Multiple designs, systems, methods and processes for controlling a system or plant using an extended active disturbance rejection control (ADRC) based controller are presented. The extended ADRC controller accepts sensor information from the plant. The sensor information is used in conjunction with an extended state observer in combination with a predictor that estimates and predicts the current state of the plant and a co-joined estimate of the system disturbances and system dynamics. The extended state observer estimates and predictions are used in conjunction with a control law that generates an input to the system based in part on the extended state observer estimates and predictions as well as a desired trajectory for the plant to follow.

9 Claims, 5 Drawing Sheets
## References Cited

### U.S. PATENT DOCUMENTS

<table>
<thead>
<tr>
<th>Patent Number</th>
<th>Date</th>
<th>Inventor(s)</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,826,887 A</td>
<td>7/1974</td>
<td>Pemberton</td>
<td>700/37</td>
</tr>
<tr>
<td>4,481,567 A</td>
<td>11/1984</td>
<td>Kaya et al.</td>
<td>700/37</td>
</tr>
<tr>
<td>4,768,143 A</td>
<td>11/1984</td>
<td>Lane et al.</td>
<td>700/37</td>
</tr>
<tr>
<td>5,229,699 A</td>
<td>7/1993</td>
<td>Chu et al.</td>
<td>318/561</td>
</tr>
<tr>
<td>5,397,973 A</td>
<td>7/1993</td>
<td>Chu et al.</td>
<td></td>
</tr>
<tr>
<td>5,680,304 A</td>
<td>10/1997</td>
<td>Wang et al.</td>
<td></td>
</tr>
<tr>
<td>5,742,503 A</td>
<td>4/1998</td>
<td>Yu</td>
<td></td>
</tr>
<tr>
<td>6,122,555 A</td>
<td>9/2000</td>
<td>Lu</td>
<td>700/9</td>
</tr>
<tr>
<td>6,128,541 A</td>
<td>10/2000</td>
<td>Junk</td>
<td></td>
</tr>
<tr>
<td>6,510,353 B1</td>
<td>1/2003</td>
<td>Gudz et al.</td>
<td>700/37</td>
</tr>
<tr>
<td>6,546,295 B1</td>
<td>4/2003</td>
<td>Pyotra et al.</td>
<td></td>
</tr>
<tr>
<td>6,564,194 B1</td>
<td>5/2003</td>
<td>Koda et al.</td>
<td>706/13</td>
</tr>
<tr>
<td>6,631,299 B1</td>
<td>10/2003</td>
<td>Patel</td>
<td></td>
</tr>
<tr>
<td>6,658,305 B1</td>
<td>12/2003</td>
<td>Gudmundsson et al.</td>
<td></td>
</tr>
<tr>
<td>6,980,869 B1</td>
<td>12/2005</td>
<td>Chandhoke</td>
<td></td>
</tr>
<tr>
<td>7,765,254 B1</td>
<td>1/2011</td>
<td>GAHINET et al.</td>
<td></td>
</tr>
<tr>
<td>8,180,462 B2</td>
<td>5/2012</td>
<td>Gao et al.</td>
<td>700/44</td>
</tr>
<tr>
<td>2003/0139825 A1</td>
<td>7/2003</td>
<td>Lund</td>
<td></td>
</tr>
</tbody>
</table>

### OTHER PUBLICATIONS


* cited by examiner
EXTENDED ACTIVE DISTURBANCE REJECTION CONTROLLER

CROSS-REFERENCE TO RELATED APPLICATIONS


This work was supported at least in part by NASA under NASA contract number GT3-52387. Accordingly, the United States government may have certain rights herein.

TECHNICAL FIELD

The subject matter described herein relates to controllers, systems, and methods for feedback control of various systems.

BACKGROUND

A feedback (closed-loop) control system 10, as shown in Prior Art FIG. 1, is widely used to modify the behavior of a physical process, denoted as the plant 110, so it behaves in a specific desirable way over time. For example, it may be desirable to maintain the speed of a car on a highway as close as possible to 60 miles per hour in spite of possible hills or adverse wind; or it may be desirable to have an aircraft follow a desired altitude, heading and velocity profile independently of wind gusts; or it may be desirable to have the temperature and pressure in a reactor vessel in a chemical process plant maintained at desired levels. All of these industrial tasks are accomplished today by using traditional feedback control, and the above are examples of what automatic control systems are designed to do, without human intervention.

The key component in a feedback control system is the controller 120, which determines the difference between the actual output of the plant and the desired output as small as possible as rapidly as possible. Today, controllers are employed in a large number of industrial control applications and in areas like robotics, aeronautics, astronautics, motors, motion control and thermal control, just to name a few.

INDUSTRIAL APPLICABILITY

The present system and method has potential applicability to a wide range of different industrial and commercial applications. The following brief synopsis is intended only to provide background on some exemplary applications to assist a person of ordinary skill in the art in understanding this disclosure more fully. Additional background information on a select number of exemplary industrial applications are also provided.

BACKGROUND OF THE INVENTION

Classical Control Theory provides a number of techniques for an engineer to use in controller design. Existing controllers for linear, time invariant, and single-input single-output plants are categorized in one taxonomy into three forms: the proportional/integral/derivative (PID) controllers, transfer function based (TFB) controllers, and state feedback (SF) controllers. The PID controller is defined by the equation:

\[ u = K_p + K_i \int e(t) dt + K_d \frac{de(t)}{dt} \]  

where (u) is the control signal and (e) is the error between the set point and the process output (y) being controlled. As commonly used, the notation of a dot (•) above a variable indicates the use of a derivative of the variable where the order corresponds to the number of dots.

The PID-type of controller has been employed in engineering and other applications since the early 1920s. It is an error-based controller that does not require an explicit mathematical model of the plant 110.

Transfer Function Based Controller

The transfer function based (TFB) controller is given in the form of

\[ U(s) = G_p(s)E(s) \]  

where U(s) and E(s) are Laplace Transforms of u and e defined above, and n(s) and d(s) are polynomials in s. The TFB controller can be designed using methods in control theory based on the transfer function model of the plant, G(s). A PID controller can be considered a special case of a TFB controller because it has an equivalent transfer function of s.

\[ G_c(s) = k_p + k_i \frac{1}{s} + k_d s \]  

where (k_p), (k_i), and (k_d) are constants selected to improve the controller's performance. The State Feedback (SF) Controller

The State Feedback (SF) controller can be defined by

\[ u = -Kx \]  

where u is the control input, r is the setpoint for the output to follow and x is the state vector associated with the system, based on the state space model of the plant 110:

\[ \dot{x} = Ax + Bu(t) \]  

where x(t) is the state vector, y(t) is the output vector, u(t) is the input or control vector, A is the state matrix, B is the input matrix, C is the output matrix and D is the feed-through matrix. When the state x is not accessible, a state observer (SO):

\[ \dot{x} = (A+L)\hat{x} + Bu(t) + Ly(t) \]
is usable to estimate the state $x$, where the $'$ symbol is used to denote an estimate or observed value of a given variable and $L$ denotes the observer coefficient matrix.

State Observers

Observers extract real-time information of a plant’s internal state from its input-output data. The observer usually presumes precise model information of the plant, since performance is largely based on its mathematical accuracy. Most prior art closed loop controllers require both types of information. Such presumptions, however, often make the method impractical in engineering applications, since the challenge for industry remains in constructing these models as part of the design process. Another level of complexity is added when gain scheduling and adaptive techniques are used to deal with nonlinearity and time variance, respectively.

