

Prospects for observing ultracompact binaries with space-based gravitational wave interferometers and optical telescopes

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ABSTRACT

Space-based gravitational wave interferometers are sensitive to the galactic population of ultracompact binaries. An important subset of the ultracompact binary population are those stars that can be individually resolved by both gravitational wave interferometers and electromagnetic telescopes. The aim of this paper is to quantify the multimessenger potential of space-based interferometers with arm-lengths between 1 and 5 Gm. The Fisher information matrix is used to estimate the number of binaries from a model of the Milky Way which are localized on the sky by the gravitational wave detector to within 1 and 10 deg² and bright enough to be detected by a magnitude-limited survey. We find, depending on the choice of GW detector characteristics, limiting magnitude and observing strategy, that up to several hundred gravitational wave sources could be detected in electromagnetic follow-up observations.

Key words: gravitational waves – binaries: close – white dwarfs – Galaxy: stellar content.

1 INTRODUCTION

A variety of detector concepts for space-based gravitational wave interferometers have been proposed, the most well-studied concept being Laser Interferometer Space Antenna (LISA) (Bender et al. 1998). It was understood early on that the most numerous source class radiating in the band covered by LISA-like detectors will be the galactic population of ultracompact binaries (UCBs) comprised of pairs of stellar remnants: white dwarfs, neutron stars or black holes. The gravitational radiation from these UCBs will be the dominant signal in the frequency band covered by LISA-like detectors.

Early estimates of the composite signal from the UCBs (Evans et al. 1987; Hils, Bender & Webbink 1990; Hills & Bender 1997) demonstrated that the signals of the vast majority of the galactic binaries will overlap and be unresolvable from one another, forming a limiting foreground (or ‘confusion noise’) for space-based gravitational wave detectors. Later studies based on population synthesis (Nelemans et al. 2001; Benacquista, DeGoes & Lunder 2004; Edlund et al. 2005; Timpano, Rubbo & Cornish 2006; Ruitter et al. 2010) have borne this expectation out. Detailed data analysis studies have shown that $\sim 10^4$ individual binaries could be resolved out

of the foreground by a gravitational wave observatory like LISA (Timpano et al. 2006; Crowder & Cornish 2007; Littenberg 2011; Nissanke et al. 2012).

A subset of the resolvable binaries will be detectable electromagnetically. The purpose of this work is to assess the multimessenger potential for different space-based detectors spanning the trade-space of future mission designs. This builds off previous work (Cooray, Farmer & Seto 2004; Nelemans 2006, 2009) demonstrating the feasibility of follow-up observations for high-frequency UCB sources. We estimate the total number of multimessenger sources by beginning with a population synthesis model of the galaxy (Nelemans et al. 2004), complete with optical magnitudes. From this we produce a magnitude-limited source catalogue, then estimate how well each system will be localized on the sky by different gravitational wave detector configurations. Using hundreds of Monte Carlo realizations over the spatial distribution of the galaxy and the UCB orientations, we find from a few to hundreds of sources that can be observed both electromagnetically and gravitationally.

The information encoded about the UCBs in each of the two spectra is highly complementary, enabling tests of general relativity, full measurement of the physical parameters enabling constraints on binary synthesis channels, and new methods of probing the close interaction dynamics of the compact stars (Cutler, Hiscock & Larson 2003; Stroeer, Vecchio & Nelemans 2005).

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2 DETECTORS

For a gravitational wave observatory, the limiting sensitivity as a function of frequency is dominated at low frequencies by acceleration noise S_a , while the ‘floor’, where the detector is most sensitive, is dominated by position measurement noise S_x . Table 1 contains the parameters used for the detector configurations in this study. These parameters can be used to compute the noise power spectral density

$$S_n(f) = S_{\text{gal}}(f) + (4/3) \sin^2 u [(2 + \cos u)S_x + 2(3 + 2 \cos u + \cos 2u)S_a / (2\pi f)^4], \quad (1)$$

where $u = 2\pi f \ell / c$ and S_{gal} is the contribution to the instrument noise from the unresolved UCB foreground (Timpano et al. 2006).

