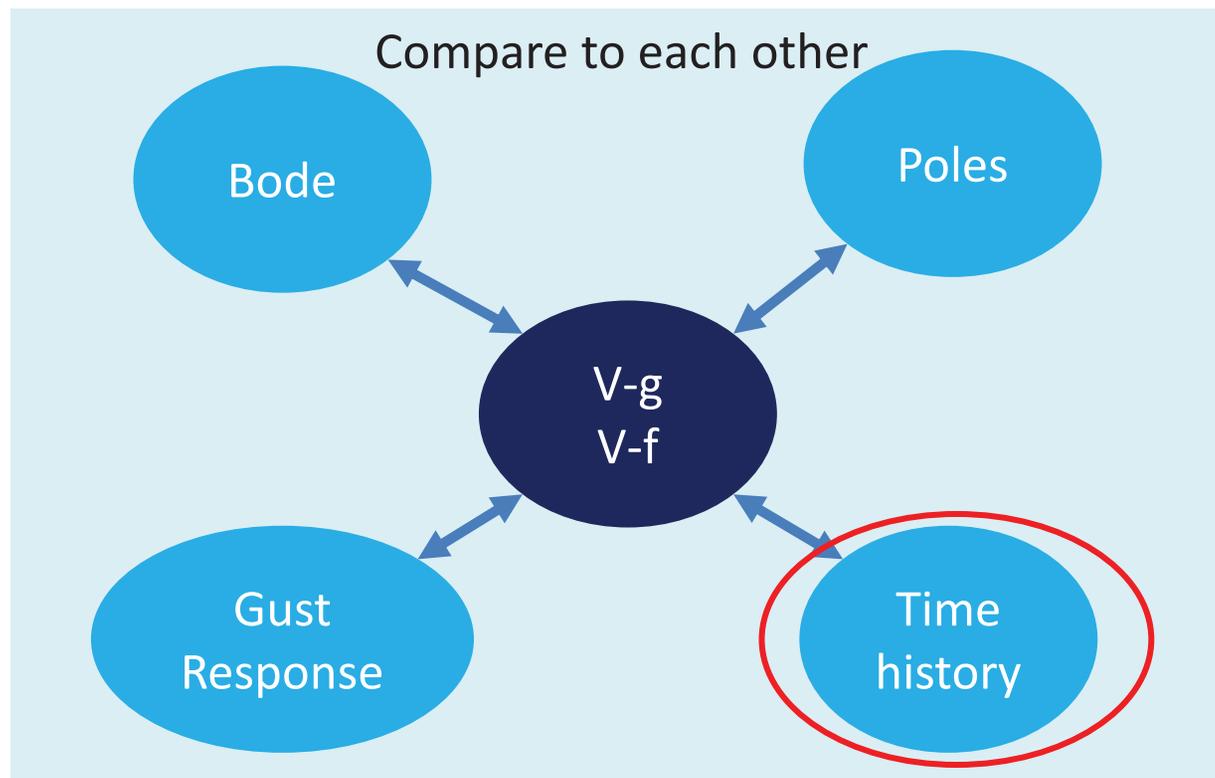
- At speed below flutter speed, amplitudes of two distinct modes visible
- At flutter speed only flutter mode is visible
- Frequency is the same as predicted from the V-f analysis

