Short Abstract

An Upper Bound on Orbital Debris Collision Probability
When Only One Object has Position Uncertainty Information

Upper bounds on high speed satellite collision probability, P_c, have been investigated. Previous methods assume an individual position error covariance matrix is available for each object. The two matrices being combined into a single, relative position error covariance matrix. Components of the combined error covariance are then varied to obtain a maximum P_c. If error covariance information for only one of the two objects was available, either some default shape has been used or nothing could be done. An alternative is presented that uses the known covariance information along with a critical value of the missing covariance to obtain an approximate but useful P_c upper bound.

Long Abstract

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When Only One Object has Position Uncertainty Information

Joseph H. Frisbee, Jr.

There are various avenues along which an upper bound on the high speed satellite collision probability has been pursued. Typically, for the collision plane representation of the high speed collision probability problem, the predicted miss position in the collision plane is assumed fixed. Then the shape (aspect ratio of ellipse), the size (scaling of standard deviations) or the orientation (rotation of ellipse principal axes) of the combined position error ellipse is varied to obtain a maximum P_c. Regardless as to the exact details of the approach, previously presented methods all assume that an individual position error covariance matrix is available for each object and the two are combined into a single, relative position error covariance matrix. This combined position error covariance matrix is then modified according to the chosen scheme to arrive at a maximum P_c. But what if error covariance information for one of the two objects is not available?

When error covariance information for one of the objects is not available the analyst has commonly defaulted to the situation in which only the relative miss position and velocity are known without any corresponding state error covariance information. The various usual methods of finding a maximum P_c do no good because the analyst defaults to no knowledge of the combined, relative position error covariance matrix. It is reasonable to think, given an assumption of no covariance information, an analyst might still attempt to determine the error covariance matrix that results in an upper bound on the P_c. Without some guidance on limits to the shape, size and orientation of the unknown covariance matrix, the limiting case is a degenerate ellipse lying along the relative miss vector in the collision plane. Unless the miss position is exceptionally large or the at-risk object is exceptionally small, this method results in a maximum P_c too large to be of practical use. For example, assuming that the miss distance is equal to the current ISS alert volume along-track (+ or -) distance of 25 kilometers and that the at-risk area has a 70 meter radius.
The maximum (degenerate ellipse) $P_c$ is about 0.00136. At 40 kilometers, the maximum $P_c$ would be 0.00085 which is still almost an order of magnitude larger than the ISS maneuver threshold of 0.0001. In fact, a miss distance of almost 340 kilometers is necessary to reduce the maximum $P_c$ associated with this degenerate ellipse to the ISS maneuver threshold value. Such a result is frequently of no practical value to the analyst. Some improvement may be made with respect to this problem by realizing that while the position error covariance matrix of one of the objects (usually the debris object) may not be known the position error covariance matrix of the other object (usually the asset) is almost always available. Making use of the position error covariance information for the one object provides an improvement in finding a maximum $P_c$ which, in some cases, may offer real utility. The equations to be used are presented and their use discussed.