The configurations we will highlight correspond to the classic LISA design ($\ell = 5$ Gm), as well as two shorter arm-length configurations ($\ell = 2$ and 1 Gm) in order to cover a variety of plausible mission configurations. The 1 Gm configuration is similar to the evolved Laser Interferometer Space Antenna (eLISA) mission being considered by the European Space Agency (Amaro-Seoane et al. 2012). We use an observation time of 2 yr for each configuration.

This suite of detectors provides a broad palette to illustrate the observational capabilities of these instruments with regards to the UCBs. A classic depiction of the performance for these interferometers is a plot of the average sensitivity curve in strain spectral density versus frequency (Larson, Hiscock & Hellings 2000), as shown in Fig. 1. The eLISA concept is the only one which uses a 4-link configuration. The Doppler ranging between each spacecraft

Table 1. Gravitational wave detector configurations used in this study. Configuration 1 corresponds to eLISA. Configuration 5 is the classic LISA design. All simulations were for 2 yr mission lifetimes.

Config.	ℓ (m)	$\sqrt{S_a}$ ($\text{m s}^{-2} \text{Hz}^{-1/2}$)	$\sqrt{S_x}$ ($\text{m Hz}^{-1/2}$)	Links
1	1×10^9	4.5×10^{-15}	11×10^{-12}	4
2	2×10^9	3.0×10^{-15}	10×10^{-12}	6
3	5×10^9	3.0×10^{-15}	18×10^{-12}	6

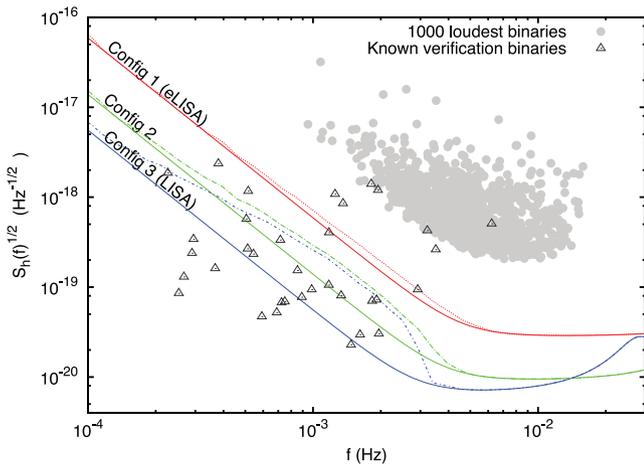


Figure 1. Sensitivity curve for each of the detector configurations in Table 1. The solid lines show the sensitivity set by the measurement noise while the dashed curves include an estimate of the UCB confusion-limited foreground. Overplotted are the brightest UCBs in our simulated catalogue (grey circles), and the known verification binaries (black triangles).

in the constellation is accomplished using two laser links. Thus, the 4-link design is a single-vertex interferometer, while the 6-link designs allow for three (coupled) interferometers. This difference accounts for an additional improvement in the 6-link sensitivity curves by a factor of $\sim \sqrt{2}$ at frequencies where the UCBs are found.

3 DISCOVERING NEW VERIFICATION BINARIES

The focus of this work is to study the population of detectable UCBs in the context of multimessenger astronomy. We will focus on the sources detected via GWs which could potentially be identified electromagnetically. There is a separate class of UCBs, the ‘verification binaries’, which are known low-frequency GW sources with AM CVn serving as the archetype. There are ~ 30 known verification binaries, ~ 5 – 10 of which could be identified by the GW detectors considered here, with sources still being discovered (Roelofs et al. 2007; Brown et al. 2011; Nelemans 2011). This study does not include the known verification binaries in the galaxy catalogues. Furthermore, many of the AM CVn systems would *not be localized well enough by the GW measurement alone* to warrant simple electromagnetic follow-up observations.