## Bode Plot of Surface to Leading Edge Accelerometer



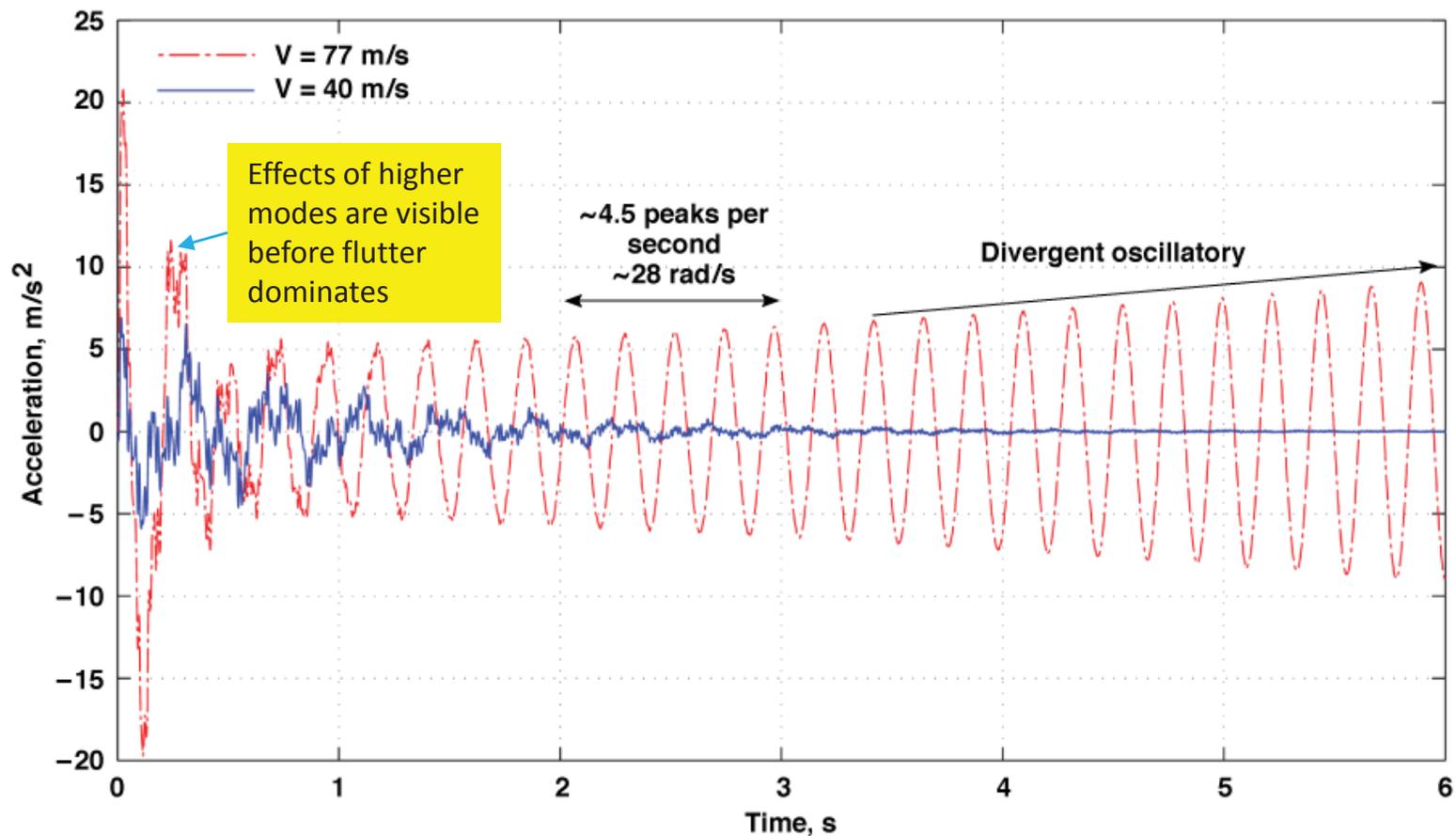
# State Space Model Verification

- We verify that the state space models correlate with what was predicted from the V-g and V-f analyses