Disturbance Observers and Disturbance Rejection

Recently, disturbance rejection techniques have been used to account for uncertainties in the real world and successfully control complex nonlinear systems. The premise is to solve the problem of model accuracy in reverse by modeling a system with an equivalent input disturbance that represents any difference between the actual plant $P$ and a derived/selected model $P_d$ of the plant, including external disturbances $w$. An observer is then designed to estimate the disturbance in real time and provide feedback to cancel it. As a result, the augmented system acts like the model $P_d$ at low frequencies, making the system behave like $P_d$, and allowing a controller to be designed for $P_d$.


Extended State Observer (ESO)

In this regard, the extended state observer (ESO) is quite different. Originally proposed by Han, J. (1999), “Nonlinear Design Methods for Control Systems.” Proc. 14th IFAC World Congress, in the form of a nonlinear UIO and later simplified to a linear version with one tuning parameter by Guo, Z. (2003). “Scaling and Parameterization Based Controller Tuning,” Proc. of ACC, 4989-4996, the ESO combines the state and disturbance estimation power of a UIO with the simplicity of a DOB, One finds a decisive shift in the underlying design concept as well. The traditional observer is based on a linear time-invariant model that often describes a nonlinear time-varying process. Although the DOB and UIO reject input disturbances for such nominal plants, they leave the question of dynamic uncertainty mostly unanswered. The ESO, on the other hand, addresses both issues in one simple framework by formulating the simplest possible design model $P_d = 1/s^2$ for a large class of uncertain systems. The design model $P_d$ is selected to simplify controller and observer design, forcing $P$ to behave like the design model $P_d$ at low frequencies rather than $P_u$. As a result, the effects of most plant dynamics and external disturbances are concentrated into a single unknown quantity. The ESO estimates this quantity along with derivatives of the output, giving way to the straightforward design of a high performance, robust, easy to use and affordable industrial controller.
Active Disturbance Rejection Control (ADRC)


What is needed is a control framework for application to systems throughout industry that are complex and largely unknown to the personnel often responsible for controlling them. In the absence of required expertise, less tuning parameters are needed than current approaches, such as multi-loop PID, while maintaining or even improving performance and robustness.

INDUSTRIAL APPLICATION

A selection of industrial applications is presented herein to provide a background on some selected applications for the present extended active disturbance rejection controller. Tracking Control

Tracking Control refers to the output of a controlled system meeting design requirements when a specified reference trajectory is applied. Often times, it refers to how closely the trajectory is applied. Oftentimes, it refers to how closely the tracking control refers to the output of a controlled system meeting design requirements when a specified reference trajectory is applied. Often times, it refers to how closely the trajectory is applied. Oftentimes, it refers to how closely the output matches the reference input at any given point in time. This measurement is known as the error $e=r-y$.

Control problems are categorized in two major groups; point-to-point control and tracking control. Point-to-point applications usually call for a smooth step response with small overshoot and zero steady state error, such as when controlling linear motion from one position to the next and then stopping. Since the importance is placed on destination accuracy and not on the trajectory between points, conventional design methods produce a bandwidth limited controller with inherent phase lag in order to produce a smooth output. Tracking applications require a good tracking of a reference input by keeping the error as small as possible at all times, not just at the destination. This is particularly common in controlling a process that never stops. Since the importance is placed on accurately following a changing reference trajectory between points, phase lags are avoided as much as possible because they may lead to significant errors in the transient response, which lasts for the duration of the process in many applications. A common method of reducing phase lag is to increase the bandwidth, at the cost of increased control effort (energy) and decreased stability margin.

Various methods have been used to remove phase lag from conventional control systems. All of them essentially modify the control law to create a desired closed loop transfer function equal to one. As a result, the output tracks the reference input without much phase lag and the effective bandwidth of the overall system is improved. The most common method is model inversion where the inverse of the desired closed loop transfer function is added as a prefilter. Another method proposed a Zero Phase Error Tracking Controller (ZPETC) that cancels poles and stable zeros of the closed loop system and compensates for phase error introduced by un-cancelable zeros. Although it is referred to as a tracking controller, it is really a prefilter that reduces to the inverse of the desired closed loop transfer function when unstable zeros are not present. Other methods consist of a single tracking control law with feed forward terms in place of the conventional feedback controller and prefilter, but they are application specific. However, all of these and other previous methods apply to systems where the model is known.

Model inaccuracy can also create tracking problems. The performance of model-based controllers is largely dependent on the accuracy of the model. When linear time-invariant (LTI) models are used to characterize nonlinear time-varying (NTV) systems, the information becomes inaccurate over time. As a result, gain scheduling and adaptive techniques are developed to deal with nonlinearity and time variance, respectively. However, the complexity added to the design process leads to an impractical solution for industry because of the time and level of expertise involved in constructing accurate mathematical models and designing, tuning, and maintaining each control system.

There have been a number of high performance tracking algorithms that consist of three primary components: disturbance rejection, feedback control, and phase lag compensation implemented as a prefilter. First, disturbance rejection techniques are applied to eliminate model inaccuracy with an inner feedback loop. Next, a stabilizing controller is constructed based on a nominal model and implemented in an outer feedback loop. Finally, the inverse of the desired closed loop transfer function is added as a prefilter to eliminate phase lag. Many studies have concentrated on unifying the disturbance rejection and control part, but not on combining the control and phase lag compensation part, such as the RIC framework. Internal model control (IMC) cancels an equivalent output disturbance. B. Francis and W. Wonham, "The Internal Model Principle of Control Theory," Automatica, vol 12, 1976, pp. 457-465. E. Schrijver and J. Van Dijk, "Disturbance Observers for Rigid Mechanical Systems: Equivalence, Stability, and Design," Journal of Dynamic Systems, Measurement, and Control, vol. 124, December 2002, pp. 539-548 uses a basic tracking controller with a DOB to control a multivariable robot. The ZPETC has been widely used in combination with the DOB framework and model based controllers.

Thus, having reviewed prior art in tracking control, the application now describes example systems and methods of tracking control employing predictive ADRC.

Web Processing Applications

Web tension regulation is a challenging industrial control problem. Many types of material, such as paper, plastic film,
cloth fabrics, and even strip steel are manufactured or pro-
cessed in a web form. The quality of the end product is often
greatly affected by the web tension, making it a crucial vari-
able for feedback control design, together with the velocities
to the various stages in the manufacturing process. The ever-
increasing demands on the quality and efficiency in industry
motivate researchers and engineers alike to explore better
methods for tension and velocity control. However, the highly
nonlinear nature of the web handling process and changes in
operating conditions (temperature, humidity, machine wear,
and variations in raw materials) make the control problem
challenging.

Accumulators in web processing lines are important ele-
ments in web handling machines as they are primarily respon-
sible for continuous operation of web processing lines. For
this reason, the study and control of accumulator dynamics is
an important concern that involves a particular class of prob-
lems. The characteristics of an accumulator and its operation
as well as the dynamic behavior and control of the accumu-
lator carriage, web spans, and tension are known in the art.

Both open-loop and closed-loop methods are commonly
used in web processing industries for tension control pur-
poses. In the open-loop control case, the tension in a web span
is controlled indirectly by regulating the velocities of the
rollers at either end of the web span. An inherent drawback
of this method is its dependency on an accurate mathematical
model between the velocities and tension, which is highly
nonlinear and highly sensitive to velocity variations. Never-
theless, simplicity of the controller outweighs this drawback
in many applications. Closing the tension loop with tension
feedback is an obvious solution to improve accuracy and to
reduce sensitivity to modeling errors. It requires tension
measurement, for example, through a load cell, but is typically
justified by the resulting improvements in tension regulation.