3.1 Binary selection

The UCB population model is essentially identical to that found in Nelemans et al. (2004), so the positions and ages of the systems are based on the Boissier & Prantzos (1999) Galactic model. We use the white dwarf cooling tracks based on Hansen (1999) as shown in the appendix of Nelemans et al. (2004). We convert the luminosities to V -band magnitudes using zero-temperature white dwarf radii and simple bolometric corrections based on the effective temperature. This should suffice for this initial estimate of the potentially detectable population, but can be improved using detailed WD cooling models in the future. We determine the absorption as in Nelemans et al. (2004) based on the Sandage (1972) model, but correcting for the fact that the dust is more concentrated than the stars, so we use 120 pc as scaleheight for the absorption.

To construct the magnitude-limited catalogue, we begin with the entire binary population in the synthesized galaxy. The limiting apparent magnitude of a telescope is a function of the aperture D , the exposure time t and the properties of the detector used for imaging and photometry (Howell 1989; Schaeffer 1990). A rudimentary fit to the limiting magnitude m using a telescope of aperture D (in m) and for exposure time t (in s) is given by $m = 19.6 D^{0.073} t^{0.025}$. Using commercial CCD detectors, a $D = 0.5$ m telescope will reach a photometric magnitude $m \simeq 21$ in $t \sim 75$ s, where as a $D = 1.0$ m telescope will reach the same magnitude in $t \sim 20$ s. This paper examines the role of small to large aperture telescopes by examining a broad range of limiting magnitudes; lower bounds of $m = 18$ – 24 were chosen as the electromagnetic cutoff. All-sky survey instruments such as Large Synoptic Survey Telescope (LSST) could further improve the number of candidates. The single exposure limit for LSST is expected to be $m \simeq 24$, whereas the magnitude limit of the final stacked image is expected to be around $m \simeq 27$ (Ivezik et al. 2011).

3.2 Gravitational wave detector response

From the magnitude-limited catalogue, we determine the number of ‘bright’ UCBs that will be well measured by the GW detector. To do so, we must first estimate the confusion noise for each

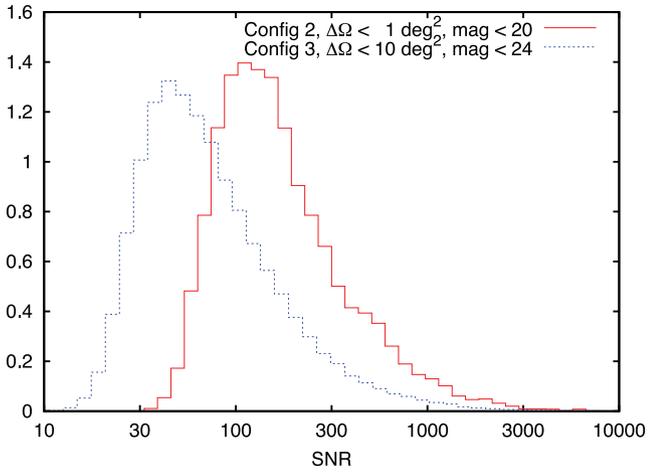


Figure 2. Distribution of SNRs for binaries which satisfy both GW and EM criteria for Config 2, $\Delta\Omega < 1 \text{ deg}^2$, and $m < 20$ (red, solid) and the most lenient set; Config 3 (LISA), $\Delta\Omega < 10 \text{ deg}^2$, and $m < 24$ (blue, dotted). Cornish & Crowder (2005) found good agreement between the Fisher approximation and the posteriors for $\text{SNR} > 10$.

configuration. The instrument response to the galactic foreground is constructed by generating and co-adding waveforms for each source in the full simulated galaxy catalogue using the fast-slow decomposition in Cornish & Littenberg (2007). The confusion noise, S_{gal} , is empirically determined from the simulated data by iteratively removing sources brighter than a running estimate of the background. This procedure is first discussed in Timpano et al. (2006) with an improved implementation used here as in Nissanke et al. (2012).