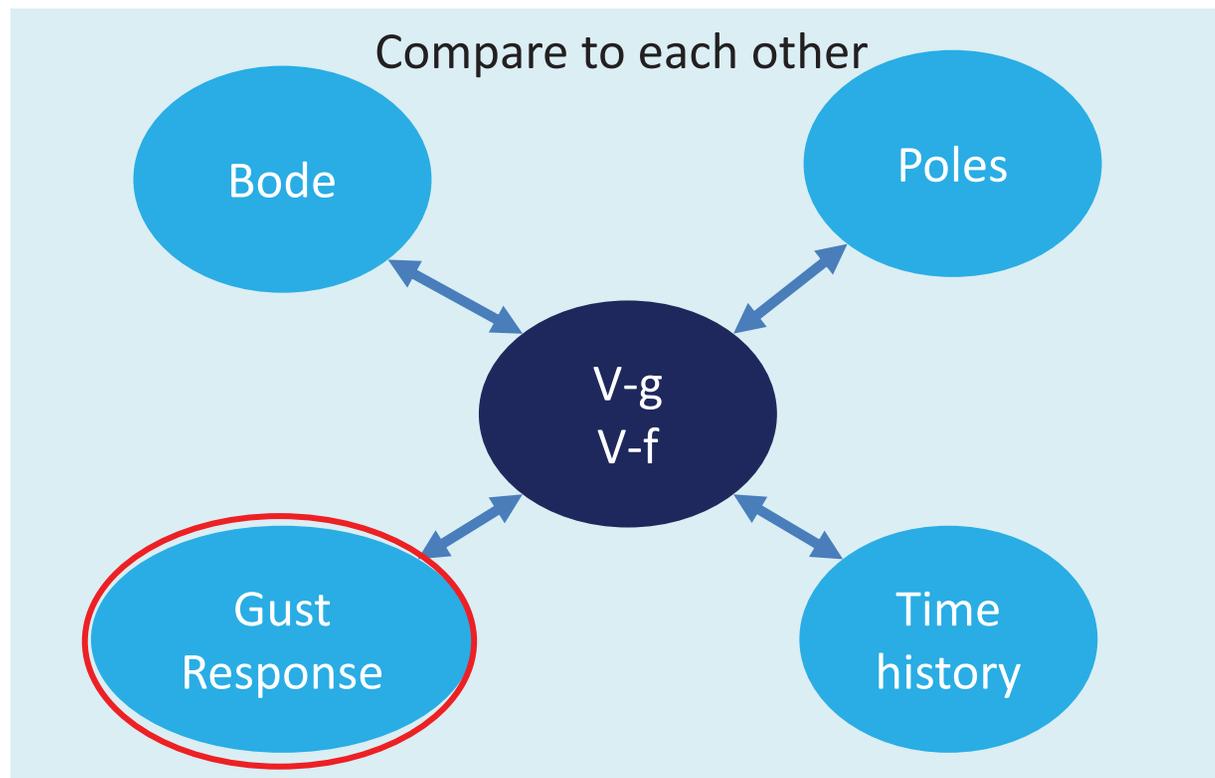
# Impulse to State Space Model

- Flutter is apparent in model designed past flutter speed
  - Divergent oscillatory
- Model at lower speed is damped after impulse



# State Space Model Verification

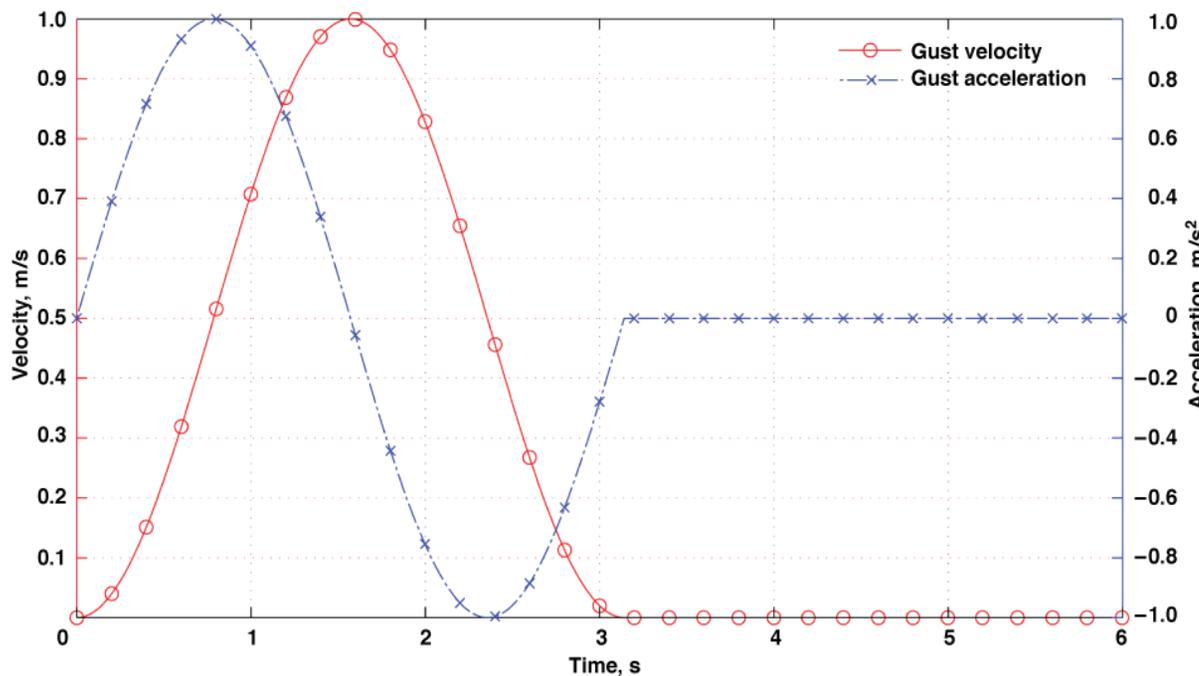
- We verify that the state space models correlate with what was predicted from the V-g and V-f analyses



# 1-cos Gust Model

- Gust inputs to structure are designed with gust modes and 1-cos gust input structure

## 1-cos Gust Input Model



## Gust mode approximation

$$g_{wash} = -\exp\left(\hat{i} \frac{2k}{\bar{c}} (x_{c.p.} - \bar{x}_{gust})\right)$$