Most control systems will unavoidably encounter disturb-
ances, both internal and external, and such disturbances
have been the obstacles to the development of high perfor-
ance controller. This is particularly true for tension control
applications and, therefore, a good tension regulation scheme
must be able to deal with unknown disturbances. In particular,
tension dynamics are highly nonlinear and sensitive to velo-
city variations. Further, process control variables are highly
dependent on the operating conditions and web material char-
acteristics. Thus, what are needed are systems and methods
for control that are not only overly dependent on the accuracy
of the plant model, but also suitable for the rejection of sig-
ificant internal and external disturbances.

Jet Engine Control Applications

A great deal of research has been conducted towards the
application of modern multivariable control techniques on
aircraft engines. The majority of this research has been to
control the engine at a single operating point. Among these
methods are a multivariable integrator windup protection
Integrator Windup Protection Technique Applied to a Turbo-
fan Engine Control,” AIAA Guidance Navigation and Control
Conf.), a tracking filter and a control mode selection for
“Tracking Filter Selection And Control Mode Selection For
Model Based Control.” AIAA 30th Joint Propulsion Conference
and Exhibit), an H\textsubscript{\infty} method and linear quadratic Gauss-
ian with loop transfer recovery method (Watts, S. R. and S.
Garg (1995).” A Comparison Of Multivariable Control
Design Techniques For A Turbofan Engine Control.” Int. J.
Gas Turbine and Aeroengines Congress and Expo.),
and a performance seeking control method (Adibhatla, S. and
K. L. Johnson (1993). “Evaluation of a Nonlinear Psc Algo-

SUMMARY OF INVENTION

The present system, method, process, and device for con-
AIAA/SAG/ASEME/ASEE
90th Joint Propulsion Conference and Exhibit). Various
schemes have been developed to reduce gain scheduling

Various

methods and have even been combined with integrator windup protection and H\textsubscript{\infty}
Engine Control Design Using Robust Multivariable Control

Conventionally, there have been a limited number of con-
rol techniques for full flight operation (Garg, S. (1997). “A
Simplified Scheme for Scheduling Multivariable Control-
ors.” IEEE Control Systems; and Polley, J. A., S. Adibhatla
Control for Full Conference on Decision and Control Flight
Operation.” Gas Turbine and Exp). However, there has been
no development of tuning a controller for satisfactory perfor-
mance when applied to an engine. Generally, at any given
operating point, models can become inaccurate from one
engine to another. This inaccuracy increases with model com-
plicity, and subsequently design and tuning complexity. As a
result, very few of these or similar aircraft design studies have
led to implementation on an operational vehicle.

The current method for controlling high performance jet
engines remains multivariable proportional-integral (PI) con-
Closed-Loop Nyquist and Bode Arrays.” Int. J. on Control,
30:5, 773-802, and Polley, J. A., S. Adibhatla and P. J. Hoff-
mans.” AIAA Guidance Navigation and Control
Conference and Exhibit). Although the controller is designed by
implementing Bode and Nyquist techniques and is tunable,
a problem remains due to the sheer number of tuning param-
eters compounded by scheduling.

In a second embodiment the model is used to provide an
enhanced control law that provides improved performance. In
Another aspect, the ADRC controller further
includes a model of the plant dynamics. The model is used in
one embodiment to improve the state estimator or predictor.
In a second embodiment the model is used to provide an
enhanced control law that provides improved performance. In
both aspects of the embodiments discussed in this paragraph, the
generalized disturbance captures model errors and dis-
crepancies and allows the system to eliminate errors caused
by model errors, discrepancies, and assumptions. In the multi-
tiple embodiments, a variety of models are available for use
by one of ordinary skill in the art to achieve the desired
performance relative to the type of plant being controlled.

In yet another aspect, the ADRC controller further includes
a non-linear, or discrete time optimal control law. The non-
linear control law of the present embodiment utilizes non-
linear control laws to improve the overall performance of an
ADRC controller as compared to an ADRC controller with a
linear proportional-derivative based control law.

In still another aspect, additional knowledge of derivative
plant output gathered from high quality sensor information or
direct measurement sensors (e.g. velocity and acceleration
sensors) is used to reduce the order of the system state and
disturbance estimator.

**BRIEF DESCRIPTION OF THE DRAWINGS**

The accompanying figures depict multiple embodiments of
a device and method for the separation of constituents from a
flow. A brief description of each figure is provided below.
Elements with the same reference numbers in each figure
indicate identical or functionally similar elements. Addition-
ally, the left-most digit(s) of a reference number identifies
the drawings in which the reference number first appears.

FIG. 1 is a block diagram of a Prior Art feedback controller.

FIG. 2 is a block diagram of an ADRC configuration for a
2nd order plant.

FIG. 3 is a block diagram of a predictive ADRC configu-
ration.

FIG. 4 is a block diagram of a 2 degree of freedom transfer
function based (TFB) control structure.

FIG. 5 is a block diagram of an implementation of a
PADRC configuration based on output prediction.

The present invention is described with reference to block
diagrams and operational flow charts. It is to be understood
that the functions/acts noted in the blocks may occur out of
the order noted in the operational illustrations. For example,
two blocks shown in succession may in fact be executed
substantially concurrently or the blocks may sometimes be
executed in the reverse order, depending upon the function-
ality/acts involved, including for example executing as asyn-
chronous threads on a processor. Although some of the dia-
grams include arrows on communication paths to show a
primary direction of communication, it is to be understood
that communication may occur in the opposite direction to
the depicted arrows.

**DETAILED DESCRIPTION**

Multiple embodiments of a system, device, and method for
the control of systems are presented herein. Those of ordinary
skill in the art can readily use this disclosure to create alter-
native embodiments using the teaching contained herein.

**Lexicon**

The following terms used herein have the meanings as
follows.

As used herein the term “computer component” refers to a
computer and elements of a computer, such as hardware,
firmware, software, a combination thereof, or software in
execution. For example, a computer component can include
by way of example, a process running on a processor, a
processor, an object, an executable, an execution thread, a
program, and a computer itself. One or more computer com-
ponents can in various embodiments reside on a server and
the server can be comprised of multiple computer compo-
nents. One or more computer components are in some cases
referred to as computer systems whereby one or more com-
puter components operate together to achieve some function-
ality. One or more computer components can reside within a
process and/or thread of execution and a computer compo-
nent can be localized on one computer and/or distributed
between two or more computers.

“Software”, as used herein, includes but is not limited to,
one or more computer readable and/or executable instruc-
tions that cause a computer or other electronic device to
perform functions, actions and/or behave in a desired manner.
The instructions may be embodied in various forms like rou-
tines, algorithms, modules, methods, threads, and/or pro-
grams. Software may also be implemented in a variety of
executable and/or loadable forms including, but not limited to,
a stand-alone program, a function call (local and/or remote),
a servlet, an applet, instructions stored in a memory, part
of an operating system or browser, and the like. It is to be
appreciated that the computer readable and/or executable
instructions can be located in one computer component and/
or distributed between two or more communicating, co-
operating, and/or parallel processing computer components
and thus can be loaded and/or executed in serial, parallel, mas-
vatively parallel and other manners. It will be appreciated by
one of ordinary skill in the art that the form of software may
be dependent on, for example, requirements of a desired
application, the environment in which it runs, and/or the
desires of a designer/programmer or the like.