With the confusion noise incorporated into the detector sensitivity curves, we determine how well the GW detector can measure the source parameters of a UCB waveform, using the well-worn Fisher information matrix Γ_{ij} (Cutler & Flanagan 1994), the inverse of which approximates the covariance matrix. There is no shortage of literature highlighting shortcomings of the Fisher to approximate GW parameter errors e.g., Vallisneri (2008). However, given the scope of the problem we are addressing (hundreds of Monte Carlo’s of thousands of detectable binaries), more rigorous parameter estimation studies would be impractical [recently, Vallisneri (2011) has

proposed a way around this dilemma]. On the other hand, the UCBs in which we are most interested – those that can be well localized on the sky – are shown in Fig. 2 to have sufficiently high signal-to-noise ratios (SNRs) where the Fisher provides a good estimate of the true errors (Cornish & Crowder 2005).

For a binary to be considered ‘well localized’, we require that the 95 per cent confidence interval of the sky-location posterior distribution function subtends an area on the celestial sphere below some threshold $d\Omega$. To bracket the capabilities of ground-based optical telescopes, we perform the analysis with $d\Omega \leq 1$ and $\leq 10 \text{ deg}^2$. We estimate the area of the sky-location error ellipse using the full covariance matrix found by inverting Γ_{ij} (Lang & Hughes 2008).

The number of well-localized, bright binaries is computed for 625 realizations where we Monte Carlo over the orientation of each binary, as well as their location within the Galaxy. For the orientation, we draw the inclination i from a uniform distribution $\cos i = U[-1, 1]$, and the polarization angle ψ and initial phase φ from $U[0, 2\pi]$. We find that up to several hundred GW sources will be viable candidates for electromagnetic follow-up searches, depending on the depth of EM survey and the GW detector characteristics (See the left-hand panel of Figs 3 and 4).

3.3 EM detection strategies

We now consider how to select candidates for follow-up observations from the full GW catalogue. Pointing telescopes at all of the GW sources localized within the adopted threshold would be inefficient, as we find that between 10^3 and 10^4 GW sources in the full catalogue will meet the $d\Omega \leq 10 \text{ deg}^2$ threshold, while $\lesssim 10$ per cent are likely to be brighter than $m = 24$, and only $\lesssim 1$ per cent pass the $m \leq 20$ cut.

Additional considerations need to be made to increase the efficiency of follow-up observing campaigns. We illustrate two simple ways to isolate the GW sources that may be electromagnetically observable. These suggestions are supported by calculations shown in Table 2.

First, the large majority of UCB sources are confined within the galactic plane. Conversely, the magnitude-limited catalogues sample the local galaxy, which is much more uniformly distributed on the celestial sphere. Therefore, as a rough cut on the GW catalogue,