## Gust velocity

$$w_g(t) = \frac{W_{g,max}}{2} (1 - \cos(Gt))$$

## Gust acceleration

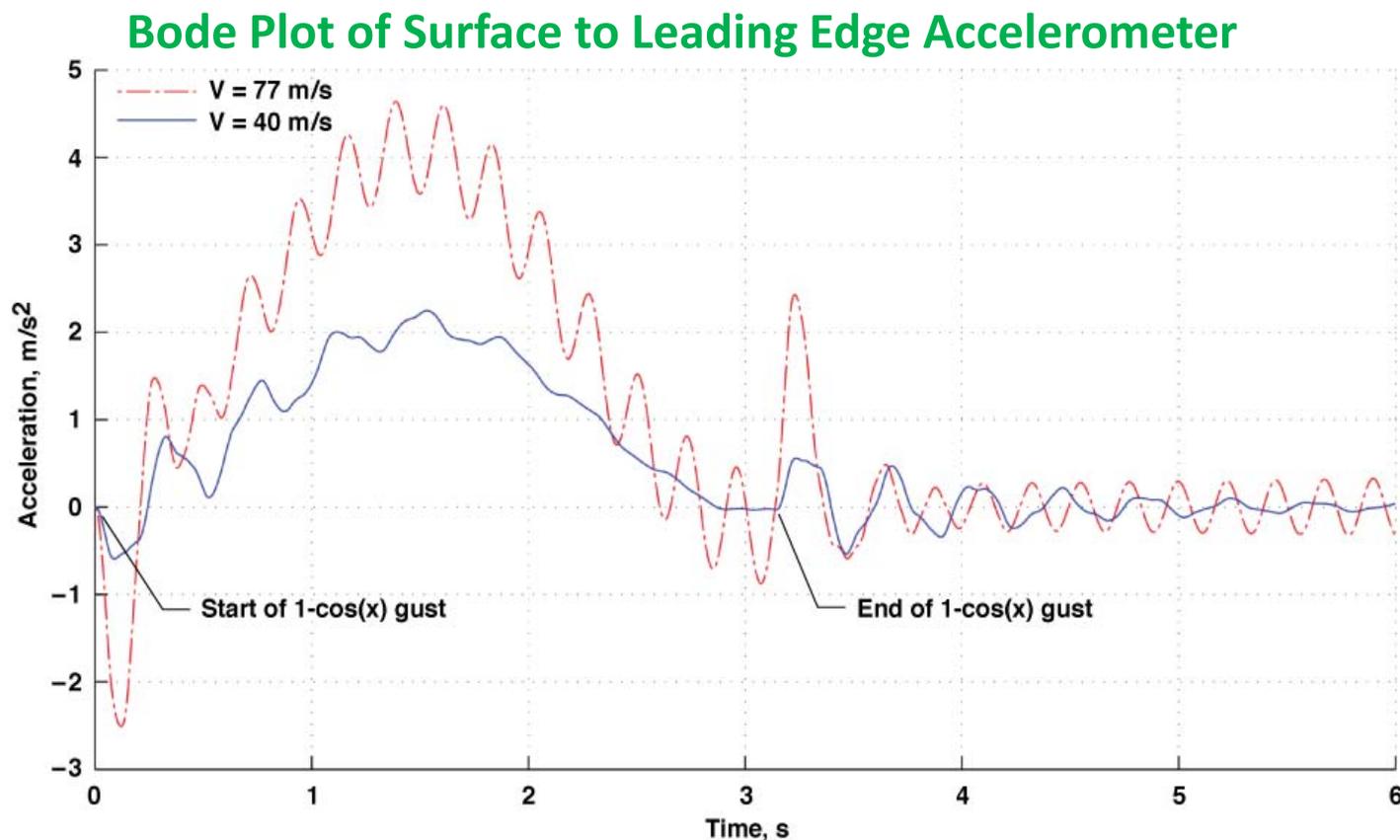
$$\dot{w}_g(t) = \frac{W_{g,max}}{2} \sin(Gt) G$$

## Gust frequency

$$G = 2 \frac{\dot{W}_{g,max}}{W_{g,max}}$$

# Gust Input to State Space Model

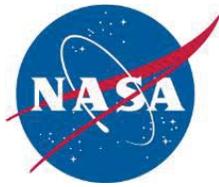
- The response of wing to 1-cos gust is expected
  - Low frequency gust response and high frequency oscillations from flutter are seen to be superimposed





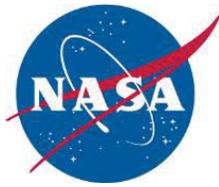
# Conclusions

- Several first step verification and validation studies were presented for a new aeroservoelastic tool
- More verification and validation is needed to assess the state space models including
  - An experimental flutter test and active flutter suppression
- This work further supports independent flutter analysis conducted by Dr. Conyers in his dissertation



# Future Work

- Improvements will be made to include rigid body modes in the tool
- Input structure will be made more user friendly
- Would like to look into transitioning to use as an open tool for students



# Questions?