“Computer communications”, as used herein, refers to a
communication between two or more computers and can be,
for example, a network transfer, a file transfer, an applet
transfer, an email, a hypertext transfer protocol (HTTP) mes-
sage, an object, an object transfer, a binary large object
(BLOB) transfer, and so on. A computer communication can
occur across, for example, a wireless system (e.g., IEEE
802.11), an Ethernet system (e.g., IEEE 802.3), a token ring
system (e.g., IEEE 802.5), a local area network (LAN), a wide
area network (WAN), a point-to-point system, a circuit
switching system, a packet switching system, and so on.

An “operable connection” is one in which signals and/or
actual communication flow and/or logical communication
flow may be sent and/or received. Usually, an operable con-
nection includes a physical interface, an electrical interface,
and/or a data interface, but it is to be noted that an operable
connection may consist of differing combinations of these
or other types of connections sufficient to allow operable con-

As used herein, the term “signal” may take the form of a
continuous waveform and/or discrete value(s), such as digital
value(s) in a memory or register, present in electrical, optical
or other form.

The term “controller” as used herein indicates a method,
process, or computer component adapted to control a plant
(i.e. the system to be controlled) to achieve certain desired
goals and objectives.

As used herein the following symbols are used to describe
specific characteristics of the variable. A variable with a “+”
overtop the variable label, unless other indicated to the con-
trary, indicates that the variable is an estimate of the actual
variable of the same label. A variable with a “—” overtop the
variable label, unless otherwise indicated, indicates that the
variable is an nth derivative of that variable, where n equals
the number of dots.

To the extent that the term “includes” is employed in the
detailed description or the claims, it is intended to be inclusive
in a manner similar to the term “comprising” as that term is interpreted when employed as a transitional word in a claim.

To the extent that the term “or” is employed in the claims (e.g., A or B) it is intended to mean “A or B or both”. When the author intends to indicate “only A or B but not both”, then the author will employ the term “A or B but not both”. Thus, use of the term “or” in the claims is the inclusive, and not the exclusive, use.

Active Disturbance Rejection Control (ADRC)

Active Disturbance Rejection Control ("ADRC") is a method or process and, when embedded within a computer system, a device (i.e., a controller that processes information and sends signals using the ADRC algorithms), that effectively rejects both unknown internal dynamics of a system of an arbitrary order to be controlled, or plant 110, and external disturbances in real time without requiring detailed knowledge of the plant 110 dynamics, which is required by most existing control design methods.

The only information needed to configure an ADRC controller is knowledge of the relative order of the plant 110 and the high frequency gain of the plant 110. In comparison, traditional model-based control methods require accurate mathematical models to conform with prevailing model-based design methods. In some embodiments, the ADRC controller leverages knowledge gained from model-based controller designs to improve control performance. The ADRC approach is unique in that it first forces an otherwise nonlinear, time-varying and uncertain plant 110 to behave as a simple linear cascade integral plant that can be easily controlled. To this end, the unknown, nonlinear, and time-varying internal dynamics is combined with the external disturbances to form what is denoted as the generalized disturbance, which is then estimated and rejected, thus greatly simplifying the control design.

The embodiments of the ADRC controller design are founded on the basic theory that uncertainties in dynamics and unknown external disturbances in the operating environment are a key challenge to be dealt with in control engineering practice. Traditional control systems are incapable of providing a solution to controlling and mitigating the impact of these uncertainties. The ADRC controller in contrast is specifically adapted to mitigate and eliminate the impact of these uncertainties, thus providing a dynamically simplified system for control.

One previous major characteristic of the ADRC controller is the parameterization of the controller as a function of control loop bandwidth. This parameterization makes all of the ADRC controller parameters functions of the control loop bandwidth. In comparison, traditional model-based control methods require accurate mathematical models to conform with prevailing model-based design methods. In some embodiments, the ADRC controller leverages knowledge gained from model-based controller designs to improve control performance. The ADRC approach is unique in that it first forces an otherwise nonlinear, time-varying and uncertain plant 110 to behave as a simple linear cascade integral plant that can be easily controlled. To this end, the unknown, nonlinear, and time-varying internal dynamics is combined with the external disturbances to form what is denoted as the generalized disturbance, which is then estimated and rejected, thus greatly simplifying the control design.

The corresponding state observer is

$$\begin{align*}
\dot{z} &= A_x z + B_x u + E f \\
y &= C_x z
\end{align*}$$

where

$$A_x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C_x = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The corresponding state observer is

$$\begin{align*}
\dot{z} &= A_x z + B_x u + L(y_m - C_x z) \\
f &= v_z
\end{align*}$$

where $y_m$ is the measured system output, $B_x = [0 \ 0 \ 0]^T$, $V = [0 \ 0 \ 1]$, and $L = [1, 1, 1]^T$ is the observer gain vector. With the parameterization technique as described more fully in U.S. patent application Ser. No. 10/351,664, L is obtained by solving the equation $\lambda(s) = \lambda I - A_x + L C_x$, that is, all the eigenvalues of the observer are placed at $-\omega_c$ with

$$l_1 = 3 \omega_c, l_2 = 3 \omega_c^2, l_3 = \omega_c^3$$

With a well-tuned extended state observer, the generalized disturbance $f$ is rejected by the control law

$$u = (u_0 - z_3)b$$

which reduces the plant 110 to a unity gain, double integral plant

$$\tilde{f} = u_a$$

that can be easily controlled with linear proportional-derivative (PD) with feedforward

$$u_a = g_{pd}(\tilde{e}, \omega_c)\tilde{e}$$

where $g_{pd}(\tilde{e}, \omega_c)$ is the parameterized linear PD controller with the loop bandwidth of $\omega_c$.

$$g_{pd}(\tilde{e}, \omega_c) = \omega_c^2 + 2 \omega_c \tilde{e}$$

with the error term defined as

$$e = y - y_m$$

where the desired trajectory $(r)$ is provided by the user.

Note that the linear PD control law $206$ in Equation (14) corresponds to the common PD gains of

$$k_p = \omega_c^2$$

and $k_d = 2 \omega_c$.

Equation (17)
Using these details, the ADRC controller is generalized in its various embodiments to control an arbitrary n th-order system \( y^{(n)} = f(y, y', \ldots, y^{(n-1)}, t) + bu \), in which the external disturbances and unknown internal dynamics are rejected. In a similar manner, the plant 110 is reduced to a cascaded integral plant \( y^{(n)} = u \), which is then controlled by a generalized proportional-derivative (PD) control law, also described as a linear PD control law 206 similar to Equation (14).

Use of Enhanced Sensor Information to Improve Control Performance

In other embodiments of the ADRC controller, the linear PD control law 206 is updated to enhance performance by using direct measurement of the system output states \( y_m \) instead of the estimated states \( z_i, z_2 \). Specifically, when the measured output \( y_m \) is sufficiently noise free and \( y_m \) is obtainable, either via direct measurement or from \( y_m \), the control law in Equation (14) can be implemented with

\[
e^{-r}y_m - \hat{y}_m
\]

That is, the output measurement and its derivative are used in the control law, instead of their estimates, \( z_1 \) and \( z_2 \). Similarly for the n th-order plant 110, the output measurement \( y_m \) and its derivatives from the \((n-1)\)th to the \(n\)th-order available, can also be used directly in the PD control law, also referred to as the linear PD control law 206, instead of their estimates from the Extended State Observer (ESO) 204. Using the direct measurement of the output measurement state \( y_m \) and its derivatives reduces the phase lag and improve control system performance, if the output measurement and its derivatives are not corrupted by noise.