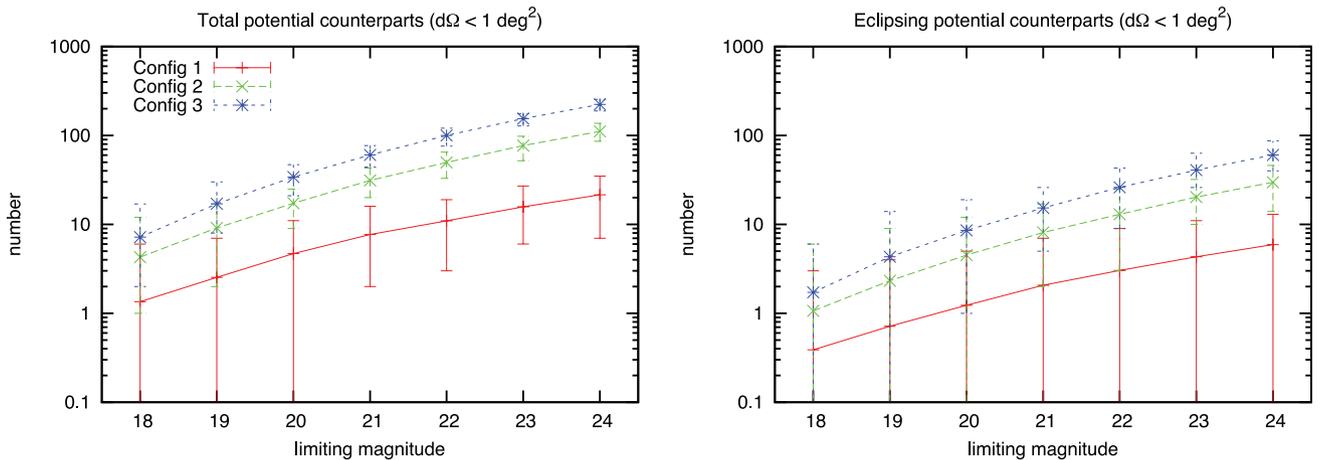


Figure 3. Number of binaries with sky location resolved to within 1 deg^2 for each configuration as a function of limiting magnitudes. The left-hand panel shows the total number of candidates, the right-hand panel the subset of eclipsing binaries. The error bars represent the full range after Monte Carlo’ing over the location and orientation of each UCB system. Even the modest detection abilities (magnitudes $m \sim 19$) of small aperture telescopes could yield several electromagnetic counterparts; larger telescopes with deeper magnitude grasp will have significantly more sources that can be surveyed.

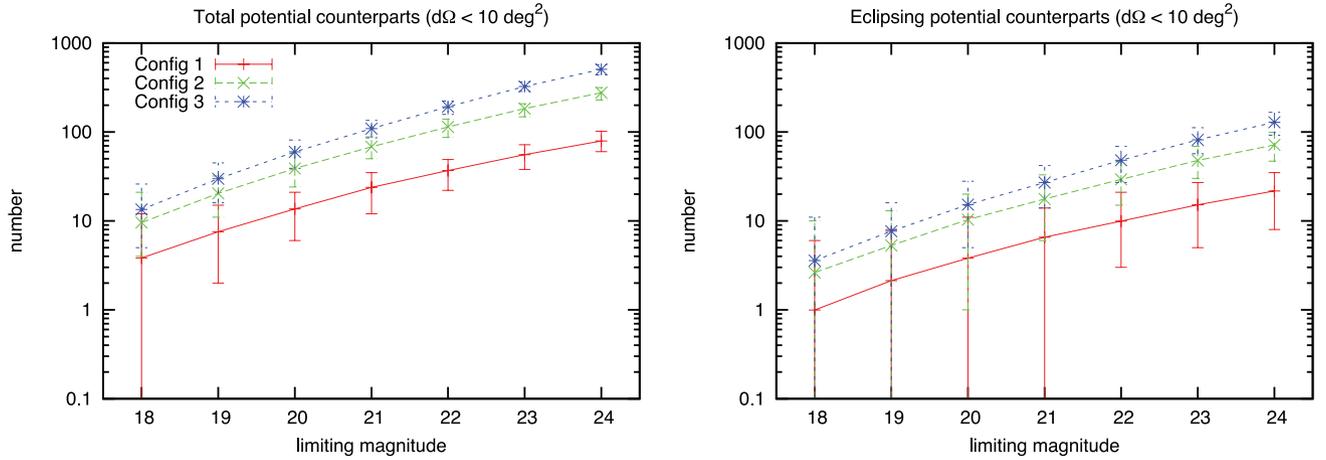


Figure 4. Same as Fig. 3, except that here we use an angular resolution threshold for the GW detector of $d\Omega \leq 10 \text{ deg}^2$.