Multiple Extended States

The Extended State Observer 204 in alternative embodiments includes more than one extended states, such as embodiments where the 2nd order plant 202 is replaced with a higher order plant 110. In one embodiment, when augmented with two extended states, \( x_3 = f \) and \( x_4 = f \), the extended plant state space model can be written as follows.

\[
\begin{align*}
\dot{x} &= Ax + Bu + E\hat{f} \\
y &= Cx
\end{align*}
\]

TABLE I

<table>
<thead>
<tr>
<th>Type</th>
<th>( \Phi )</th>
<th>( \Gamma )</th>
<th>( L_p )</th>
<th>( L_v )</th>
</tr>
</thead>
</table>
| Euler      | \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}
\] | \[0\] | \[\frac{3(1 - \beta)}{(1 - \beta^2)}\] | \[1 - \beta^2\] |
| ZOH        | \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}
\] | \[\frac{\beta^2}{2}\] | \[\frac{3(1 - \beta)}{(1 - \beta^2)}\] | \[1 - \beta^2\] |
This discrete implementation of ADRC is also generalizable to n-th-order plants [10].

Predictive Active Disturbance Rejection Controller

The thrust of the many of the embodiments presented here is the addition of the predictive functions to extend the capabilities of the original components of the ADRC. This extends the ADRC controller and generates embodiments of predictive versions of the ADRC, such as the predictive ADRC [300], depicted in block diagram form in FIG. 3. In this embodiment, the linear PD control law [206] in the predictive ADRC [300] is now a predictive control law [306]. Similarly, the observer, or more specifically the extended state observer [204] from the traditional ADRC [200] is now embodied as a predictive state and disturbance observer [304]. In one embodiment the predictive state and disturbance observer [304] is a predictive extended state observer (PESO) where the prediction occurs within the bounds of the ESO [204] to where the estimated states \( z_i(t) \) are predicted to form the predictive state and disturbance observer [304], and outputs the prediction of the estimated states \( z_i(t+\tau) \). In a second embodiment, a state predictor method is used to extend the design of the ESO [204] to estimate the states \( z_i(t) \) and form the predictive state and disturbance observer [304].

In other embodiments, other estimators and predictors, either in an integrated form or in combination are used to generate similar information as that generated by the predictive state and disturbance observer [304] and used by the embodiments predictive ADRC [300]. One of these other embodiments adopts a disturbance observer (DOB) estimator structure to estimate the disturbance and a state observer and a predictor element that together form the predictive state and disturbance observer [304]. In a second other embodiment, the predictive state and disturbance observer [304] utilizes an unknown input observer (UIO) estimator. Other estimators, including combinations of estimators such as the combination of a state observer and a DOB, are suitable for estimation of both the plant output and the known part of the general disturbance \( f(t) \) or adaptability using the disclosure provided herein to those of ordinary skill in the art to serve as a predictive state and disturbance observer [304].

The challenge in constructing the various embodiments of a predictive controller within the general purpose ADRC controller architecture is how the various signals used by the ADRC algorithm are predicted and used. The signals used by the ADRC algorithm originate from a multitude of sources in the context of Predictive ADRC [300], the knowledge of \( f(t) \) can be used to help predict its future value, \( f(t+\tau) \). The corresponding control law for Equation (23) and Equation (24) are

\[
\dot{e}(t) = g(e(t), e(t), \omega_e) + f(t) - f(t+\tau)
\]

and

\[
\dot{e}(t) = g(e(t), e(t), \omega_e) + f(t) - f(t+\tau) - g(t+\tau)
\]

respectively.

Described below are various methods to obtain the prediction of \( e(t) \) and \( \dot{e}(t) \), respectively, so that these predictive ADRC [300] control laws can be implemented.

**Prediction of \( e(t) \) and \( \dot{e}(t) \)**

Two embodiments of methods for predicting the values of error \( e(t) \) and its derivative are presented herein.

**Method 1:** Using Taylor Series, the prediction of \( e(t) \) and \( \dot{e}(t) \) can be obtained as \( e(t+\tau) = e(t) + \dot{e}(t)(\tau) + \frac{1}{2} \ddot{e}(t)(\tau)^2 \) or

\[
e(t + \tau) = e(t) + \dot{e}(t)\tau + \frac{1}{2} \ddot{e}(t)\tau^2
\]

and \( \dot{e}(t+\tau) = \dot{e}(t) + \ddot{e}(t)\tau \). Here \( \dot{e}(t) \) can be obtained directly from \( \dot{r} \), or from the second-order ESO (RESO) using RESO. Here \( \dot{e}(t) \) can be obtained directly from the future reference signals \( r(t+\tau) \) and \( \dot{r}(t+\tau) \), which are generally known, and the predicted output of the ESO \( z_i(t+\tau) \) and \( z_i(t+\tau) \), which need to be obtained approximately.

**Method 2:** Since \( e(t+\tau) = r(t+\tau) - z_i(t+\tau) \) and \( \dot{e}(t+\tau) = \dot{r}(t+\tau) - \dot{z}_i(t+\tau) \), \( e(t+\tau) \) and \( \dot{e}(t+\tau) \) can be obtained directly using the future reference signals \( r(t+\tau) \) and \( \dot{r}(t+\tau) \), which are generally known, and the predicted output of the ESO \( z_i(t+\tau) \) and \( z_i(t+\tau) \), which need to be obtained approximately.

**Prediction of \( \dot{r}(t) \) in ESO and RESO**

The primary method of predicting \( \dot{r}(t) \) is the 1st or 2nd-order Taylor Series approximation, i.e.

\[
\dot{r}(t+\tau) = \ddot{r}(t)\tau + \frac{1}{2} \dddot{r}(t)\tau^2
\]
which requires the 1st and/or 2nd order derivatives of \( \hat{y} \). To this end, a new form and a new way of implementation of extended state observer (ESO) 204 are introduced first, followed by the discussion on how to use them to obtain the predicted estimate off in Equation (27) and Equation (28).

Reduced-Order ESO

In the observer-based control methods, the phase lag introduced by the observer decreases the phase margin of the control loop and is therefore undesired. To reduce the phase lag in the ESO 204, an embodiment of a reduced-order ESO (RESO) described here.

Consider the plant 202 in Equation (9), if the derivative of the measured output \( y_m (\dot{y}_m) \), is given, a RESO can be constructed with \( z_1 \) and \( z_2 \) estimating \( x_3 - \hat{y} \) and \( x_3 - \hat{f} \) respectively. The correspondence between the augmented plant state space model and the RESO is shown as follows.

\[
\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} f
\]
\[
y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x
\]

The shadowed blocks show how the gain matrices of the RESO are obtained from the augmented plant state space model. Equivalently, the RESO can be represented as

\[
\begin{aligned}
\dot{z} &= A_z z + B_z u + L (y_m - C_z z) \\
\dot{f} &= V z
\end{aligned}
\]

where

\[
A_z = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}
\]

\( C_z = [1 0], V = [0 1] \), and \( l_1 = 2 \omega_n l_2 = \omega_n^2 \).

With RESO replacing the ESO 204, the corresponding linear PD control law 206 in ADRC 400 is

\[
\begin{aligned}
u &= (g_m (e, \dot{e}, \omega_n) + r - z_3) / b \\
u &= (g_m (e, \dot{e}, \omega_n) + r - \dot{y}_m) / b
\end{aligned}
\]

The first part is a Luenberger observer for the double integral plant in Equation (13), and the second part shows that the estimation of the general disturbance can be obtained by integrating the observer error with a gain of \( l_3 \). This implementation is equivalent to the ESO 204.

When implemented in ZOH current estimator form, the Luenberger observer is
\[ z[k + 1] = \Phi z[k] + \Gamma u[k] + L_s(y_m[k] - z_1[k]) \]
\[ z[k] = z[k] + L_c\left( y_m[k] - z_1[k] \right) \]

where
\[ \Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \frac{T^2}{2} \\ bT \end{bmatrix}, \quad L_s = \begin{bmatrix} 1 - \beta_1 \\ (1 + \beta_1 - 2\beta_1)/T \end{bmatrix}, \quad L_c = \begin{bmatrix} 2 - 2\beta_1 \\ (1 + \beta_2 - 2\beta_1)/T \end{bmatrix} \]

and
\[ \beta_1 = e^{-\frac{3}{2}t_0T} \cos\left( \frac{\sqrt{3}}{2}T_0T \right), \quad \beta_2 = e^{-\frac{3}{2}t_0T}. \]

Similarly, the RESO in Equation (30) can be divided into two parts:

\[ z_1 = b\dot{u} + h(y_m - z_1) \]
\[ z_2 = \int \dot{t}_2(y_m - z_1) \, dt \]

where the first part is a first order Luenberger observer, and the second part is an integrator. When implemented in ZOH current estimator form, this Luenberger observer is

\[ z_1[k + 1] = \Phi z_1[k] + \Gamma u[k] + L_s(y_m[k] - z_1[k]) \]
\[ z_1[k] = z_1[k] + L_c\left( y_m[k] - z_1[k] \right) \]

where \( \Phi = 1 \), \( \Gamma = \hat{\Gamma}, \quad \hat{L}_c = \hat{L}_c = 1 - \beta \), and \( \beta = e^{-\frac{2}{T_0T}}. \)

\section*{Prediction of Chaotic Behavior}

With the new implementation of ESO 204, \( \hat{t} = \int b(y_m - z_1) \, dt \), \( \hat{t}_1 = \int b(y_m - z_2) \, dt \), and this enables us to determine \( \hat{f}(t+\Delta t) \) based on the 1st or 2nd-order Taylor series approximation shown in

\[ \hat{f}(t+\Delta t) = f(t) + f'(t)\Delta t \]

or

\[ \hat{f}(t+\Delta t) = \int \dot{f}(y_m - z_1) \, dt + f'(y_m - z_1) \Delta t \]

\section*{State Predictor Method for Predictive Extended State Observer}

Another embodiment for forming a predictive state and disturbance observer 304 module is to use a state predictor method to extend the performance of a baseline ESO 204 to predict state estimates \( t_0 \) seconds in the future. The basic state predictor method approach was published by T. Oguchi, H. Nijmeijer, \textit{Prediction of Chaotic Behavior}, IEEE Trans. on Circuits and Systems-I, Vol. 52, No. 11, pp. 2464-2472, 2005, which is hereby incorporated by reference. The state predictor method is combined in the embodiments presented herein with the regular ESO 204 of the form

\[ \dot{z} = Az + Bu + L(y - \hat{y}) \]
\[ \hat{y} = Cz \]
respectively.

A state and disturbance observer 504, shown in FIG. 5, accepts the predicted output \( y(t+\tau) \) and the control commands \((u(t))\) to estimate the future states \((z(t+\tau))\) where \((n)\) is the number of estimated states plus an estimate of the generalized disturbance \((f)\). The state and disturbance observer 504 in one embodiment is an ESO 204. In a second embodiment, the state and disturbance observer 504 is a disturbance observer (DOB) structure. In yet another embodiment, the state and disturbance observer 504 is an unknown input observer (UID). It is apparent that the combination of the state and disturbance observer 504 in combination with the prediction module 508 provides a similar output and performance a similar function within the framework of an output predictive ADRC controller 300 as the predictive ADRC controller 500 of FIG. 5. In this manner, the various embodiments of the predictive ADRC controller 300 and output predictive ADRC controller 500 are correlated.

Also shown in FIG. 5, a control law 506 is provided that uses the estimated future states of the plant 110 coupled with the desired trajectory \((r(t+\tau))\) to generate a current control input \((u(t))\). The embodiment of the control law 506 depicted in FIG. 5 incorporates the additive inverse of the estimated future values of the estimated disturbance \((f)\).

Combine the Output Prediction with RESO

With the RESO configuration, the prediction is obtained as:

\[
\hat{y}(t+\tau) = y_m(t) + f_m(t) + \hat{f}(t+\tau) 
\]

and the new RESO is constructed as:

\[
\{ \dot{z}(t+\tau) = A_z z(t+\tau) + B_z u(t) + L_z (\hat{y}(t+\tau) - C_z z(t+\tau)) \\
\dot{\hat{f}}(t+\tau) = V_z \hat{f}(t+\tau) \}
\]

where

\[
A_z = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_z = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, L_z = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}
\]

Combine the Output Prediction with Regular ESO

To compensate for the delay, \(y(t)\) is replaced with the approximately predicted output \(\hat{y}(t+\tau)\) as the input to the ESO 204. Again, the approximated prediction, \(\hat{y}(t+\tau)\), is obtained from the Taylor Series approximation. The 1st-order Taylor series approximation and the 2nd-order Taylor series approximation of \(\hat{y}(t+\tau)\) are as shown in the following equations respectively.

The ideas is that since the predicted output \(\hat{y}(t+\tau)\) as shown in the following equations from the Taylor Series approximation. The 1st-order Taylor series approximation and the 2nd-order Taylor series approximation of \(\hat{y}(t+\tau)\) are as shown in the following equations respectively.

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, L = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}
\]

The ideas is that since the predicted output \(\hat{y}(t+\tau)\) as shown in the following equations from the Taylor Series approximation. The 1st-order Taylor series approximation and the 2nd-order Taylor series approximation of \(\hat{y}(t+\tau)\) are as shown in the following equations respectively.

\[
L = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}
\]

The ideas is that since the predicted output \(\hat{y}(t+\tau)\) as shown in the following equations from the Taylor Series approximation. The 1st-order Taylor series approximation and the 2nd-order Taylor series approximation of \(\hat{y}(t+\tau)\) are as shown in the following equations respectively.

\[
l_1 = 2\omega_0 \omega_2 - \omega_0^2, \quad l_2 = -2\omega_1 \omega_2 - \omega_0^2 \]

Then the predicted system output and the current control signal are used as the inputs to the ESO 204 as follows.

\[
\{ \dot{z}(t+\tau) = A_z z(t+\tau) + B_z u(t) + L_z (\hat{y}(t+\tau) - C_z z(t+\tau)) \\
\dot{\hat{f}}(t+\tau) = V_z \hat{f}(t+\tau) \}
\]

which will provide both the predicted states and the disturbance estimation in \(z(t+\tau)\).