Table 2. Multimessenger candidates will be a minority of the spatially well-resolved GW signals. Here we enumerate the fraction of binaries that will make good candidates for electromagnetic follow-up observations. Plain numbers in the table correspond to the $d\Omega \leq 10 \text{ deg}^2$, while those in parentheses correspond to the 1 deg^2 threshold. We tabulate the average number of binaries \bar{N} that meet the sky-resolution requirements (column 2), and then the fraction which are significantly out of the Galactic plane ($|b| > 20^\circ$ – column 3), or have d_L (inferred directly from f , \dot{f} and \mathcal{A}) measured to within 20 per cent (column 4), the idea being that near-by binaries are more likely to be optically detectable – proximity can be inferred by $|b|$ and/or determined through d_L . The columns then repeat for the $m \leq 20$ and $m \leq 24$ magnitude-limited catalogues.

Config	Full galaxy			$m \leq 20$			$m \leq 24$		
	\bar{N}	$ b > 20^\circ$	$\frac{\Delta d_L}{d_L} < 20 \text{ per cent}$	\bar{N}	$ b > 20^\circ$	$\frac{\Delta d_L}{d_L} < 20 \text{ per cent}$	\bar{N}	$ b > 20^\circ$	$\frac{\Delta d_L}{d_L} < 20 \text{ per cent}$
1	1100 (215)	0.02 (0.03)	0.74 (0.96)	13 (4)	0.49 (0.50)	0.28 (0.70)	80 (22)	0.24 (0.26)	0.47 (0.88)
2	5500 (1600)	0.01 (0.01)	0.45 (0.84)	39 (17)	0.49 (0.47)	0.13 (0.28)	277 (111)	0.22 (0.22)	0.25 (0.52)
3	9400 (2900)	0.01 (0.01)	0.35 (0.74)	60 (34)	0.49 (0.47)	0.09 (0.16)	507 (224)	0.21 (0.21)	0.15 (0.31)

any binaries that are well localized but out of the galactic plane are good candidates. These are additionally attractive sources, as there will be less optical background and extinction against which the observing campaign will have to compete. We find that between ~ 20 and ~ 50 per cent of the well-localized binaries in the 20th to 24th magnitude-limited catalogues have galactic latitudes $|b| \geq 20^\circ$, while that fraction is reduced to ~ 1 per cent for the full GW catalogue. A uniform distribution of stars on the celestial sphere would have 66 per cent of the stars with $|b| \geq 20^\circ$.

The other strategy for identifying optical counterparts relies on estimates of the distance to the galactic binary. Typical UCB sources will undergo very little evolution of their orbital period during a space-borne GW detector’s lifetime. Without measurement of the rate of change of the gravitational wave frequency \dot{f} the GW observation only constrains the overall amplitude of the signal without decoupling the chirp-mass \mathcal{M} and the luminosity distance d_L (Schutz 1986; Stroeer & Vecchio 2006). However, because we are preferentially selecting from the highest SNR systems, between ~ 20 and 90 per cent of the multimessenger sources, we sufficiently constrain \dot{f} and \mathcal{A} to measure d_L to within 20 per cent, depending on the detector configuration. However, astrophysical effects such as tides may impact the orbital evolution and thus bias the distance estimate.

For the remaining systems in the GW catalogue, we can use reasonable priors on the mass and mass ratio of white dwarf binaries to put meaningful constraints on d_L from the amplitude measurement alone. Using only the posteriors estimated by the Fisher approximation for the amplitude and frequency, and priors on the masses constructed from the population synthesis simulation, we draw values for f , \mathcal{A} and \mathcal{M} and find the most likely (ML) luminosity dis-

tance. The resulting distribution of ML luminosity distances $d_{L, \text{ML}}$ is strongly peaked between 0 and 8 kpc – the distance to the galactic centre – for the magnitude-limited catalogues. The $d_{L, \text{ML}}$ distributions for the full well-localized catalogue with no magnitude cut is more uniformly distributed over a larger range.

Our final consideration pertains to the expected optical light curves for UCB systems identified in the GW catalogue. The population synthesis galaxy in our study is restricted to detached white dwarf binaries, as opposed to interacting AM CVn systems. Without mass transferring from one star to the other in the binary, photometric variability is not guaranteed. The systems in the GW catalogue that are best constrained are typically those at the high-frequency end of the population. This is to our advantage, because the shorter period binaries have a higher probability of eclipsing one another during an orbital cycle.

We can put an additional cut on our EM/GW catalogue by requiring the binaries to be eclipsing. (See the right-hand panel of Figs 3 and 4.) From simple geometrical arguments (Cooray et al. 2004), the minimum inclination angle with respect to our line of sight that will produce eclipsing light curves is

$$\cos(i_{\text{min}}) \sim 0.3(f/3.5 \text{ mHz})^{2/3}, \quad (2)$$

assuming all binary constituents have mass $M_{\text{total}} \sim 0.5 M_\odot$ and radius $R_{\text{WD}} \sim 10^4 \text{ km}$. If we only consider binaries in the multimessenger catalogue with inclination angle less than i_{min} , we reduce the total number of candidates by a factor of ~ 3 . Nevertheless, we still find upwards of ~ 100 candidates for the large GW detector configurations and deep, wide-field, optical surveys. Requiring eclipsing light curves significantly degrades the multimessenger potential for

the 1 Gm configuration and catalogues limited to 20th magnitude and dimmer – such EM follow-up surveys could come up empty.

4 DISCUSSION

We conclude that space-based gravitational wave detectors will be useful observatories for discovering new UCBs in the galaxy that could be observed electromagnetically, though deep, wide-field, optical surveys may be required to produce large catalogues. We reach this verdict by considering a range of plausible near-future space-based gravitational wave detector concepts, and assess their measurement capabilities for magnitude-limited catalogues of UCBs. Magnitudes for the constituents of each binary were derived from the population synthesis simulations, and the gravitational wave measurement capabilities were estimated using the Fisher information matrix. Any UCBs that were brighter than our chosen magnitude limits (18–24) and located on the sky by the gravitational wave detector to within angular resolution $d\Omega$ were considered multimessenger candidates. We estimated the multimessenger catalogue sizes for both $d\Omega \leq 1$ and 10 deg^2 .

At the pessimistic end, we consider magnitude 18 limited catalogues, and single-vertex interferometers with 1 Gm arm-lengths. The best scenario considered the classic LISA design and an optical telescope limited at 24th magnitude. The number of multimessenger candidates was anywhere from zero to several hundreds over that range of detector capabilities. If we put on the additional constraint that the sources must be eclipsing to allow for electromagnetic observation, the counts were reduced by a factor of ~ 3 .

While most of the known verification binaries are AM CVn-type stars, our study only considered detached white dwarf binaries, thus providing a very complimentary catalogue of UCB multimessenger systems.

This work considered a conservative approach to finding multimessenger UCBs, with competing criteria that strongly affect the expected population of systems detectable in both spectra. Electromagnetic detections are most strongly affected by the magnitude limit of the detection survey, a function of telescope aperture. By contrast, the gravitational wave detection catalogues of UCBs are expected to have thousands of systems in them; most will be too faint to be detectable by any electromagnetic survey. However, the gravitational wave localization criterion is a strong constraint on the multimessenger catalogue. We find that wide-field surveys ($d\Omega \leq 10 \text{ deg}^2$) yield more candidates than more narrow fields of view ($d\Omega \leq 1 \text{ deg}^2$) by 50–100 per cent for the full catalogues, and by a factor of 2–4 for the eclipsing binaries.

We have estimated the number of UCB multimessenger candidates without considering what could be done with joint GW and EM observations. Our follow-on effort will consider the science yield from joint observations of both the known verification binaries – mostly mass-transferring systems – and the close, detached binaries that will be discovered by space-borne gravitational wave detectors.

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