Equivalent 2 dof Transfer Functions

When the 1st-order Taylor series approximation is used to obtain \(\hat{y}(t+\tau)\), the equivalent 2dof transfer functions are:

\[
\begin{align*}
C(s) &= \frac{1}{s^3 + (l_1 + l_2)(s + k_p) + (l_2 + k_p + k_d)} \\
H(s) &= \frac{s^3 + l_1 s^2 + l_2 s + l_3 (k_p + k_d + \omega_0^2)}{s (s^2 + (l_1 + l_2)(s + k_p) + (l_2 + k_p + k_d))}
\end{align*}
\]

When the 2nd-order Taylor series approximation is used to obtain \(\hat{y}(t+\tau)\), the equivalent 2dof transfer functions are:

\[
\begin{align*}
C(s) &= \frac{1}{s^4 + (l_1 + l_2)(s^2 + k_p s + \omega_0^2) + (l_2 + k_p + k_d)} \\
H(s) &= \frac{s^4 + l_1 s^3 + l_2 s^2 + l_3 (k_p s + \omega_0^2) + l_3 s + \omega_0^2}{s (s^3 + (l_1 + l_2)(s + k_p) + (l_2 + k_p + k_d))}
\end{align*}
\]
Equivalent 2 dof Transfer Functions

With the above the 1st-order Taylor Series approximation $y(t+\tau)$ is used as the input to the RESO, the equivalent 2 dof transfer functions are:

$$C(s) = \frac{1}{b} \left[ \frac{\left(b_p + k_d l_1 + l_2\right) s^2 + \left(2k_p + k_d l_1 + l_2\right) s + k_d l_2}{s^2 + (l_1 + k_d) s + k_d l_2} \right] (1 + \tau)$$  

Equation (62)

$$H(s) = \left( \frac{s^2 + l_1 s + l_2}{s^2 + (l_1 + k_d) s + k_d l_2} \right) \left( \frac{2k_p + k_d l_1 + l_2}{s + (l_1 + k_d) s + k_d l_2} \right) \left( \frac{1 + \tau}{1 + \tau} \right)$$  

Equation (63)

Combine the Output Prediction with RESO

With the RESO configuration, the prediction is obtained as

$$\hat{y}(t+\tau) = \hat{y}_d(t+\tau) + \hat{y}_d(t) \text{ and the new RESO is constructed as}$$

$$\begin{cases}
\hat{x}(t+\tau) = A_1 \hat{x}(t) + B_1 u(t) + L(\hat{y}(t+\tau) - C_1 \hat{y}(t+\tau)) \\
\hat{y}(t+\tau) = V \hat{y}(t+\tau)
\end{cases}$$  

Equation (64)

where

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix},$$

$$C_1 = [0 1], V = [0 1], \text{ and } l_1 = 2 \omega_0 \lambda_1 \omega_0^2,$$ which again provides the predicted state and disturbance estimation in $z(t+\tau)$.

Equivalent 2 dof Transfer Functions

With the above the 1st-order Taylor Series approximation $y(t+\tau)$ is used as the input to the RESO, the equivalent 2 dof transfer functions are

$$C(s) = \frac{1}{b} \left[ \frac{\left(b_p + k_d l_1 + l_2\right) s^2 + \left(2k_p + k_d l_1 + l_2\right) s + k_d l_2}{s^2 + (l_1 + k_d) s + k_d l_2} \right] (1 + \tau)$$  

Equation (66)

$$H(s) = \left( \frac{s^2 + l_1 s + l_2}{s^2 + (l_1 + k_d) s + k_d l_2} \right) \left( \frac{2k_p + k_d l_1 + l_2}{s + (l_1 + k_d) s + k_d l_2} \right) \left( \frac{1 + \tau}{1 + \tau} \right)$$  

Equation (67)

A Parameterized Nonlinear PD Control Law, Replacing the Linear PD Controller

The parameterized linear PD control law $u_{pd}(e,e,w)$ used above can be replaced by the following parameterized Nonlinear PD (NPD) control law $u_{pd}(e,e,w)$:

$$y = \omega_0^2 \dot{e} + \omega_0 \dot{e}_d,$$

Equation (68)

$$x = \begin{cases}
\omega_0 \dot{e} + \sqrt{R^2 + R^2} & y > R \\
\omega_0 (\omega_0 e + 2\dot{e}) & y \leq R
\end{cases}$$  

Equation (69)

$$g_{pd}(e, \dot{e}, \omega_0) = \begin{cases} 
R \text{ sign}(y), & |y| > R \\
x, & |y| \leq R
\end{cases}$$  

where

$$\text{sign}(y) = \begin{cases} 
1, & x \geq 0 \\
-1, & x < 0
\end{cases}$$

Equation (70)

Equation (68) shows that, within the region $|x| \leq R$ and $|y| \leq R$, this parameterized NPD control law $u_{pd}(e,e,w)$, which is the same as the parameterized linear PD control law $u_{pd}(e,e,w)$ in Equation (15); outside this region, however, the NPD control law $u_{pd}(e,e,w)$ is nonlinear. This new NPD control law $u_{pd}(e,e,w)$ can be used to replace the linear PD control law $u_{pd}(e,e,w)$ in the original ADRC algorithm to achieve better performance. In one embodiment the NPD control law $u_{pd}(e,e,w)$ is used in conjunction with a predicted state estimate $\hat{z}_n(t+\tau)$. In a second embodiment, the NPD control law $u_{pd}(e,e,w)$ is used only the current state estimate ($\hat{z}_n(t)$).

A General Form of Reduced-Order Multiple Extended-state ESO for an $n$th-Order Plant

Although the Predictive ADRC has been presented with an ESO or RESO with only one extended state, the same technique can be readily extended to a general $n$th-order plant with an ESO or RESO that has multiple extended states. To facilitate such development, we present here a general form of the reduced-order multiple extended states ESO for an $n$th-order plant of the form

$$x^{(0)} = f(x^{(0)}, \ldots, x^{(n-1)}, \dot{z}, u) + bu$$  

Equation (69)

Let the $n$ states be $x_1, x_2, \ldots, x_n$, $x_n = x_n^{(n-1)}$, and the $k$ extended states be $x_{n+1}, x_{n+2}^{(n-1)}, \ldots, x_{n+k} = x_{n+k}^{(n-1)}$. The plant $\dot{x}$ in Equation (69) can now be expressed in the augmented form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \\ \dot{x}_{n+1} \\ \vdots \\ \dot{x}_{n+k} \end{bmatrix} = \begin{bmatrix} \ddots \\ \ddots \\ \ddots \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \\ X_{n+1} \\ \vdots \\ X_{n+k} \end{bmatrix}$$

where

$$\ddots$$

and

$$X_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$X_{n+1} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_{n+1} \\ x_{n+2} \\ \vdots \\ x_{n+k} \end{bmatrix}$$
Equation (70)

\[ x = Ax + Bu + Ef(z) \]

Equation (71)

\[ z = Az + Bz + Lu + L_0(z_{m-h+1} - z) \]

where

\[ \dot{z} = vz \]

or in state space form

or equivalently

\[ \begin{cases} \dot{z} = A_z z + B_z u + E' z \med {m-h+1} \\ y = C_z x \end{cases} \]

where

\[ A_z = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 \end{bmatrix}, \quad B_z = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

and the nonzero element in \( B_z \) is in the \( n^\text{th} \) row.

Assume that \( y_m, y_m, \ldots, y_m \) are measured or otherwise available as \( y_m, y_m, \ldots, y_m \), then the approximate states \( x_1, x_2, \ldots, x_{n+h} \) are available (If \( m-h \), the last state can be obtained from its definition: \( x_n+1 = y_m \)). Also assume \( f(z) \) does not change rapidly and can be handled by the observer. The ESO 204 order can be reduced to \( m \), where \( h < m < n \).

Thus, when \( m > h \) the nonzero elements of \( B_z \) and \( V \) are in the \((m-h)^\text{th}\) and \((m-h+1)^\text{th}\) columns, respectively.

For tuning simplicity, the observer gains \( l_1, \ldots, l_m \) are chosen as

\[ l_i = \begin{bmatrix} m+1 \\ i \end{bmatrix} \]

which denotes the Binomial Coefficient

\[ \binom{m}{i} = \frac{m!}{i!(m-i)!} \]

Also, to make the transfer function derivation easier, define

\[ l_0 = \begin{bmatrix} m+1 \\ 0 \end{bmatrix} \]

With the state \( z_{m-h+1} \) in the ESO 204 to estimate \( f \), a control law, \( u = - (l_0 + z_{m-h+1}) \), is applied to reduce the plant to a pure...
n\textsuperscript{th}-order cascade integral plant $y^{(n)} = u_0\tau$, which can be easily controlled using the parameterized linear PD control law 206 of the form

$$u_0 = \sum_{i=1}^{n-1} k_i y^{(i-1)} - \sum_{i=1}^{n-h} k_i y^{(i-1)} - \sum_{i=1}^{n-h} k_{i-1} h \cdot \tau$$

Equation (72)

where

$$k_i = \left( \frac{m}{i-1} \right) y^{(i-1)}$$

0\$\leq i \leq m+1$, and the middle term,

$$\sum_{i=1}^{n-h} k_i y^{(i-1)}$$

is corresponding to the order reduction (it disappears when the inputs of ESO 204 are $y_m$ and $u$). In the case where the measured output $y_m$ is clean and its derivatives can be obtained from $y_m$, ADRC 200 can also be applied as

$$u_0 = \sum_{i=1}^{n-1} k_i y^{(i-1)} - \sum_{i=1}^{n-h} k_i y^{(i-1)} - \sum_{i=1}^{n-h} k_{i-1} h \cdot \tau$$

Equation (73)

The corresponding transfer function representation of ADRC 400, in a 2 dof structure (as shown in FIG. 4) is

$$H(s) = \frac{\psi_{m-1}}{\psi_{m}}$$

Equation (74)

$$\psi_{m} = \sum_{i=1}^{n-1} k_i y^{(i-1)} + \sum_{i=1}^{n-h} k_i y^{(i-1)} - \sum_{i=1}^{n-h} k_{i-1} h \cdot \tau$$

Equation (75)

$$G_s(s) = \frac{1}{b} \sum_{i=1}^{n-1} k_i y^{(i-1)} + \sum_{i=1}^{n-h} k_i y^{(i-1)} - \sum_{i=1}^{n-h} k_{i-1} h \cdot \tau$$

where

$$\psi_{m-1} = \sum_{i=1}^{n} i \cdot \tau$$

0\$\leq x \leq y$, ADRC with Partially Known Plant Dynamics

ADRC controller does not require the detail plant model information. However, if the whole or part of the plant dynamics is known, various embodiments of the ADRC controller can be modified to utilize the information to estimate and reject disturbance more effectively, either by control law modification or ESO 204 modification. With a part of the plant dynamics known, the known part of the generalized disturbance can be generated and rejected in the control law directly, while the rest of the generalized disturbance is estimated by ESO 204 and then rejected in the linear PD control law 206 as previously shown in the original ADRC 200 framework. The other way to utilize the partially known plant dynamics is adapting the dynamics into ESO 204, where ESO 204 still estimates the generalized disturbance.

Consider the second-order plant 202 in Equation (8), with the knowledge of the plant information, part of the generalized disturbance can be generated and rejected in the control law directly, while the rest of the generalized disturbance is estimated by ESO 204 and then rejected in the linear PD control law 206 as previously shown in the original ADRC 200 framework. The other way to utilize the partially known plant dynamics is adapting the dynamics into ESO 204, where ESO 204 still estimates the generalized disturbance.

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where \( g(e, \dot{e}, u_k) \) is the parameterized PD controller or NPD controller defined in Equation (15) or Equation (68) respectively. Of course, any combination of the above three methods are available for use by one of ordinary skill in the art.

INDUSTRIAL APPLICATIONS

The present active disturbance rejection controller, and its associated processes, methods and devices as disclosed herein possesses a number of unique attributes and industrial applicability, including for example, utility as a controller for tracking control, web processing applications, and jet engine control applications as described in greater detail above.

For example, in the case of tracking or motion control applications, the various predictive methods presented herein, particularly the ones that use the knowledge of the plant dynamics, offer reduced phase lag in the observer and compensate for the time delay in the plant itself, all of which especially benefit tracking applications that employ a desired output trajectory, sometimes called motion profile.

CONCLUSION

While various embodiments of the present system and method for feedback control of systems have been described above, it should be understood that the embodiments have been presented by the way of example only, and not limitation. It will be understood by those skilled in the art that various changes in form and details may be made therein without departing from the spirit and scope of the invention as defined. Thus, the breadth and scope of the present invention should not be limited by any of the above described exemplary embodiments.

What is claimed is:

1. A controller, comprising:
   a predictive state and disturbance observer module configured to receive a sensor signal and a disturbance adjusted control signal, and to output extended state estimate data representing a state of a plant, an extended state of a plant system dynamic and an external disturbance of the plant, and a control module configured to output a control signal based on the extended state estimate data, the predicted extended state estimate, a trajectory and a trajectory prediction; and
   wherein the extended state of the plant system dynamic is adapted to cause an input-output characteristic of the output and an associated input of the plant to behave as a double-integral plant with a scaling factor.

2. The controller of claim 1, wherein the predictive state and disturbance observer module comprises a system output predictor and an extended state observer.

3. The controller of claim 2, wherein the control module comprises an observer model of a dynamic of the plant.

4. The controller of claim 2, wherein the control module comprises a non-linear control law given by an equation:

   \[
   y = \omega_2 e + \omega_3 e,
   \]

   \[
   x = \begin{cases} 
   e_0 \gamma + \frac{R(R + 8|y| - R)}{2} \text{sign}(y), |y| > R \\
   e_0 (e_0 \gamma + 2e),
   \end{cases} \quad |y| \leq R
   \]

   \[
   \delta_{asp}(e, \dot{e}, u_k) = \begin{cases} 
   R \text{sign}(x), |x| > R \\
   x, \quad |x| \leq R
   \end{cases}
   \]

   \[
   \text{sign}(x) = \begin{cases} 
   1, x \geq 0 \\
   -1, x < 0
   \end{cases}
   \]

   and where \( \omega_2 \) is a frequency of the controller, \( e \) is an error, \( R \) is a maximum control signal, and \( \delta_{asp} \) is the control signal.

5. The controller of claim 2, wherein the sensor signal represents at least a first derivative of the output of the plant, and wherein the extended state observer is a reduced order extended state observer.

6. The controller of claim 1, wherein the predictive state and disturbance observer module comprises a predictive extended state observer.

7. The controller of claim 1, wherein the predictive state and disturbance observer module comprises an observer model of a dynamic of the plant.

8. The controller of claim 1, wherein the control module further comprises an additive inverse model of a function \((f)\) that comprises an estimate of a dynamic of the plant and the external disturbance.

9. A computer-implemented method for controlling a plant, comprising:
   estimating an extended state for the plant using a sensed output of the plant, wherein the extended state comprises a state of the plant and a total disturbance of the plant, and wherein the extended state is adapted to cause an input-output characteristic of the plant to behave as a double-integral plant with a scaling factor;
   predicting a change in the extended state to generate a state prediction and an extended state prediction;
   specifying a trajectory for an output of the plant to follow, wherein the trajectory comprises a desired trajectory and a prediction of a future desired trajectory;
   applying a control law to the desired trajectory and the prediction of the future desired trajectory, the extended state, and the extended state; and
   generating a control output based on application of the control law.

* * *